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A family-network model for wealth distribution in societies

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Abstract

A model based on first-degree family relations network is used to describe the wealth distribution in societies. The network structure is not a priori introduced in the model, it is generated in parallel with the wealth values through simple and realistic dynamical rules. The model has two main parameters, governing the wealth exchange in the network. Choosing their values realistically, leads to wealth distributions in good agreement with measured data. The cumulative wealth distribution function has an exponential behavior in the low and medium wealth limit, and shows the Pareto-like power-law tail for the upper 5% of the society. The obtained Pareto indexes are in good agreement with the measured ones. The generated family networks also converge to a statistically stable topology with a simple Poissonian degree distribution. On this family network many interesting correlations are studied, and the main factors leading to wealth diversification and the formation of the Pareto law are identified.

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1. Introduction

Since the seminal work by Vilfredo Pareto [1], it is known that the wealth distribution in capitalist economies shows a very peculiar and somehow universal functional form. In the range of low income, the cumulative distribution of wealth (the probability that the wealth of an individual is greater than a given value) may be fitted by an exponential or log-normal decreasing function, while in the region containing the richest part of the population, generally less than 5% of the individuals, this distribution is well characterized by a power-law (see for example Ref. [2] for a review). This empirical behavior has been confirmed by a number of recent studies on the economy of several corners of the world. The available data are coming from so far apart as Australia [3], Japan [4,5], the US [6], continental Europe [7,8] or the UK [9]. The data are also spanning so long in time as ancient Egypt [10], Renaissance Europe [11] or the 20th century Japan [12]. Most of these data are based on the declaration of income of the population, which is assumed to be proportional to the wealth. There are however some other databases obtained from different sources like for instance the area of the houses in ancient Egypt [10], the inheritance taxation or the capital transfer taxes [13]. The results mostly back Pareto's conjecture on the shape of the wealth distribution. The interesting problem that remains to be answered is the origin of the peculiar functional trend.

The answer to this question is a long-standing problem, which even motivated some of the initial Mandelbrot's and Simon's work 50 years ago. Let $P_>(w)$ be the probability of having a wealth higher than w. Pareto's law then establishes that the tail of $P_>(w)$ decays as

$$P_>(w) = \int_w^\infty P(w')\,\mathrm{d}w' \sim w^{-\alpha}\,,$$

where α is the so-called Pareto index and $P(w)$ the normalized wealth distribution function. Typically, the presence of power-law distributions is a hint for the complexity underlying a system. It is however important to notice that in spite of what happens with most exponents in statistical physics, α may change in time depending on the economical circumstances [5,12], making thus impossible the definition of some sort of universal scaling in this problem. This aspect is a key characteristic that any model on wealth distribution should be able to reproduce.

Economical models are essentially composed of a group of agents placed on a lattice that interchange money following pre-established rules. The system will eventually reach a stationary state where some quantities, as for instance the distribution $P_>(w)$, may be measured. Following these ideas, Bouchaud and Mézard [14] and Solomon and Richmond [15,16] separately proposed a very general model for wealth distribution. This model is based on a mean field type scenario with interactions among all the agents and on the existence of multiplicative fluctuations acting on each agent's wealth. Their results on the wealth distributions adjust well to the phenomenological $P_>(w)$. Roughly, the same conclusions were obtained by Scaffeta [17], who considered a nonlinear version of the model and from other regular lattice-based models as those in Refs. [18,19]. This kind of models defined on pre-established regular lattices is however unable, by construction, to account for the complexity of the interaction network observed in real economical systems.

In parallel to the previous efforts to characterize economical systems, the study of complex networks has experienced a burst of activity in the last few years (see Ref. [20] for a recent review). Social networks, in particular, are of paramount interest for economy since everyday economical transactions actually produce a network of this type. The topology of the economical network can indeed condition the output of any economical model running on it. Such effect has been documented for example in Refs. [12,21,22], in which the models described above were simulated on small-world or scale-free networks. One of the main characteristics of social networks is the positive correlation existing between the node degrees [23,24], i.e., the high-connected individuals commonly tend to connect with other well-connected people. The way of constructing this type of networks is precisely the main topic of a recent work by Boguña and coworkers [25]. In what follows, we are going to use a somewhat similar approach to grow our working network.

Boguña's method is based on the existence of hidden variables characterizing each agent state. In this work, and in the spirit of Ref. [26], we present a simple economic model where those hidden variables are identified with the wealth of each agent. This introduces a coupling between the dynamics of the network structure and the evolution of the wealth distribution. Each value of the external parameters thus determines not only the final wealth distribution but also the structure of the underlying interchange network.

This paper is organized as follows. In Section 2 we introduce our model, in which the agents are identified as families linked by first-degree family relationship. In Section 3 we present computational results on this model. For a wide range of the parameters of the model we study both the wealth distribution and the structure of the underlying network. In Section 4 we discuss our results from several viewpoints. In this Section the results are compared with real data on wealth distribution, the correlation between the wealth and connectivity of the agents is studied, and the dynamics leading to wealth diversification is investigated. Section 5 is then dedicated to conclusions.

2. The family-network model

In modeling the wealth distribution in societies we identify as main entities (agents) the families. In the framework of our model, the families are nodes in a complex network, and the links of this network are first-degree family relations. Beside its links, each node is characterized by its "age", $A(i)$, and wealth, $w(i)$. The age of a node is proportional to the simulation time-steps elapsed from its birth ($A(i) = t - t_b(i)$, where $t_b(i)$ denotes the time-step when node i was born), and the wealth is a positive real number that will change in time. We consider both the total wealth of the system, W_t, and the number of families (nodes), N, conserved. The structure of the network is not a priori fixed, and will also change during the evolution of the system. Initially, we start with nodes arranged on a regular hierarchical network (as sketched in Fig. 1) where the age of node i is simply $N + 1 - i$. In this manner, node 1 will be the oldest and node N the youngest one. It is worth mentioning here that the final statistics of wealth distribution and the final network topology are rather independent on how the initial network topology was chosen. We verified this by choosing several other qualitatively different initial network structures.

Initially, we also assign random wealth to each node according to a uniform distribution on the $(0, 1)$ interval. In this manner we constructed the start-up society with a simple network structure (family relationships) and randomly distributed wealth values. The time evolution of the system is then chosen to be as simple as possible, but capturing the realistic wealth exchange processes between families. For each simulation time step, the dynamics is as follows:

(1) The oldest node (let this be j) is taken away from the system. The wealth of this node is uniformly redistributed between its first neighbors (nodes that are linked to it), and all its links are deleted.
(2) Node j is reintroduced in the network with age $A(j) = 0$. It is linked to two randomly selected nodes (let these be k and l) that have wealth greater than a minimal value q. The wealth q is taken away from the wealth of the selected k and l nodes, and it is redistributed in a random and preferential manner in the society. The preferential redistribution is realized by splitting the $2q$ wealth into s parts

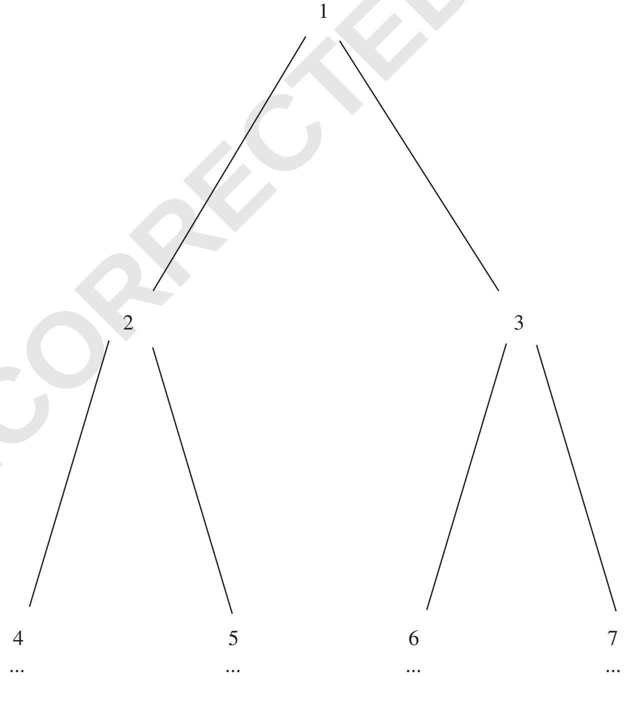

Fig. 1. Initial structure of the network.

and choosing the nodes which will benefit from these parts with a probability proportional to their actual wealth. This preferential redistribution will favor a rich-get-richer effect. After the redistribution of the $2q$ amount, a p part ($p<1$) of the remaining wealth of nodes k and l is given as start-up wealths for node j. After these wealth redistribution processes, the wealth of nodes k and l will thus be $w'(k)=[w(k)-q](1-p)$ and $w'[l]=[w(l)-q](1-p)$, respectively. Node j will start with $w(j)=p[w(k)+w(l)-2q]$ wealth.

(3) The age of all nodes is increased by unity.

Let us now explain the socio-economic phenomena that are modeled by the above dynamics. Step 1 models the inheriting process following the death of one family. The wealth of this family is redistributed among its first-degree relatives (children). Step 2 models the formation of a new family. In order to create a new family two other families have to raise one child. For raising a child a minimum amount of wealth is needed (q). This cost is paid to the society (for food, clothes, services, etc.), and the members of the society will benefit unevenly from it. Families with bigger wealth control more business, so they will naturally benefit more. The preferential redistribution of the $2q$ wealth models this uneven profit, and it is the main ingredient necessary to reproduce the Pareto distribution. Finally, when a new family is born, it is linked by first-degree relations to two existing families and gets a given part (p) of the parents wealth as start-up money. The time scale of the simulation is governed by the time needed to change all nodes, which we call one generation or one Monte Carlo step (MCS). By fixing N and W_t, and studying the thermodynamic limit $N\to\infty$, the model becomes essentially a two-parameter model (q and p), which is suitable for extended computer simulations.

Although very simple in nature, the chosen dynamics incorporates, we believe, the main socio-economic factors that influence the redistribution of wealth between families. As time passes, the families will be able to gather more and more wealth due to the $2q$ wealth redistribution process in the society. When their wealth becomes big enough they can create new families, and donate a part of this wealth to the new family. This process is costly and will therefore lower their wealth. Very poor nodes will not likely reach the q threshold and will not be able to create new families, becoming isolated nodes. There is no clear determinism however, since the redistribution in step 2 is realized in a random manner, and the selection of the two nodes to which the new family links is also random. So in principle there is the chance that nodes that start with low wealth will become very rich, or rich nodes do not increase their wealth as expected. The actual way how the preferential redistribution in step 2 is implemented is by dividing the $2q$ value in many (usual several hundred) equal parts, and each part is assigned to a randomly chosen node, biased proportionally with the wealth of the node. To do this biased redistribution, the use of a BKL-type [27] Monte Carlo algorithm is very helpful. Another possibility (leading to the same results) for doing this preferential redistribution would be to select s nodes with the same probability, independent of their wealth, and then to split the $2q$ amount between the selected nodes proportionally with their actual wealth. It is also important to note that in realizing step 1, one can get to a situation where the selected node has no links (a family dies out without children). For simplicity reasons, in this case we have also chosen to redistribute the wealth of the node in the whole society by using the same preferential rule.

Of course, this model is a rough description of the reality and it should be viewed only as a first ''mean-field'' approximation. In real societies, the number of families and also the total amount of wealth should not be considered fixed. Many other social aspects could be of interest, the actual value of q and p should vary from family to family within quite broad distributions, the nodes must not die out according to their age and many cultural and religious factors can influence the dynamics of the underlying social network. In spite of all the neglected effects we will see that this simple model is able to reproduce the observed wealth distributions and generates reasonable first-degree family relation networks. The main advantage of this model is that the network structure on which the wealth exchange is realized is not a priori put in the system. The network forms and converges to a stable topology in time, together with the wealth diversification in the system and the appearance of the Pareto distribution.

3. Results of the model

Extensive computer simulations were done to study the wealth distribution and the generated family network for various values of the model parameters. In order to minimize the statistical fluctuations, we averaged over 100 realizations for each parameter's values. The model as defined above has several parameters: N, the number of nodes in the network, W_{t}, the total wealth of the system, t the number of simulation steps done to reach a given state, the number s giving the parts on which the $2q$ wealth is divided, and the value of the q and p wealth exchange parameters. By simple simulations it is easy to show that the results are independent of the chosen value of s, provided that s is big enough ($s \geqslant 10$ gives already stable results). In the results that will be presented we always used $s = 100$. We will argue in the following that the main free parameters are q and p, since the model converges rapidly both as a function of time and as a function of the number of nodes to a stable limiting distribution and network structure.

It is easy to realize that the chosen value of W_{t} will not change the nature of the results, but it simply rescales the values of the wealth. A simple computer exercise will also convince us that the above-defined family-network model converges in time very quickly to a statistically invariant state both for the wealth distribution and network structure. Results for a relatively big lattice ($N = 10,000$) and for realistic $p = 0.3$ and $q = 0.7$ values are presented in Fig. 2. We see that roughly after 5 MCS, both the cumulative wealth distribution and the first two moments of the degree distribution converge to their stable limit.

On the other hand, one can also check that the model has a well-defined thermodynamic limit. As N increases, we obtain again that both the cumulative wealth distribution and the statistical properties of the network reach a stable limit.

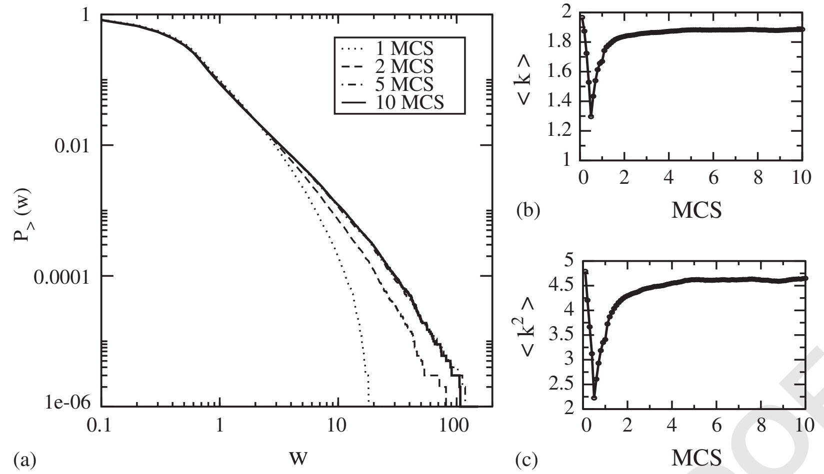

Fig. 2. Time evolution of the cumulative wealth distribution function (a), average degree of the nodes (b), and average square of the degree of the nodes (c), Simulations were done on a network with 10,000 nodes, $p = 0.3$ and $q = 0.7$.

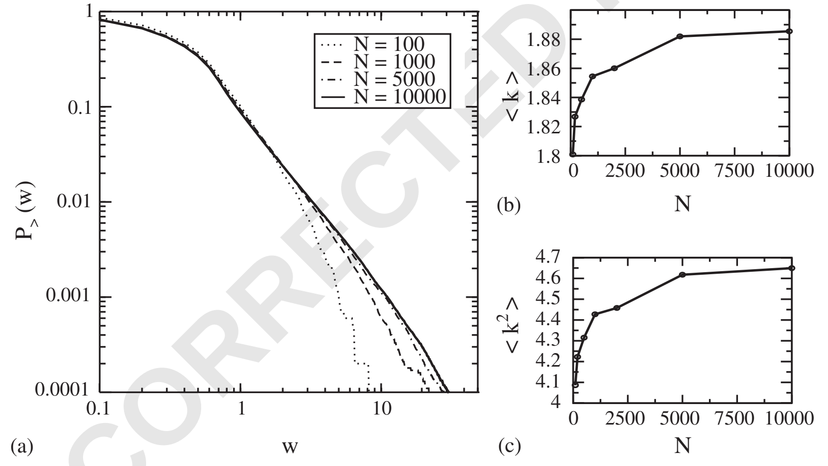

Fig. 3. Effects of the network size on the final results. The stable (after 10 MCS) cumulative wealth distribution function (a), average degree (b), and average square degree of the nodes (c), all for different network sizes. Simulations with $p = 0.3$ and $q = 0.7$.

Characteristic results for this variation are presented in Fig. 3. As we can see from the figure, for reasonably big lattices $N \approx 10,000$, a stable limit is reached.

We will study now the influence of the p and q wealth-exchange parameters. Since we have verified that the model converges relatively quickly to a stable limit, we will consider in all simulations 10 MCS. The number of nodes in the network will be chosen $N = 10,000$, which ensures that the thermodynamic limit is approached. The

p parameter can theoretically vary in the (0,1) interval; we consider however, a realistic variation in the (0.1–0.3) interval. Since start, from wealth values distributed randomly and uniformly on the (0,1) interval, the minimal q value needed to raise a child should thus also be in the (0,1) interval, otherwise no new family could be linked to the network. First, we present our results on the $P_>(w)$ cumulative wealth distribution curves. For two fixed values of p ($p = 0.1$ and $p = 0.3$) the curves are given in Fig. 4.

The curves in Fig. 4 suggest that the good scale-free Pareto tail is obtained for q values in the (0.7–0.9) interval, and we will thus focus in the following on this parameter region. It is also evident that results for $p = 0.3$ have a better trend. The Pareto index (power-law exponent) in this region varies in the (1.7–2.5) interval, depending on the chosen p and q values and fitting intervals. The $P_>(w)$ curves have the right shape, they show the power-law trend for the rich nodes and the exponential behavior in the low and medium wealth limit (Fig. 5). Moreover, one can also observe that in good agreement with the reality, roughly 5–10% of the nodes have wealth in the Pareto regime.

The network generated by the model is a simple exponential one. Considering the realistic $q \in (0.7–0.9)$ and $p \in (0.1–0.3)$ parameter region, in Fig. 6 we present results obtained for the $P(k)$ degree distribution (probability distribution that one node has a given number of links). From the degree distribution we conclude that the network is an exponential one. The most probable connectivity of a node is around 2, and we obtained that in this parameter region $\langle k \rangle$ varies between 1.8–1.9, which are reasonable values for real first-degree family relation networks. No relevant clusterization was observed in these networks.

It is also instructive to study different kinds of correlations in the generated networks. First, one can study the correlation between the k connectivity of the nodes, and mean connectivity of the neighbors $\langle k_{nn} \rangle$ for nodes with k links. If there exists a positive degree–degree correlation, i.e., if well-connected nodes tend to connect with other well-connected ones, then $\langle k_{nn}(k) \rangle$ must increase with k. In the relevant parameter region results in this sense, are plotted in Fig. 7. For low values of

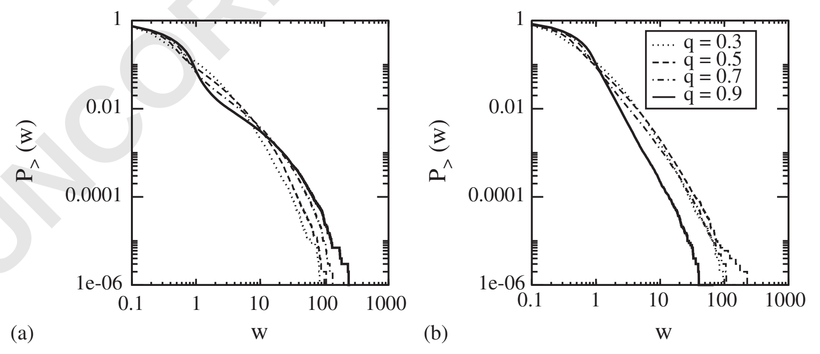

Fig. 4. Cumulative distribution functions for different p and q values, (a) $p = 0.1$ and (b) $p = 0.3$. Different curves are for different q values, as sketched on the legend of (b). The results are after 10 MCS and for $N = 10,000$.

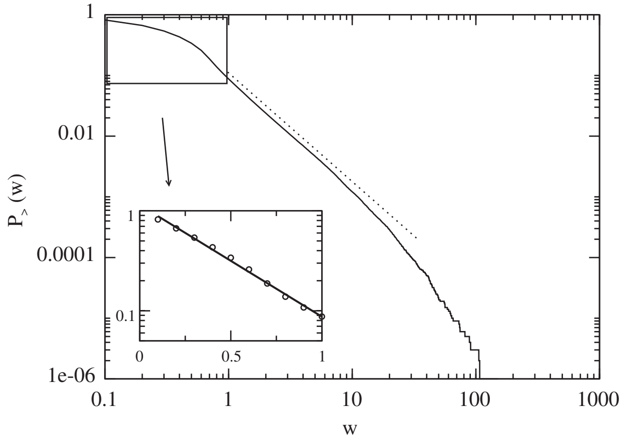

Fig. 5. The shape of the obtained cumulative wealth distribution function for $p = 0.3$, $q = 0.7$ ($N = 10{,}000$ and results after 10 MCS). The tail is approximated by power-law with exponent $\alpha = 1.80$, and the initial part of the curve has an exponential trend. The inset shows this initial trend on log-normal scale

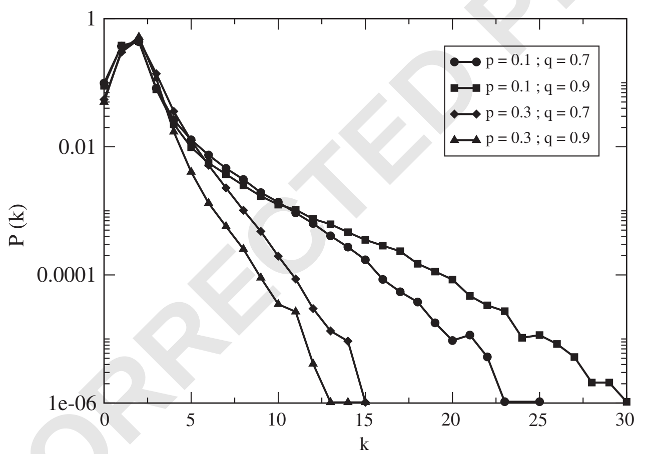

Fig. 6. The degree distribution of the obtained networks on a log-normal scale for various values of q and p. (Simulations after 10 MCS and with $N = 10{,}000$ nodes).

p there are no obvious correlations, but as p increases one can observe a positive correlation effect, $\langle k_{nn}(k)\rangle$ increases roughly linearly with k. This means that, if the new family gets a bigger portion of the parents' wealth, the number of links parents and children have are positively correlated. The effect is simply understandable, taking into account that for higher values of p the wealth of the parents and children should be also correlated, creating similar conditions for accepting links.

The correlation between the wealth w of one node and the average wealth of the neighbors $\langle w_{nn}\rangle$, should follow a similar trend. Indeed, as expected, this correlation

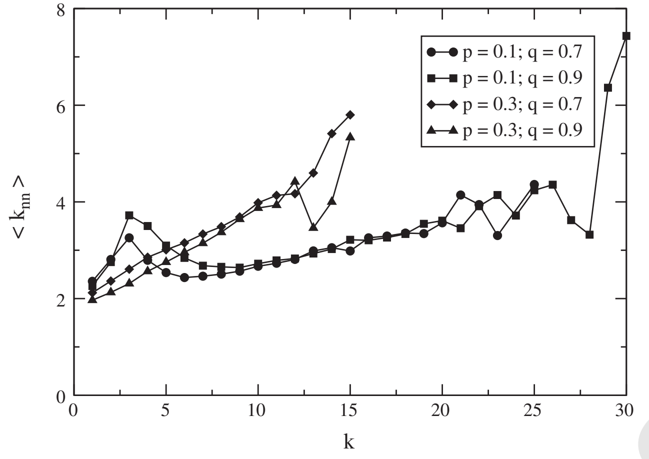

Fig. 7. Mean connectivity of the neighbors ($\langle k_{nn} \rangle$) as a function of the connectivity of the nodes for various values of q and p. (Results obtained after 10 MCS and $N = 10,000$.)

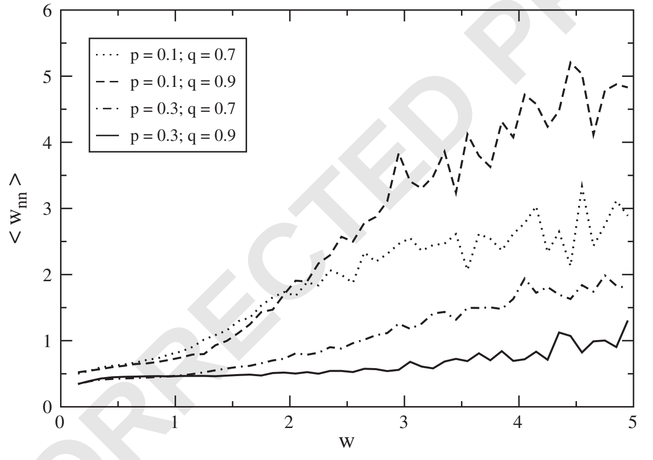

Fig. 8. Average wealth of the neighbors as a function of the wealth of the nodes. Different values of the q and p parameters are considered. (Results after 10 MCS and for $N = 10,000$.)

also has an increasing trend as p is increased (Fig. 8a). This positive correlation effect is more clear again for not too high wealth values, since in the high w limit there are few nodes and the statistics is poor. A similar correlation trend can be observed if one studies the correlation between the wealth of the nodes and the total wealth of the neighbors. In Fig. 8 we plotted the results only for $w \leqslant 5$, since for higher values of w the curves are very noisy due to the poor statistics.

Finally, one can study the correlation between the wealth and connectivity of a node, either by plotting $\langle k(w) \rangle$ (the average number of links for nodes with wealth around w in a given dw interval) as a function of w, or by simply calculating $c(w, k) =$

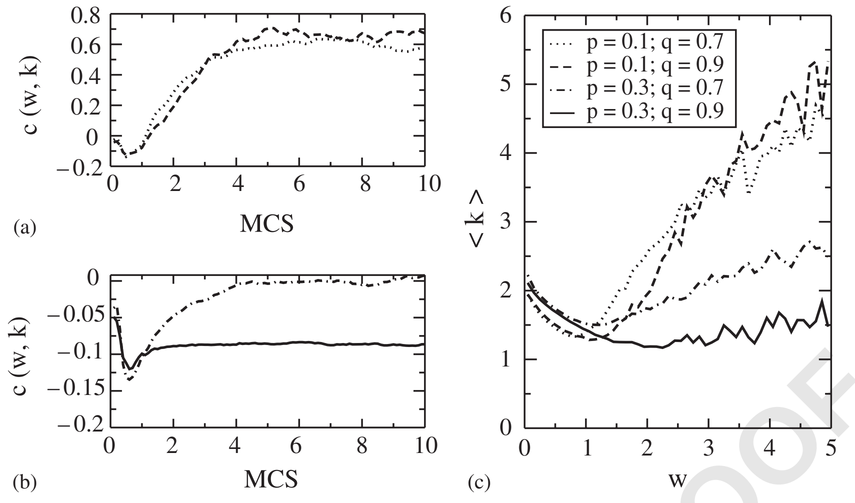

Fig. 9. Results for the correlation between the connectivity and wealth of the nodes. Figs. (a) and (b) show results for $c(w,k)$ as a function of time considering the relevant p and q values. Fig. (c) illustrates the trend for the mean connectivity of nodes with different wealth values. For all the figures the corresponding p and q values are given on the legend of (c), and we considered $N = 10,000$ nodes.

$\langle w \cdot k \rangle - \langle w \rangle \langle k \rangle$. In Figs. 9a and 9b we plot the values of $c(w,k)$ as a function of time, and in Fig. 9c we show the $\langle k(w) \rangle$ curves. (For constructing the curves in Fig. 9c we used boxes of size $\mathrm{d}w = 0.1$.) From Figs. 9 and 9b one notices again, that both the network structure and wealth distribution approach quickly (less than 5 MCS) a statistically stable limit. It is interesting to observe that the $c(w,k)$ correlation is stronger for low p values, which makes sense since as p increases the availability of a wealthy node to accept more links decreases. As p increases the $c(w,k)$ trend suggests that we deal with a clear anticorrelation between the wealth and number of links of a node, which means that nodes which do not get too many links will in general become wealthy. The trend of the $\langle k(w) \rangle$ curves (Fig. 9c) suggests similar conclusions, but here we can also see this correlation effect differentiated as a function of the w value. In the low and medium wealth limit, there is a clear anti-correlation between wealth and number of links, while for the wealthy nodes (much fewer in number) there is a positive correlation trend. In Fig. 9c, we plotted again the data only for $w \leqslant 5$, since for higher wealth values the curves are rather noisy due to poor statistics.

4. Discussion and comparison with real data

Let us now analyze real wealth distribution data in societies in order to check the quantitative agreement with our results. We use estimates for the distribution of personal wealth in the United Kingdom (available on the Internet) [13], based on inheritance tax, capital transfer tax and other data (the methods used for getting these estimates are also described in Ref. [13]). Plotting the cumulative wealth distribution for a chosen year (2001 in our case), one gets the graph in Fig. 10 (distributions for other years are quite similar, even quantitatively).

On the data presented in Fig. 10, one can nicely identify the exponential regime for low and medium wealth values, and the Pareto power-law distribution in the high wealth limit. As emphasized in the introduction, the Pareto tail describes the upper 5% of the society. The UK-2001 data suggests a Pareto index $\alpha = 1.78$ (Fig. 10). An immediate comparison with the distribution obtained for our family model (Fig. 5) shows that for the reasonable $q = 0.7$ and $p = 0.3$ parameters the model offers a fair description. The Pareto index for these parameters is around $\alpha = 1.8$, in the low and medium wealth limit the $P_>(w)$ curve is exponential, and the Pareto law is valid for the upper 5–10% of the society. Concerning the shape of the $P_>(w)$ curve, the model thus seems to work well.

The network structure generated by the model also seems to be realistic. The exponential nature of the network, the most probable value of the connectivity $k_{prob} \approx 2$, and the average connectivity $\langle k \rangle \approx 1.9$ are all reasonable for real first-degree family relation networks. The correlations $\langle k_{nn} \rangle (k)$, $\langle w_{nn} \rangle (w)$, $\langle w_{nt} \rangle (w)$ and $\langle k \rangle (w)$, presented in Figs. 7–9, and described in the previous section, are also reasonable. This kind of correlations could be expected, since our model is somehow similar to the ideas of hidden variables proposed by Boguña et al. [25], and their model also generated correlated networks.

Within the proposed model we can also identify the wealth diversification mechanism that finally leads to Pareto's law. The time evolution of the $c(w, k)$ correlations (Figs. 9a and 9b), and the time evolution for the $P_>(w)$ cumulative distribution functions (Fig. 2) viewed in parallel give us important clues in this sense. In the beginning of the dynamics there is usually a strong anti-correlation effect

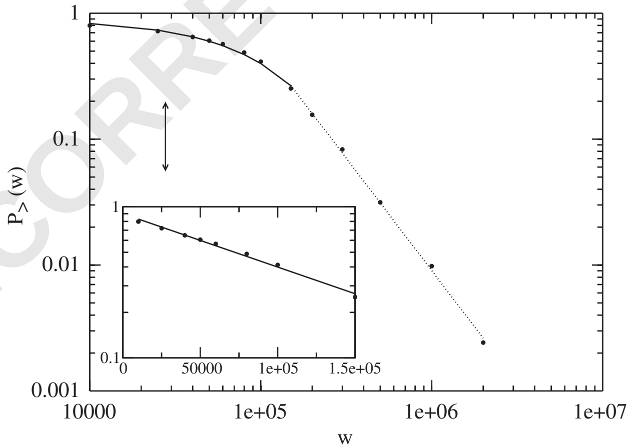

Fig. 10. Cumulative wealth distribution for the population of the United Kingdom, for the year 2001. Results obtained using the database from Ref. [13]. The power-law tail is described by an exponent $\alpha = 1.78$. The inset illustrates the initial exponential behavior of the curve, using a log-normal scale.

($\mathrm{d}[c(w,k)]/\mathrm{d}t<0$) between wealth and number of links. This means that in this regime those nodes which have fewer number of links will become wealthier. The Pareto tail here does not exist, and this is where the strong wealth diversification starts. After this initial transient regime, the $c(w,t)$ correlation will converge to a stable limit, and simultaneously the stable $P_>(w)$ cumulative distribution function with the Pareto tail is formed. The main mechanism leading to the strong wealth diversification in our model is thus the initial strong anticorrelation between the wealth and the number of links of one node.

One can also simply verify that the main necessary ingredient that will produce the power-law tail is the preferential wealth redistribution in the system. Without the preferential wealth redistribution of the $2q$ amounts, the model will not generate power-law tails for $P_>(w)$. This rich-gets-richer effect seems to be thus the main mechanism leading to power-law wealth distribution in the richer part of the societies.

5. Conclusions

We have presented a family-network model designed to explain the cumulative wealth distribution in societies. In our model the wealth exchange is realized on a first-degree family relation network, and it is governed by two parameters. The dynamics is defined through realistic rules and generates both the underlying family network and wealth distribution. The model has a stable thermodynamic limit, and the dynamics quickly lead to a network structure and wealth distribution which are stable in time. Extended computer simulations show that for reasonable parameter values both the obtained cumulative wealth distribution function and network structure are realistic: (i) in good agreement with real measurement data we were able to generate cumulative wealth-distribution functions with Pareto-like power-law tails, (ii) the obtained Pareto index is close to the measured values, (iii) the cumulative wealth distribution function for the low and medium wealth values is exponential as found in social data, (iv) the Pareto regime is valid for the upper 5% of the society, (v) the generated first-degree family relation network is realistic. We observed that in our model the initial wealth diversification is realized through a strong anticorrelation between the wealth of the nodes and their number of links. As the main mechanism leading to the formation of the Pareto power-law tail we identified the preferential redistribution of wealth in the society. In the generated networks many interesting correlations have also been revealed.

In spite of its strengths the proposed model is still a rough approximation to reality. One may argue that many important cultural, social or economic phenomena have been neglected. We consider this model as a first, mean-field-type approximation. The novel aspect of our approach is, however, that the network structure was not a priori introduced in the model, but it got formed during the postulated wealth exchange dynamics. Subscribing to the ideas presented in Ref. [26], we also feel that such type of approach should be considered for explaining many other social or economic phenomena and complex network structures.

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