

# The Rich Are Different !

*Pareto law from asymmetric interactions  
in asset exchange models*

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*Fitzgerald: "The rich are different from you and me"*

*Hemingway: "Yes, they have more money"*

## The Question

“The history of all hitherto existing society is a history of social hierarchy” (*Persky*)

**OK .... But Why ?**

## Outline of the talk

- **Background**
- **Empirical: Wealth & Income distribution in India**

SS (2005) cond-mat/0502166

- **Theoretical: Pareto Law from Kinetic Theory**

SS (2003) *Physica Scripta* T106, 59-64

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W Souma, cond-mat/0202388

Examples of Pareto index  $\alpha$  for some countries and years (Badger 1980).

Country	$\alpha$	Country	$\alpha$
England	(1843)	Perugia(city)	1.69
	(1879-80)	Perugia(country)	1.37
	(1893-94)	Ancona,Arezzo,	1.32
Prussia	(1852)	Parma,Pisa	
	(1876)	Italian cities	1.45
	(1881)	Basel	1.24
	(1886)	Paris(rents)	1.57
	(1890)	Florence	1.41
	(1894)	Peru(at the end of	1.79
Saxony	(1880)	18th century)	
	(1886)		1.51
Augsburg	(1471)		1.43
	(1498)		1.47
	(1512)		1.26
	(1526)		1.13

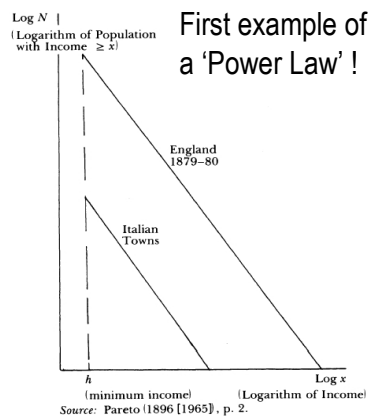
The fraction of population with income greater than  $x$ ,  $P(\geq x) \sim x^{-\alpha}$

$\alpha$  : Pareto exponent

Pareto:  $\alpha \sim 1.5$  for all societies!  
~~An universal exponent !!~~

## Pareto's Law

Pareto stated the law of income (and wealth) in actual observations of European nations



# Why is Pareto Law fascinating ?

“There is the feel of structure behind income distributions. Almost all income distributions are continuous, unimodal and highly skewed. We have no examples of uniform distributions or egalitarian distributions or strikingly trimodal distributions. Something is going on here.”

*J Persky (1992)*

## Measures of Inequality: Pareto exponent vs Gini coefficient

High resolution measure

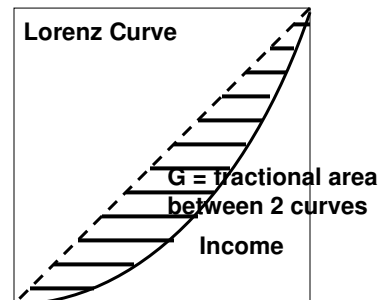
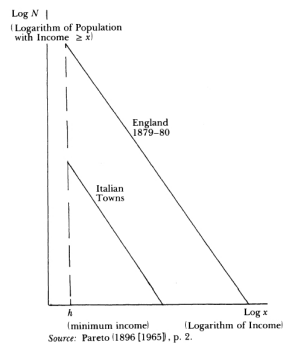
Low resolution measure

**Pareto exponent  $\alpha = 1.5$**



**Gini coefficient  $G = 0.5$**

True for most developed European nations (Yakovenko)



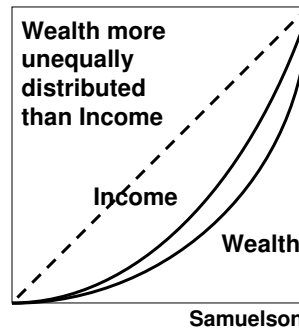
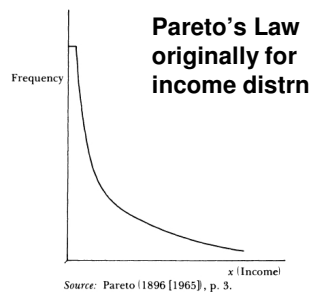
**The significance of Pareto insisting that  $\alpha = 1.5$**

This relation breaks down if the distribution is not a power law throughout

# Income vs. Wealth Distribution

**Income:** Flow of wages, dividends, interest payments, etc. over a period of time

**Wealth:** the net value of assets owned at a given point of time (both financial holdings and tangible assets like house)



➔ Pareto exponent for Wealth  $\leq$  Pareto exponent for Income

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# Rank distribution & Pareto exponent

## Pareto's Law, Power law and Zipf's Law

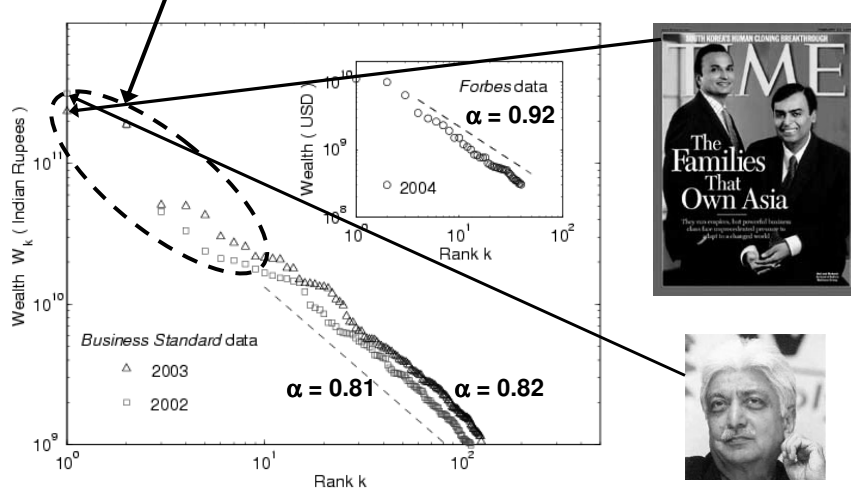
L Adamic, *preprint*

- ❑ Pareto's Law: cumulative probability distribution (exponent:  $\alpha$ )
- ❑ Power Law: probability distribution (exponent:  $1+\alpha$ )
- ❑ Zipf's Law: rank ordered distribution (exponent:  $1/\alpha$ )

To focus on a power-law tail in a small data-set, more efficient to obtain rank distribution exponent

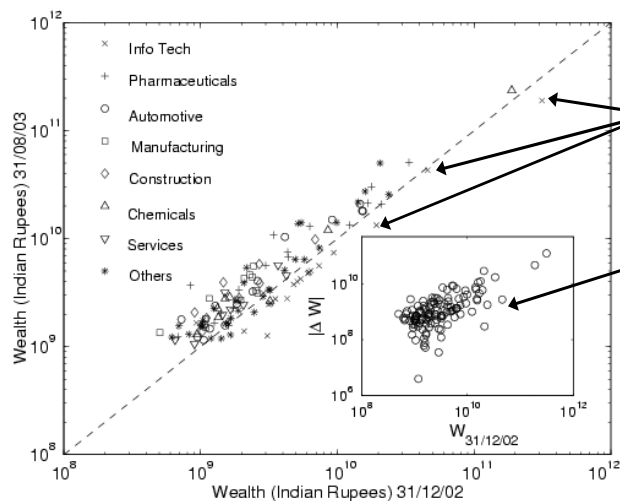
## Power-law tail of Indian wealth distribution

Data points corresponding to super-rich noisiest owing to severe under-representation



# Change in Wealth

(?) due to change in value of stocks on which the wealth is founded



Wealth according to different sectors (BSE classification)

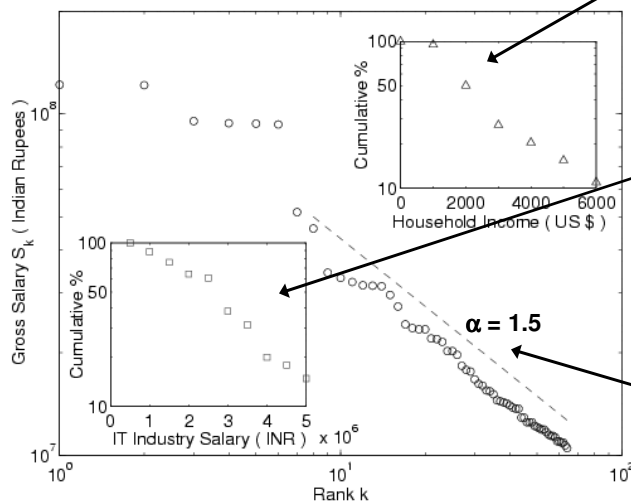
Info Tech stocks went down betn Dec 02 & Aug 03

Multiplicative stochastic models appropriate for modeling wealth evolution through asset exchange:

$$|\Delta W| \propto W$$

# Income distribution in India

IT data not publicly available



Cumulative income distribution of lower-income Indian households (1997 survey)  
*Non-monotonic!*

Cumulative salary distrn in Info Tech industry for 10 yrs experience (2002 survey)

Rank ordered salaries for highest paid company executives (2003)

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## How to explain Pareto's Law ?

**Asset exchange models: A collection of agents, each starting with some initial capital, trade with each other endlessly, unless they go bankrupt**

**The assets owned by each agent fluctuate over time – but the overall distribution among the agents can be described**

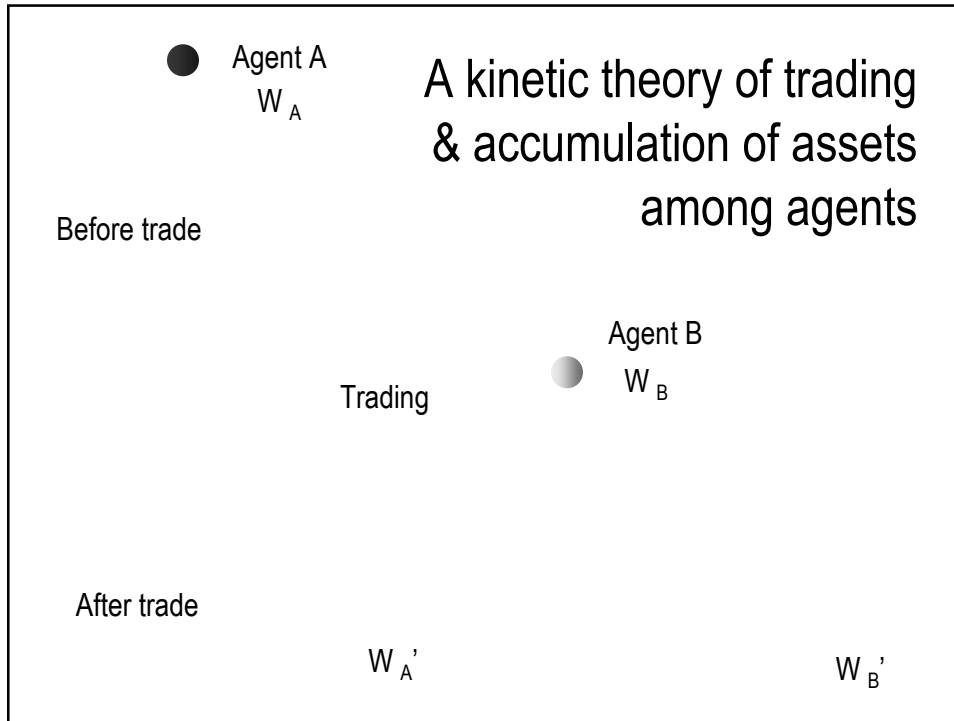
**Conservation of**

**total wealth**

**the sum of the wealth of the 2 interacting agents**

**Very similar to the kinetic theory of gas molecules!**



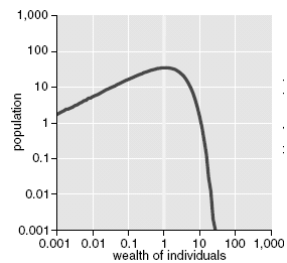
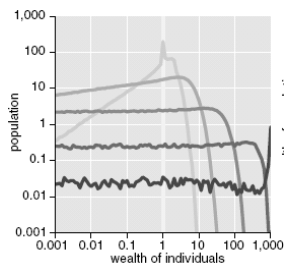


## Is perfect socialism possible ?

Suppose every agent starts off with the same initial capital...  
What happens after a few rounds of trading ?

**Random ~~Unrealistic~~ assumption**

A random fraction of the total wealth of  
the two players are exchanged as a result  
of a trade – Gibbs distribution !



**Minimum Wealth Exchange**

**A random fraction of the wealth of  
the poorer player is exchanged as a  
result of a trade**

**Condensation ! Winner takes all !!**

## But neither reproduces Pareto Law !

Models that do reproduce a Pareto Law tail are

*Chakrabarti*  
*Chatterjee*       $\alpha = 1$   
*Manna*

Randomly distributed savings propensity

*Manna et al*  
*DiMatteo, Aste et al*  
*Others*

Exchange in a network

### Claim

*The simplest method for generating Pareto Law tails*

Introduce asymmetric exchange interaction between agents

Asymmetry w.r.t. wealth

Capable of giving different values of  $\alpha$ , similar to observed values

## Clue: *The Rich Get Richer*

### *Matthew's Law*

For whosoever hath, to him shall be given,  
and he shall have more abundance:  
but whosoever hath not, from him shall be  
taken away even that he hath.

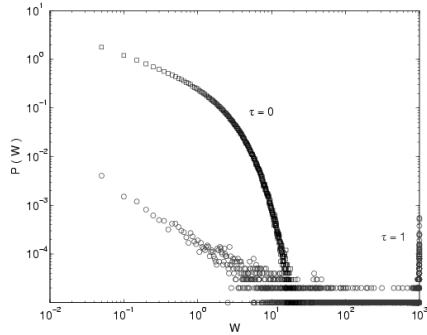
*Matthew 13:12*

Samuelson: "Savings is the greatest luxury of all"

Do the rich save more ? Savings rises rapidly as income increases

Savings fraction of agents trading with each other are functions of their relative wealth:  
poorer agent less likely to save compared to richer agent

# Asymmetric exchange model



$$W'_A = W_A + \alpha(1 - \tau[1 - \frac{W_A}{W_B}])W_B, \text{ if } W_A \leq W_B,$$

$$= W_A + \alpha W_B, \text{ otherwise}$$

$\tau = 0$ , random wealth exchange model  
 → Exponential distribution

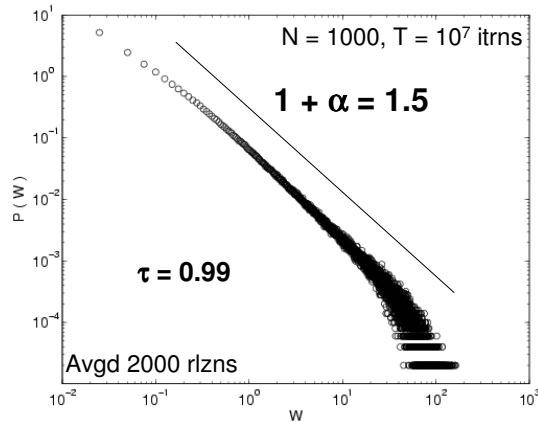
$\tau = 1$ , minimum wealth exchange model  
 → Condensation

A richer agent has more power to dictate terms of the trade than a poorer agent

By what degree an agent will use the power depends on his *thrift*  $\tau$  !

# Simplest model to give Power law

As  $\tau$  increases, exponential distribution → condensate via power law at  $\tau \rightarrow 1$

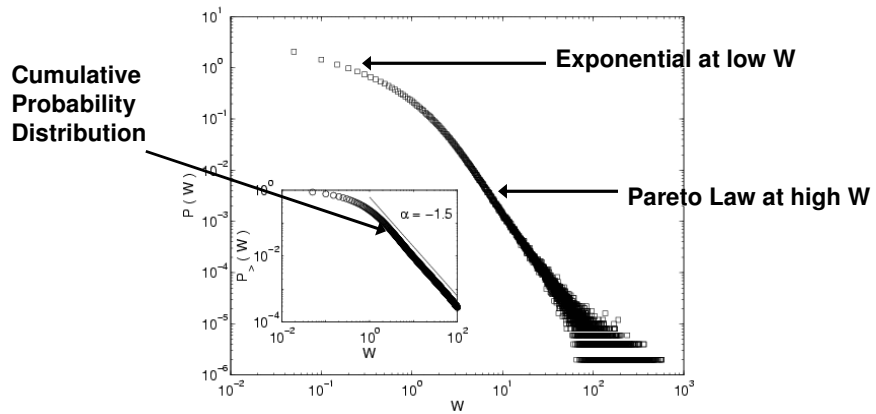


But Pareto exponent too low !

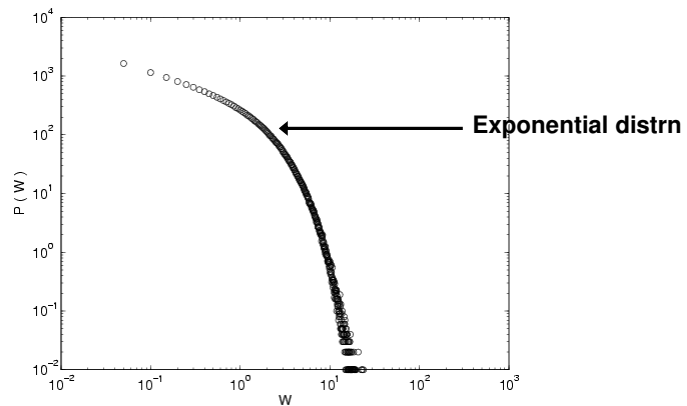
# Pareto Law reproduced

Uniform quenched random distribution of  $\tau$  among agents

➔ Pareto Law with exponent  $\alpha = 1.5$  ! (*Pareto's value*)



## Annealed random distribution of $\tau$ doesn't work !



## Obtaining other Pareto exponents

### Robust result!

Changing the nature of the distribution function for  $\tau$  *drastically* does not change the power law nature of the tail

What does change is the value of the Pareto exponent !

A mechanism to reproduce observed  $\alpha$  in various societies...

$P(\tau) = \text{uniform} \rightarrow \alpha \sim 1.5$

$P(\tau) \sim \tau \rightarrow \alpha \sim 1.3$

$P(\tau) = \tau^{-1/2} \rightarrow \alpha \sim 1.94$

$P(\tau) = \tau^{-2/3} \rightarrow \alpha \sim 2.1$

$P(\tau) = \text{U shaped} \rightarrow \alpha \sim 0.73$

Special Case (P M Gade):  $P(\tau) = \delta(0) + \delta(1) \rightarrow \alpha \sim 0.5$  (?)

## New model:

### Asymmetry in probability of winning

Asymmetry can also be introduced in the probability that an agent will gain net wealth from a trade !

Start with minimum wealth exchange model

Probability that agent  $i$  (wealth  $x_i$ ) will be gaining net wealth from a trade with agent  $j$  (wealth  $x_j$ ):

Poorer player has bargaining advantage 
$$p(i|i, j) = \frac{1}{1 + \exp\left(\beta \left[\frac{x_i}{x_j} - 1\right]\right)}$$

$1/\beta$  : indifference to relative wealth

“Fermi function”-al form

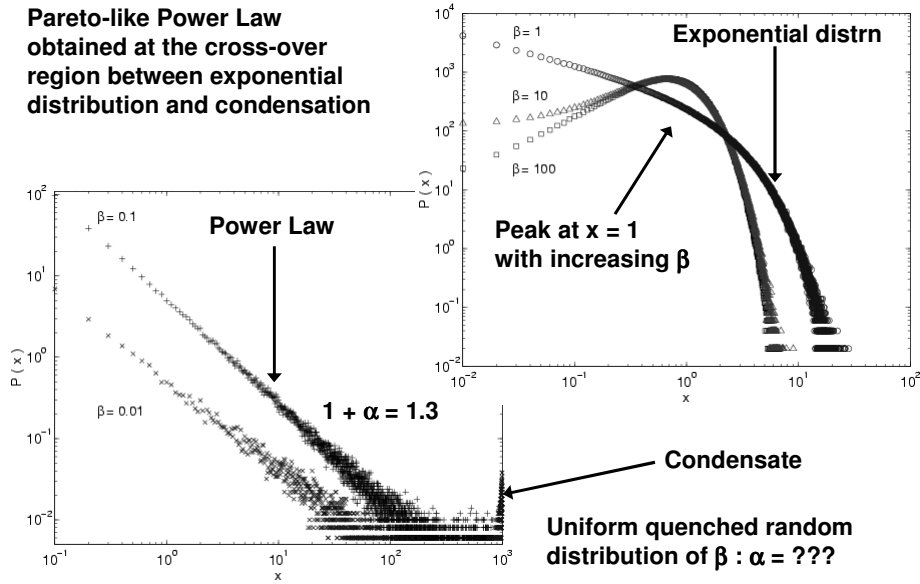
If all agents start off with equal wealth ( $x = 1$ )

$\beta = 0$  (*Uncle Scrooge*) : Condensation

$\beta \rightarrow \infty$  : Perfect Socialism

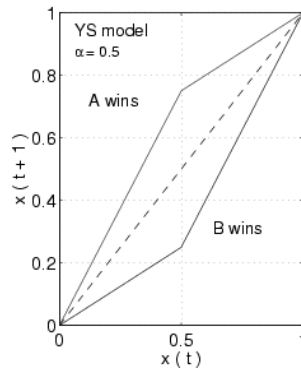
# Power Law once more !

Pareto-like Power Law  
obtained at the cross-over  
region between exponential  
distribution and condensation

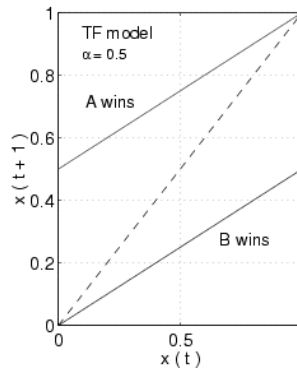


## 2-player asset exchange games: Analytical Results

minimum wealth exchange model



random wealth exchange model



$$x(t+1) = f[x(t)]$$

If  $f(0) = 0$ ,  $f(1) = 1$  and the product of the slopes of  $f$  at  $x = 0$  is less than 1,  
then irrespective of the form of  $f$ , condensation occurs !

## Conclusion

**It's easy to reproduce the observed income or wealth distribution in society through asset exchange models....**

**...as long as we remember that  
The Rich Are Different !**

**In other words, wealth-dependent asymmetry in agent-agent interaction generates power-law tails!**

**Simplest model for generating Pareto Law !!**