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Spring-block type models for crack propagation in glass plates♣

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Crack propagation in thin glass plates under high shock loading is investigated by computer simulations. The widely used spring-block type model is adapted in order to reproduce the main features of the glass breaking phenomenon. Blocks represent mesoscopic elements of the glass while the coupling between them is modeled by elastic springs with well defined breaking threshold values. The amorphous structure of glass is captured by randomly distributed blocks and spatially randomly distributed friction forces. The localized external stress is applied by increasing the spring constants in a central region of the studied system. Therefore, radial crack lines will nucleate and propagate through the system. The simulations reproduce qualitatively well the experimentally obtained radial crack lines and the dynamics induced by the shock is also revealed.

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1. Introduction

Cracking and fragmentation of materials is a common phenomena being present from astrological length-scales down to nano and atomic scales. This is the reason why the phenomenon is still subject of large scientific and industrial interest. The glass breaking phenomena we are going to investigate is in the middle of this scale. Crack propagation in glasses, inelastic deformation and failure mechanisms produced by external shock loading has attracted extensive research in the last decades [1-4]. In the early sixties a scaling law was obtained for the size-distribution of the resultant pieces [5]. From that point on the experimental research focused on many other interesting aspects. In the last years the cracking process was investigated also by high-speed camera snapshot series [6]. At the same time, many theoretical models have been created in order to understand the breaking mechanisms of glass rods and plates, focusing mainly on the case of parallel shock loading [7].

Glass breaking produced by perpendicular projectile impact was experimentally investigated in the framework of a student research project by the group of Y. Bréchet [8]. As a results of these experiments the group qualitatively revealed and classified the breaking patterns. As presented in Figure 1. the most common structure has two concentric circles and many radial cracklines initiating from the impact point.

In the present work crack propagation in thin glass plates under localized perpendicular shock loading is investigated using computer simulations. Based on the classical Burridge-Knopoff type spring-block model a very simple model is constructed, which reproduces qualitatively well the experimentally obtained radial crack lines. The objective of the present work is to investigate the applicability of the model, to find the relevant parameter values and to reveal the crack line formation and propagation dynamics.

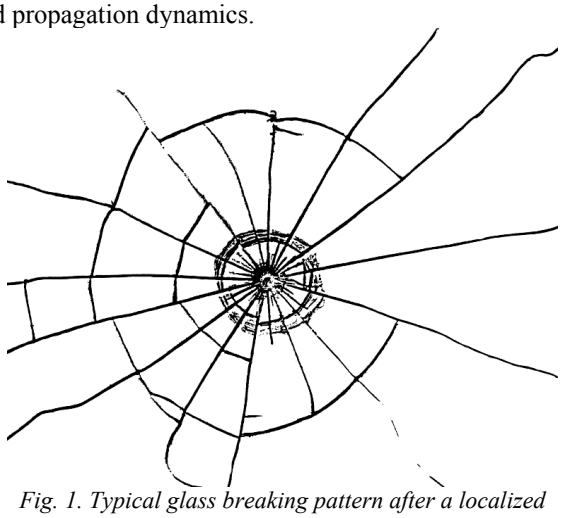

Fig. 1. Typical glass breaking pattern after a localized perpendicular shock loading [8].

2. Theoretical model

The model incorporates the main features of glasses: the amorphous structure, the elastic response to small stresses (local reorganization) and the plasticity in case of low stresses. In order to match these requirements within the framework of the spring-block some changes relative to the original Burridge-Knopoff type model is incorporated [9].

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The model built in such manner is rather similar with the spring-block stick-slip model which has been successfully used for describing fragmentation structures obtained in drying granular materials in contact with a frictional substrate [10] or drying of nanosphere suspensions [11]. The model is two dimensional; its main elements are blocks which can move hindered by friction and springs connecting them (Fig. 2). Disk shaped blocks, all with the same radius r_0, model mesoscopic elements of the glass while the coupling between them is modeled by elastic springs.

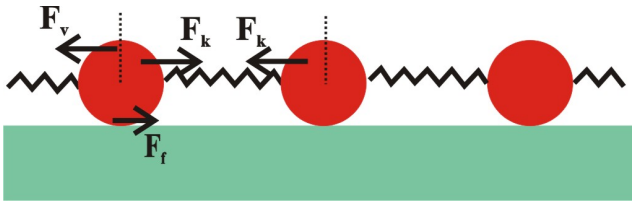

Fig. 2. Basic elements of the spring-block model.

In the model considered by us all springs have similar spring constants k, and their length is defined as the distance between the centers of the connected blocks. Fig. 3a shows the spring force which is proportional to the length of the spring ($F_k = kl$), and it has a well defined F_{k_max} breaking threshold value. In the spring force it is also included a hard-core type repulsion which forbids blocks to interpenetrate each other. This repulsion is described by the repulsive part of a Lenard-Jones-type potential. The friction (pinning) equilibrates a net force less than F_{f_max} (Fig. 3b). Whenever the total force (F_t) acting on a block exceeds F_{f_max} , the block will slip with an overdamped motion. It has to be mentioned that in order to model the amorphous structure and to maintain the disorder in the system, the F_{f_max} pinning forces are randomly assigned in the simulation space.

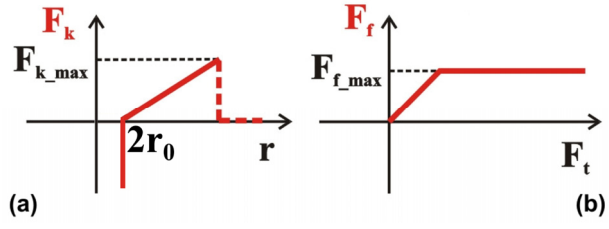

Fig. 3. Spring (a) and pinning (b) forces acting on blocks

Initially blocks are randomly distributed and connected by a network of springs (Fig. 4). By this, the initial disorder of the system is modeled. We put springs between those spheres, for which the centers can be connected without intersecting another sphere (this condition will be referred later as the geometric condition). In this way, an initially pre-stressed spring-block network is thus constructed.

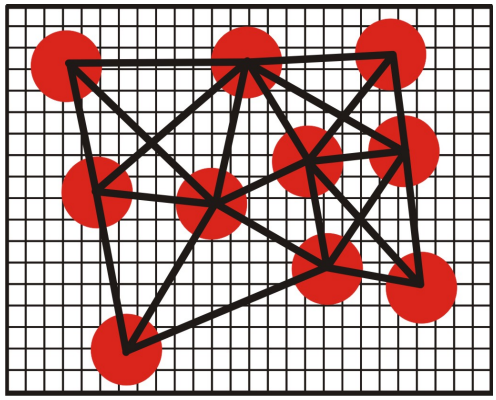

Fig. 4. The initially constructed spring-block network.

The simulation consists of two successive simulation steps. In the first one, the initial pre-stressed system is generated and relaxed to a stable configuration where the tension in each existing spring is lower than the breaking threshold F_{k_max} and the total net force acting on each block is lower in magnitude than the slipping threshold F_{f_max}. Due to the random friction this equilibrium state will include internal stresses "frozen" into the system. Then, in the second step an external stress is applied by increasing the spring constants according to the shape of the external shock loading. In order to take into account also the shock-wave propagation through the glass plate a two dimensional standing wave is used. The source of this is placed in the impact point.

The system is relaxed and during this relaxation process crack lines are nucleating and propagating in the system.

The relaxation process is realized through several relaxation steps. The time length dt for each relaxation step is taken as unity ($dt = 1$). Considering a classical molecular dynamics simulation for relaxation would be very time-consuming. Following the method used for simulating drying processes in granular media [10,11], we choose thus a simplified, overdamped dynamics. Each relaxation step may be described as follows:

(1) *Reorganization.* At the beginning of each relaxation step the spring system is reconstructed fulfilling the geometric condition. By this effect the local reorganization capability of "glass particles" (elasticity) is modeled.

(2) *Breaking of spins.* For all springs the tension is compared with the breaking threshold. If $F_k^{ij} > F_{k\_\max}$ (i and j denoting the blocks connected by the spring), then the spring is considered to be broken and it will be taken away from the system.

(3) *Recalculation of Forces.* Total forces acting on blocks are calculated: $\vec{F}^i{}_t = \sum_p d_{ip}\vec{F}_k^{ip}$, where the sum is

over all the other blocks p, d_{ip} is *1* if the blocks are connected by spring and *0* otherwise.

(4) *Motion of the blocks.* Each block is analyzed. If the magnitude of the total force $\left|\vec{F}_t^i\right|$ acting on a block is bigger than the F_{f_max} threshold, then the block will slip with an over-damped motion governed by viscosity η, and its position will be changed by: $d\vec{r}^{\,i} = \vec{F}_t^i dt/\eta$. The repulsive part of the spring forces forbids the blocks to slide on each other and the presence of viscosity eliminates unrealistic oscillations.

The (1)-(4) relaxation procedure is repeated until a relaxation step is finished without having any spring breaking or disk slipping event. Since within this algorithm it takes a long time to achieve perfect relaxation, a tolerance is introduced, and it is assumed that the relaxation is completed when the maximal slip in the system is smaller than this tolerance value.

Several types of boundary conditions can be considered. One possibility would be to use free boundary condition which can be realized in a simple manner by positioning initially the blocks inside a circle to minimize the effect of edges.

Another possibility is to consider fixed boundary conditions. This can be realized by positioning again the blocks inside a circle and considering a chain of fixed blocks on the chosen perimeter.

These fixed blocks are then connected between their neighbors with geometrically allowed springs. One can also consider periodic boundary condition and position initially the blocks inside a rectangle.

This latter condition is not applicable, because it is unrealistic to have crack lines leaving the system in one side and entering on the other side and thus self-interacting. In case of free boundaries, right in the initial simulation step the system is compacting and therefore a higher density external region is formed. Consequently, in the simulations fixed boundary conditions are used which may correspond to a glass plate fixed at its boundaries.

The model, as described above, has several parameters:

(1) The initial space filling of blocks $\rho = S/\left(N\pi R^2\right)$ (where S is the simulation area). While a continuous media is simulated, one has to deal with very high, almost close-packing space filling values.

(2) The radius of blocks r_0, considered as unity (r_0 = *1*). It defines the unit length in system.

(3) The value of the spring-breaking threshold F_{k_max}. It is set as unity (F_{k_max} = 1) defining the unit force in the system.

(4) The initial value of the spring constants, k. It is desirable to choose the value of k so that only a small number of broken springs to be found in the initially relaxed system.

(5) The parameter which governs the repulsive part of the spring force. It is set to get no repulsion at the distance $2R$ and strong hard-core type repulsion for smaller distances.

(6) The viscosity The model will only work for viscosity values chosen between reasonable limits, and for these viscosity values the final patterns are rather similar. Choosing a too small viscosity will result in unrealistic oscillations of blocks, while a too high value will make the block slip too small and increase considerably the relaxation time.

(7) The shock propagation velocity v. This is set at the same magnitude as the crack line propagation velocity.

(8) The amplitude k_0 of the shock wave at the impact point.

(9) The range (F_{f1}, F_{f2}) from which the pinning force values are randomly selected.

As one can observe only the last two parameters of the model are not adjusted by some conceptual considerations. We remain thus with two main parameters governing the generated patterns: the magnitude of the external shock loading and the disorder in the studied material.

The influence of these parameters on the final structure will be investigated by large scale computer simulations.

3. Results

The above described model can be easily implemented on computer and relatively big systems with *10000-100000* of blocks can be simulated in reasonable computational time.

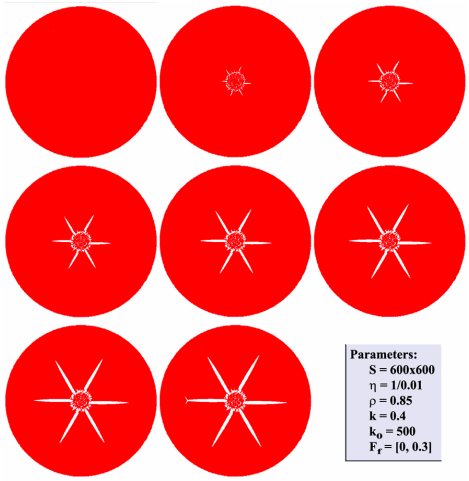

Fig. 5. Characteristic time evolution of crack propagation in glass plates due to a central impact. The structures are obtained with the parameter values specified on the figure.

First, let us take a look on the time evolution of the model. A characteristic time-sequence for the crack propagation dynamics is plotted in Figure 5. The simulation field has a diameter of D = *600* block radius. The snapshots are taken at every 4000 time steps starting from 0 to 28000.

The simulation parameters are given on the figure. As observable from the figure and in agreement with experiments, first one circle is formed around the external

shock. Then, radial crack lines are nucleating and propagating in the outside directions. It was observed that all radial crack lines are nucleated at the same time and their propagation velocities are almost identical.

Figure 6 represents the average crack-propagation velocity as a function of the simulation time. The units of the figure are derived from the arbitrary units used in the simulation ($r_0 = 1$ and $dt = 1$). Based on simulation results and in agreement with theoretical considerations it can be concluded that the radial crack-propagation velocity is a decreasing in time.

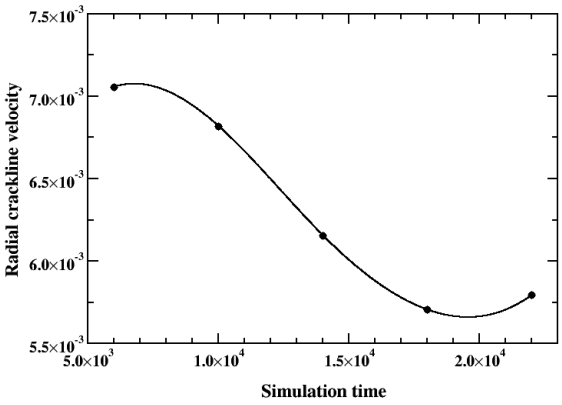

Fig. 6. Averaged radial crack line propagation velocities as a function of simulation time for the simulation presented in Fig. 5.

In order to analyze the dependence of the obtained patterns as a function of the strength of the impact, different simulations have been considered with the same parameters but for different k_0 values. Figure 7 shows two of the resulting patterns. The main difference between these snapshots consists in the radius of the central circle and the number of the radial crack lines. Namely, in case of higher shock loadings many radial crack lines have been nucleated. For these simulations the radial crack line propagation velocities have also been measured. The same value of 0.06 was obtained for both simulations. It means that the propagation velocity is almost independent on the strength of the external shock.

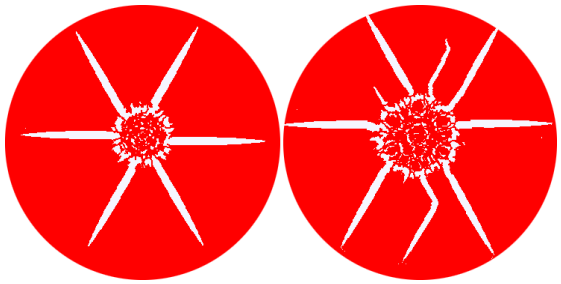

Fig. 7. Simulation results for different impact strength. On the figure in left side the k_0=500 while in the figure on the right side k_0=1000. The rest of the parameters have the same values as inFigure 5.

The effect of the disorder modeled by random pinning forces has also been studied.

In Fig. 8 two simulated patterns are presented for different (F_{f1}, F_{f2}) ranges.

For these simulations it was found that the average crack line propagation velocity is decreasing in function of the disorder in the system. It is also important to be mentioned that in case of higher disorder the crack lines are more fragmented.

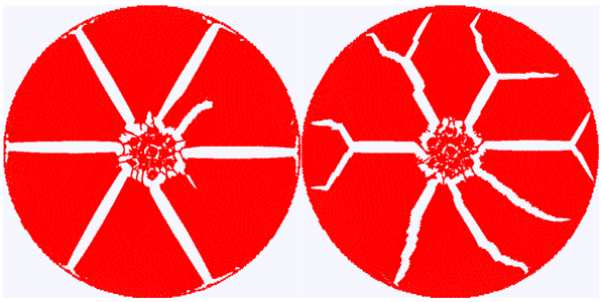

Fig. 8. Simulation results for different pinning force distributions. In the left panel the pinning force values are taken randomly and uniformly in the range [0, 0.2] while in the right panel the values are taken from [0, 0.5]. The other model parameters are $D = 400$, $\eta = 1/0.01$, $\rho = 0.85$, $k = 0.4$, $k_0 = 500$.

4. Conclusions

A mechanical spring-block type model with an over-damped relaxation dynamics was used for modeling crack propagation in thin glass plates under a central and localized shock loading.

The simulations reproduce qualitatively well the experimentally obtained radial crack lines. The relevant model parameters have been identified and their influence on the final pattern were investigated.

The introduced model is appropriate for large-scale computer simulations and for investigating theoretically the crack line formation statistics and the crack propagation dynamics in glass plates. As a continuation of our previous studies concerning the applicability of the simple spring-block type models in materials science [10,11], we have proven thus again that such simple approaches could yield valuable and important results.

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