Foreword to The Minority Game

(book of Damien Challet, Matteo Marsili, and Yi-Cheng Zhang)

By W. Brian Arthur

June, 2004

Once in a while a model problem appears that is simple to describe but offers a wealth of lessons. If the model problem is a classic — like the Ising model or the Prisoner's Dilemma — it both opens up our insight and gives us analytical pathways into an intriguing world. Challet, Marsili and Zhang's Minority Game problem is such a model. It is a classic. The game itself is easy to visualize. A number of agents wish to take some particular action, but they do not benefit if the majority of others take it too. The circumstances where this occurs may seem to be very specific, but in fact they arise *everywhere* in the economy when agents must commit in advance to something that shows diminishing returns. (Diminishing returns, that is, to the numbers committing). Financial trades must be committed to in advance, and the benefits of many strategies may be arbitraged away if others use them too. Hence the minority situation arises also in financial markets.

Damien Challet and Yi-Cheng Zhang conceived of the Minority Game in 1997. They, and their co-worker Matteo Marsili who joined them later, were intrigued not only by the problem itself, but by the possibility of applying statistical mechanics techniques to economics and game theory more generally. Over the previous two decades, physics had become adept at analyzing systems with many particles interacting heterogeneously, and markets clearly consisted of such particles — investors. But these economic "particles" reacted by attempting to forecast the outcome that would be caused by the forecasts of other particles, and to react to that in advance. They were strategic. This was a recognizable world for physicists but an intriguingly different one, and the minority game provided a pathway into it. The problem was quickly taken up by other physicists. Its formulation was simplified and progressively redefined to be closer to what physics was used to dealing with. Versions appeared, results appeared, and advances in understanding appeared. This book chronicles the progress of thought and the considerable insights gained. And it collects the papers that became the main milestones.

One distinctive feature of this work is its emphasis on phases in parameter space — different qualitative properties of the outcome that obtain under different parameter sets. Delineating phases is second nature to physicists but not to us economists, and we can learn much from this practice. In the minority game, agents align their strategies to the condition of the market: they want to choose the strategy that on average minimizes their chance of being in the majority. As a result, strategies co-organize themselves so as to minimize collective dissatisfaction. But this collective result has two phases. With few players, the recent history of the game contains some useful information that strategies can exploit. But once the number of players passes a critical value, all useful information has been used up. The properties of the outcome differ in these two regimes. Information, in fact, is central to the situation. And its role can be explicitly observed: players act as if to minimize a Hamiltonian that is itself a measure of available information. So the process

by how information gets eaten up by players is made explicit, and this is another useful pathway into exploring financial markets.

Because some information is left unexploited if few players are in the market, and all information is used up as players increase past a critical number, Challet, Marsili and Zhang point out that players may enter the market until this critical number is reached. The market will therefore display self-ordered criticality. This is an important conjecture, and not just a theoretical one. It tells us that speculative investors — technical traders, at least — will seek out thin markets where possible and avoid deep ones. Markets may therefore hover on the edge of efficiency — a significant insight that can be made explicit with the techniques used here and one that is worth further investigation.

The Minority Game grew out of my El Farol bar problem, and to fill in some pre-history I should say a few words about that. Legend is indeed correct: in 1988 on Thursday nights Galway musician Gerry Carty played Irish music at El Farol, and the bar was not pleasant if crowded. Each week I mulled whether it was worth showing up, and I mulled that others also mulled. It occurred to me that situation was something like the forecasting equivalent of prisoner's dilemma. Few showing up was best; but it was in everyone's interest to be in that few. But my motivation was not to find a forecasting prisoner's dilemma. I was interested at the time in rational expectations (or forecasts) and their meaning for economics. In solving forecasting problems, economics had found it useful to imagine that everyone had the same forecasting machine and used it. You could then ask what forecasting machine would lead to agents' actions that would produce aggregate outcomes that on average would validate the machine's forecasts. This would be the rational expectations solution — the forecasting method that "rational" agents would choose. As an analytical strategy this worked, at least in the cases studied. But it bothered me. If someone were not smart enough to figure the proper forecast they would skew the outcome. Then forecasts should deviate; then I should deviate. Then others should too, and the situation would unravel. Rational expectations may have been good theory, but it was not good science. I realized that El Farol made this plain. If we postulated a "correct" bar-attendance forecasting machine that everyone coordinated on, their actions would collectively invalidate it. Therefore there could be no such correct machine. But if no deductive forecasting solution was possible, what should agents in the economy do? The problem was behaviorally ill-defined. And so were most situations involving the future in economics, I realized. This fascinated me.

In 1992 I stripped the El Farol situation to its essentials and programmed it. I wrote the problem up and presented it at the January 1994 American Economic Association meetings in a session on complexity in the economy chaired by Paul Krugman. The paper was received politely enough, but my discussant was irritated. He pointed out the problem had a solution in mixed Nash strategies: the bar-goers could toss coins to reach a satisfactory outcome. I had thought of that, but to me that missed the point. To me, El Farol was not a problem of how to arrive at a coordinated solution (although the Minority Game very much is). I saw it as a conundrum for economics: How do you proceed analytically when there is no deductive, rational solution? Defining the problem as a game lost this — all games have at least one Nash mixed-strategy equilibrium — so I resisted any game-theoretic formulation. My paper duly appeared, but economists didn't quite know what to make of it. My colleague at Santa Fe, Per Bak, did know however. He saw the manuscript and began to fax it to his physics friends. The physics community took it up, and in the hands of Challet, Marsili and Zhang, it inspired something different than I expected — the Minority Game. El Farol emphasized (for me) the difficulties of formulating economic behavior in ill-defined problems. The

Minority Game emphasizes something different: the efficiency of the solution. This is as should be. The investigation reveals explicitly how strategies co-adapt and how efficiency is related to information. This opens an important door to understanding financial markets.

As I write this, there are now over 140 papers on the Minority Game, and a growing community of econophysicists who have become deeply immersed in the dynamics of markets — especially financial markets. Economists wonder at times whether all this work in physics is not just a lengthy exercise in physicists learning what economists know. It is certainly not — and indeed this book shows it is not. Modern physics can offer much to economics. Not just different tools and different methods of analysis, but different concepts such as phase transitions, critical values, and power laws. And not just the analysis of pattern at stasis, but the analysis of patterns in formation. Economics needs this. In fact economics is changing currently from an emphasis on equilibrium and homogeneity to an emphasis on the formation of pattern and heterogeneity. And so, economics in due course would be forced to use the kind of tools that are brought to bear in this book. Luckily physics is there to supply them.

The papers here and the text are an important part of a movement — looking at the economy under heterogeneity. Challet, Marsili and Zhang and the others described here have done much to power this movement, and I congratulate them for this. It is indeed a benefit to physicists and economists to have this work collected in one place.

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