Stochastic resonance in Ising systems

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We study by Monte Carlo techniques the evolution of finite two-dimensional Ising systems in oscillating magnetic fields. The phenomenon of stochastic resonance is observed. The characteristic peak obtained for the correlation function between the external field and magnetization, versus the temperature of the system, is studied for various external fields and lattice sizes.

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I. INTRODUCTION

A periodically modulated bistable system in the presence of noise exhibits the phenomenon of stochastic resonance (SR) [1–3]. Plotting the correlation σ between the modulation signal and the response of the system versus the noise intensity, we usually obtain a strong peak. This characteristic picture is described in the literature as the phenomenon of SR. A brief review of the theoretical aspects is given in [4]. Experimental evidence for the SR phenomenon was found in analog simulations with proper electronic circuits [3,5,6], laser systems operating in multistable conditions [7], electron paramagnetic resonance [8,9], and in a free standing magnetoelastic ribbon [10].

In the present paper we report on the possibility of obtaining SR in a bidimensional Ising system. The phenomenon of SR has already been studied in globally coupled two-state systems [11,12]. Due to the fact that the spins in the Ising model can be considered as coupled bistable elements, the proposed system is a special case of the problem treated in [11]. In contrast with all earlier works, we do not consider any external stochastic forces, just the thermal fluctuations in the system.

II. THE METHOD

Considering an Ising system on a finite square lattice at zero thermodynamic temperature, the free-energy curve versus the magnetization will have a double well form [13]. An external oscillating magnetic field will modulate the two minima in antiphase. The effect of a positive temperature in this system can be considered as a stochastic driving force, and the magnetization as a function of the time $[m(t)=\sum_i S_i^z]$ as the response function of the system. In this way all the necessary conditions are satisfied for the SR phenomenon. Due to the fact that the noise intensity (thermal fluctuations) is temperature dependent, the characteristic peak must be observed for a resonance temperature T_r in the plot of the correlation (σ) versus the temperature (T).

It is worth mentioning that recently the Ising system in oscillating fields was considered by computer simulations, studying the hysteretic response of the system [14].

We studied the SR in the proposed system by a Monte Carlo method using the well-known Metropolis algorithm [15]. The Hamiltonian of the problem is

$$H=-J\sum_{i,j} S_i^z S_j^z+\mu_B B(t)\sum_i S_i^z \ , \tag{1}$$

where the sum is referring to all nearest neighbors, $S_i^z=\pm 1$, μ_B is the Bohr magneton, and $B(t)$ the external magnetic field. We will consider $B(t)$ in a harmonic form:

$$B(t)=A\sin\left[\frac{2\pi}{P}t\right] . \tag{2}$$

The time scale was chosen in a convenient form, setting the unit-time interval equal to the average characteristic time (τ) necessary for the flip of a spin. We have taken this time interval τ as a constant, and thus independent of the temperature. Although this assumption is just a working hypothesis, we expect useful qualitative results.

Our Monte Carlo (MC) simulations were performed on square lattices with $N\times N$ spins, considering the value of N up to 200. One MC step is defined as $N\times N$ trials of changing spin orientations and corresponds to a time interval τ. (The period P of the oscillating magnetic field is also given in these τ units.) The amplitude A of the magnetic field is considered already multiplied by μ_B/k, and thus it has the dimensionality of the temperature (k is the Boltzmann constant). The temperature is given in arbitrary units. The critical temperature of the infinite system,

$$T_c=2.2692\ldots\times\frac{J}{k} \ , \tag{3}$$

is always considered as 100 units. Starting the system from a completely random configuration, to approach the dynamic thermodynamic equilibrium we considered 5000 MC steps. The correlation function between the driving field and the magnetic response of the system,

$$\sigma=\langle B(t)m(t)\rangle=\frac{1}{n}\sum_{i=1}^{n}B(t_i)m(t_i) \ , \tag{4}$$

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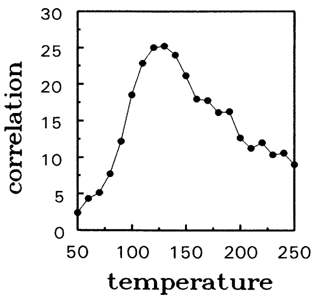

FIG. 1. Characteristic peak for the SR phenomenon in the plot of the correlation $[\sigma=\langle B(t)m(t)\rangle]$ versus the system temperature ($T_c=100$, $A=10$, $P=50$, and $N=5$).

was studied during $n=5000$ extra iterations. (The averaging in the expression for σ is as a function of time, and $t_i=\tau i$.) The correlation (4) was studied as a function of

(i) the temperature (T),
(ii) the lattice size (N),
(iii) the amplitude of the magnetic field (A), and
(iv) the period of the magnetic field (P).

III. RESULTS

Our results are summarized in Figs. 1–4. As we expected, for a fixed lattice size and a given oscillating magnetic field the curve σ versus T exhibits the characteristic peak of SR. Considering $A=10$, $P=50$, and $N=5$ a generic result is plotted in Fig. 1. From Fig. 2 we conclude that the location of the peak is not significantly influenced by the amplitude of the magnetic field. As expected, the amplitude influenced only the shape of the peak, and its height increases strongly with the amplitude. (Our results would suggest an exponential type variation.) For small lattices ($N<10$) this resonance temperature (T_r^N) exhibits a strong dependence on the lattice size, and converges to a T_r limiting value for big lattices ($N>100$):

$$T_r=\lim_{N\to\infty} T_r^N . \tag{5}$$

In Fig. 3 we present the results for three choices of P. One can immediately observe that the resonance temperature T_r is dependent on the period of the magnetic field. As we illustrated in Fig. 4 this dependence can be approximated with the curve

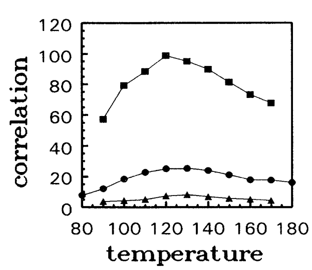

FIG. 2. Shape of the peak for three different values of the magnetic field amplitude ($A=5$ ▲, 10 ●, and 20 ■) ($T_c=100$, $P=50$, and $N=5$).

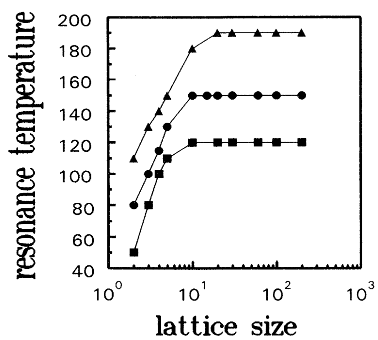

FIG. 3. Dependence of the resonance temperature versus the lattice linear size for three different values of the period (20 ▲, 50 ●, and 200 ■) ($T_c=100$ and $A=10$).

$$T_r=T_c+557.264P^{-0.6335} . \tag{6}$$

In the limit of high frequencies the resonance temperature is tending to infinity and in the limit of small ones T_r is tending to T_c. Due to the fact that the real experimental conditions work in the very low frequency limit (the periods in Fig. 4 are given in units of τ), one would expect this phenomenon to be detected at T_c.

IV. CONCLUSIONS

In conclusion, in this work we studied by MC techniques the phenomenon of SR in finite Ising systems considered in a periodic magnetic field. The thermal fluctuations were considered as a stochastic force, and the resonance was detected for a resonance temperature T_r. The dependence of T_r on the characteristics of the magnetic field and the lattice size was investigated.

Due to the sensitivity of T_r to the size of the system (Fig. 3 for small N) an interesting subject would be the study of this effect on materials containing small magnetic domains. From the computational point of view, a much more detailed study of the phenomenon, and the consideration of a three-dimensional case would also be of interest.

The theory of SR has previously been developed in the context of classical statistical physics using linear response theory and the fluctuation-dissipation relations [16–18]. In this sense, by calculating the susceptibility of

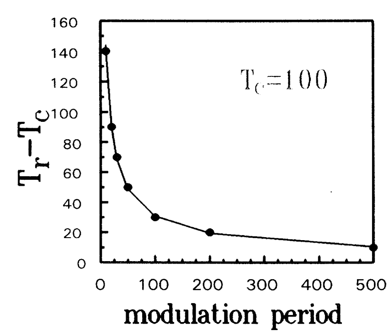

FIG. 4. Variation of the resonance temperature (T_r-T_c) for big lattices (200×200) against the period of the magnetic field. The best-fit curve indicates $T_r-T_c=557.264P^{-0.6335}$.

our model in the absence of the periodic magnetic field and studying the response of the system with the linear response theory one could perform analytical studies of the problem. We consider that such studies would be important both for statistical physics and for the phenomenon of SR.

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