CURS 7 + SEM 6 8.5

Proiectarea filtrelor. Transformata Laplace

Probleme fizice, circuite → ecuații diferențiale sau sisteme de ec. dif.

Sisteme de ec dif. → soluții în t. (dep. de t).

Transformata Laplace:

t

$f(t) \xrightarrow{Laplace}$ $s = \sigma + j\omega$ → variabilă complexă

$F(s)$ → frequency-space

$$\boxed{F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt}$$

$$\frac{df}{dt} \xrightarrow{Laplace} sF(s) - f(0)$$ → val. inițială în timp.

$$\int f(t)dt \xrightarrow{Laplace} \frac{F(s)}{s} + \frac{1}{s}\left(\int_{-\infty}^{0^-} f(t)dt\right)$$

Ec dif → Ec algebrice

Dacă inițial $i_L = 0$ și $v_C = 0$:

În timp:

•) Pt. inductor:

ohm.

$$v(t) = L \cdot \frac{di}{dt} \longrightarrow V(s) = L \cdot s \cdot I(s)$$

$$\boxed{Z_L(s) = L \cdot s} \qquad V(s) = Z_L(s) \cdot I(s)$$

$$[Z_L] = H \cdot \frac{1}{s} = \frac{H}{s} = \frac{Wb}{As} = \frac{Vs}{As} = \frac{V}{A} = \Omega$$

•) Pt. condensator:

$$v(t) = \frac{1}{C}\int i(t)dt \xrightarrow{Laplace} V(s) = \frac{I(s)}{Cs} = \frac{1}{Cs} \cdot I(s)$$

$$V(s) = Z_C(s) \cdot I(s)$$

$$Z_C(s) = \frac{1}{Cs}$$

$$[Z_C] = \frac{s}{F} = \frac{s \cdot V}{C} = \frac{V}{A} = \Omega$$

$$A^{-1}$$

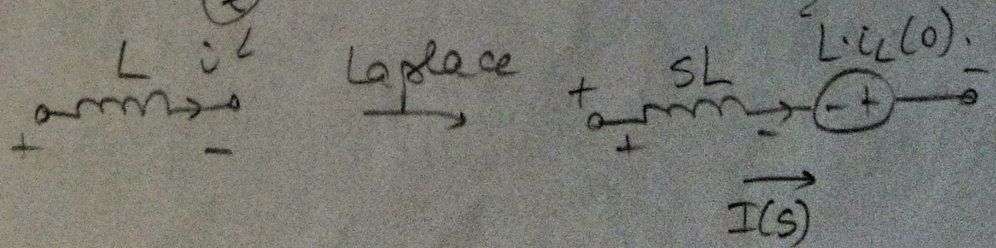
•) Pt. rezistor:

$$v(t) = R \cdot i(t) \xrightarrow{Laplace} V(s) = R \cdot I(s)$$

•) Pt. un inductor:

$$v(t) = L \cdot \underbrace{\frac{di(t)}{dt}}_{sI(s) - i_L(0)} \rightarrow V(s) = \underbrace{LsI(s)}_{\text{căderea de tensiune pe } Z_L = s \cdot L} - \overbrace{L \cdot i_L(0)}^{\text{tensiune constantă ce se opune (Lenz)}}$$

•) Circuit echivalent L:



2.

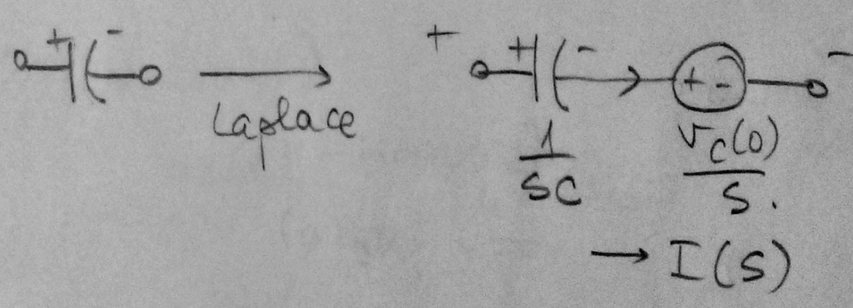
·) Pt. un condensator:

$$v(t)=\frac{1}{C}\int i(t)dt \xrightarrow[\text{Laplace}]{} V(s)=\frac{I(s)}{sC}+\frac{1}{sC}\underbrace{\overbrace{\left(\int_{-\infty}^{0} i_C(t)dt\right)}^{q_C(0)}}_{\text{sarcina acumulată în condens.}}$$

v constant (inițial).

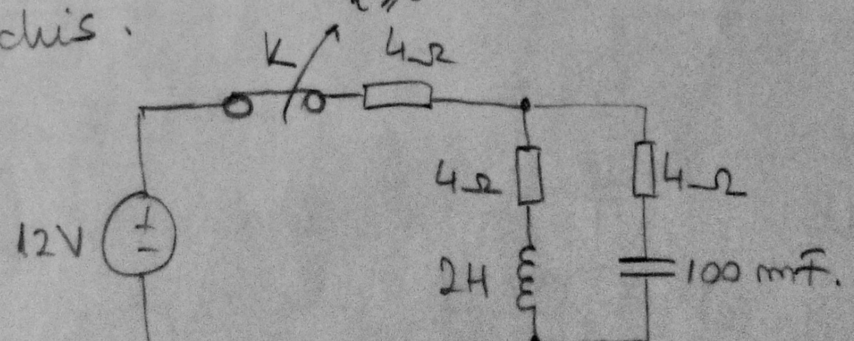
$$V(s)=\underbrace{\frac{I(s)}{sC}}_{\text{căderea de tensiune pe } Z_C}+\frac{v_C(0)}{s}$$

·) circuit echivalent C:



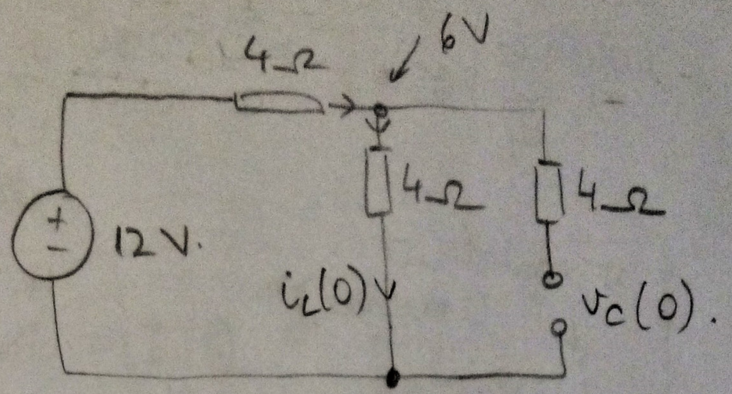
·) Exemplu:

Se cere forma lui $i_L(t)$ după ce K este deschis. $t \geq 0$.



circuitul înaintea deschiderii lui k.

$t=0^-$ - curentul, tensiunea inițiale

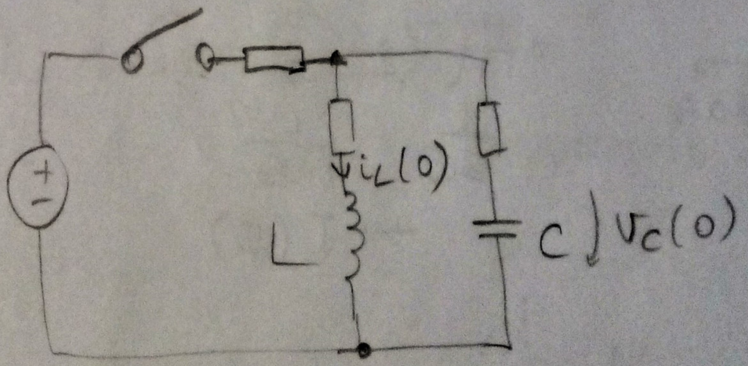


inițiale ←

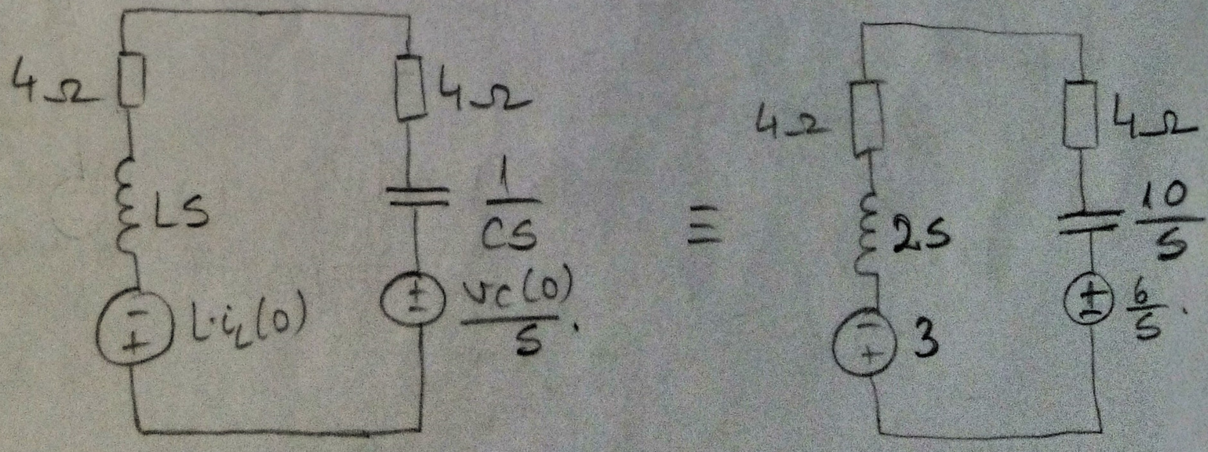
$$i_L(0) = \frac{12\,V}{4\Omega+4\Omega} = \frac{\cancel{12}^{3}}{\cancel{8}_{2}} = \frac{3}{2} = 1.5\,A$$

$$v_C(0) = 6\,V$$

pentru $t=0$



Pentru $t \geq 0$.

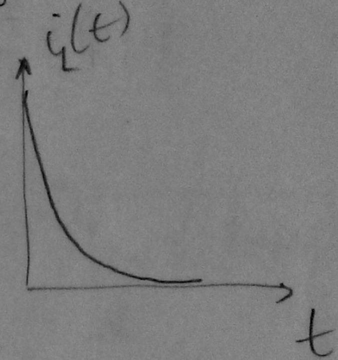


$$\frac{6}{s}+3=I(s)\left(4+4+2s+\frac{10}{s}\right) \Rightarrow$$

$$\Rightarrow I(s)=\frac{\frac{6}{s}+3}{8+2s+\frac{10}{s}}=\frac{\frac{3s+6}{\cancel{s}}}{\frac{8s+2s^2+10}{\cancel{s}}} \Rightarrow$$

$$\Rightarrow \boxed{I_L(s)=\frac{3s+6}{2s^2+8s+10}}$$

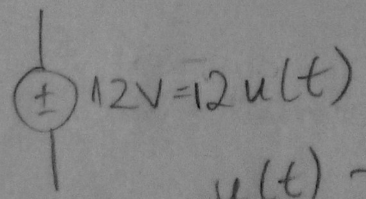
i.L.T. $\quad i_L(t)=\frac{3}{2}e^{-2t}\cos t$



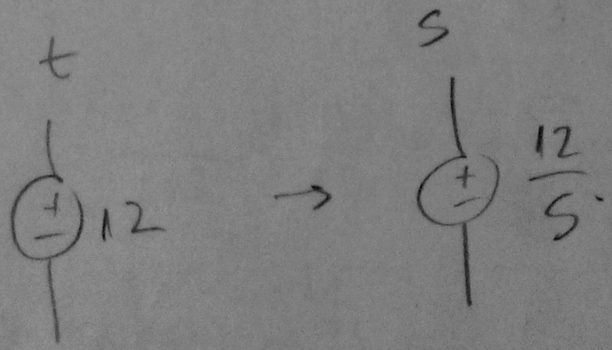
OBS:

Surse DC:

$u(t) \xrightarrow{\text{Laplace}} \frac{1}{s}$.

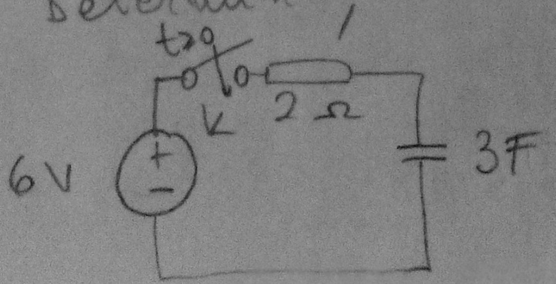


$u(t)$ - Heaviside step function

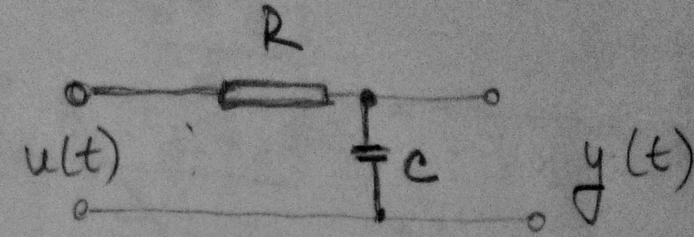


Temă:

determinați $v_C(t)$ după ce K este închis



•) FT ∃ unipol:



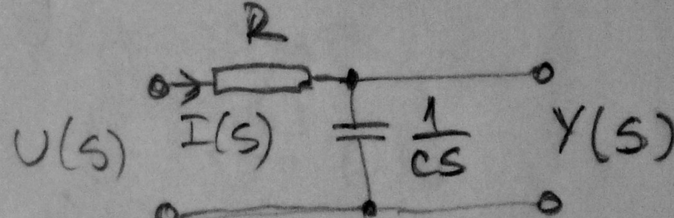
consid. $v_c(0) = 0$.

↓ Laplace

$$U(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

$$H(s) = \frac{Y(s)}{U(s)}$$

Circuitul după Laplace transf.



$$U(s) = I(s)\left[R + \frac{1}{cs}\right].$$

$$Y(s) = I(s)\frac{1}{cs}$$

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{cs}}{R+\frac{1}{cs}} = \frac{1}{cs} \cdot \frac{1}{R+\frac{1}{cs}}$$

$$\boxed{H(s) = \frac{1}{1+RC\cdot s}}$$

→ raport de 2 polinoame.

→ rădăcinile numărătorului
o zerouri

→ rădăcinile numitorului
× poli

$$H(s) = \frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

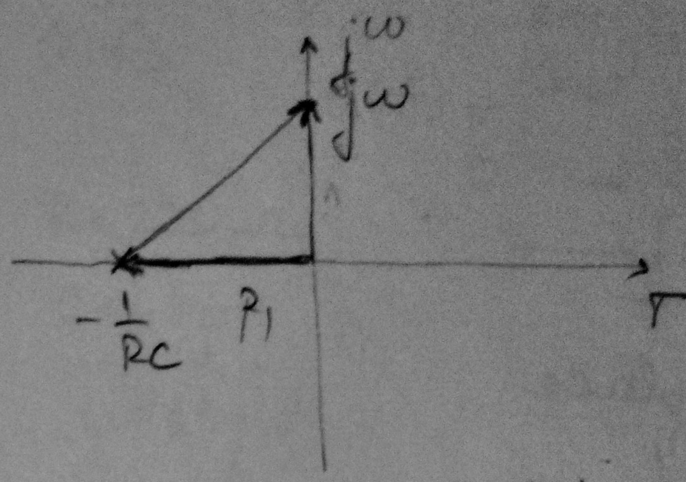
$1 + RCs = 0$

$RCs = -1$

I.V.T. $\quad h(0) = \lim_{s \to \infty} sH(s)$

F.V.T. $\sigma < 0 \quad h(\infty) = \lim_{s \to 0} sH(s)$

$s = -\frac{1}{RC} \rightarrow$ 1 pol la $\sigma = -\frac{1}{RC}$



($\sigma < 0 \Rightarrow$ sistem stabil)

$p_1 = -21.28 \times 10^3$.

$$|H(s)| = \frac{1}{|1 + RC(\sigma + j\omega)|} =$$

$R = 100\ \Omega$

$C = 470\ \mu F$

$$= \frac{1}{\sqrt{(1+RC\sigma)^2 + (RC\omega)^2}}$$

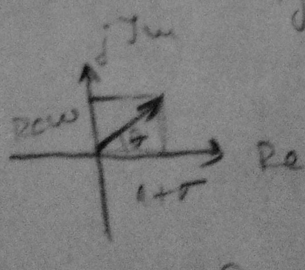
$RC = 100 \cdot 470 \cdot 10^{-6} =$

$= 47000 \cdot 10^{-6} = 47 \cdot 10^{-3}\ \Omega F$.

$$|H(s)| = \frac{1}{\sqrt{(1+\sigma 47 \cdot 10^{-3})^2 + (\omega \cdot 47 \cdot 10^{-3})^2}} =$$

$tg\,\theta = \frac{RC\omega}{1+\sigma RC}$

$\theta = arctg \frac{RC}{1+\sigma}\omega$



$$= \frac{1}{\sqrt{(1+47\sigma)^2 + (47\omega)^2}}$$

$\theta = arctg \cdot \frac{47 \cdot 10^{-6}}{1+\sigma RC}\omega$

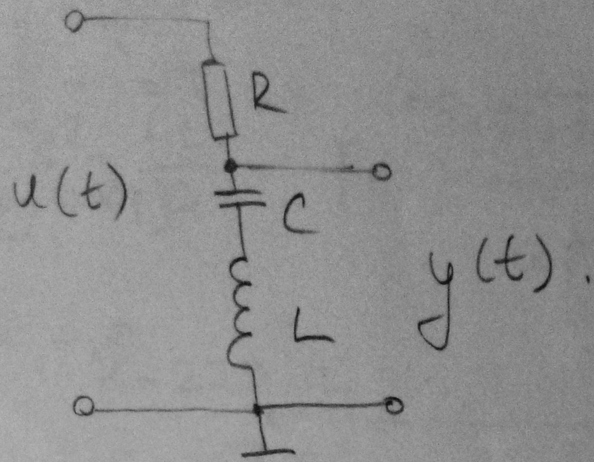
$\sigma, \omega \rightarrow 10^6$.

$$H(s) = \frac{1}{1 + \sigma + jRC\omega}$$

rolloff $|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$

$\omega \gg \omega_c \quad |H(j\omega)| = \frac{\omega_c}{\omega}$

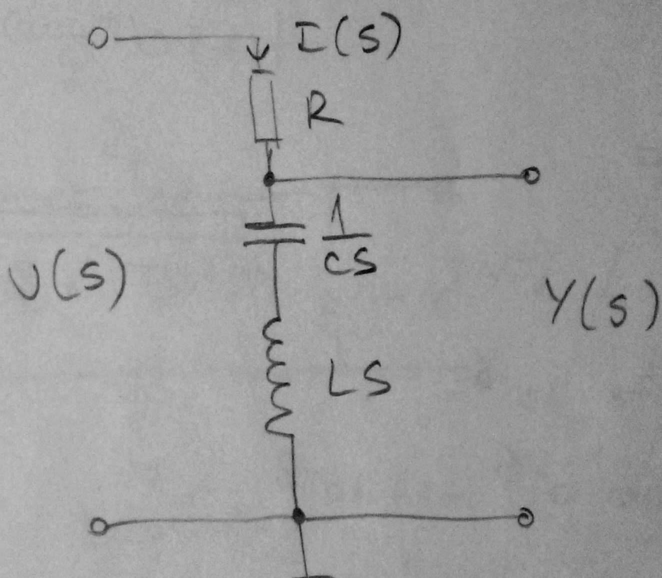
Analiza circuitului RLC serie:



$H(s) = ?$

$\left.\begin{matrix} |H(s)| \\ \angle H(s) \end{matrix}\right\} ?$

Laplace:



$$U(s) = I(s)\left[R + \frac{1}{Cs} + Ls\right]$$

$$Y(s) = I(s) \cdot \left[\frac{1}{Cs} + Ls\right]$$

2 zerouri

$$H(s) = \frac{Ls + \frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \boxed{\frac{Ls^2 + \frac{1}{C}}{Ls^2 + Rs + \frac{1}{C}}}$$

2 poli

.) zerouri:

$$Ls^2 + \frac{1}{C} = 0 \Rightarrow Ls^2 = -\frac{1}{C} \Rightarrow s^2 = -\frac{1}{LC};\ z_{1,2} = \pm j \frac{1}{\sqrt{LC}}$$

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·) poli:

$$Ls^2 + Rs + \frac{1}{C} = 0$$

$$\Delta = R^2 - \frac{4L}{C} \qquad p_{1,2} = \frac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L}$$

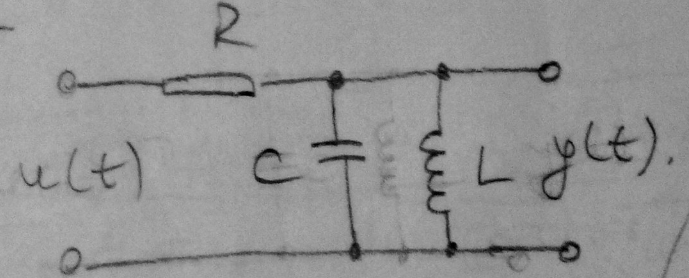
$R = 220\,\Omega$

$C = 47\,pF$

$L = 54\,\mu H$

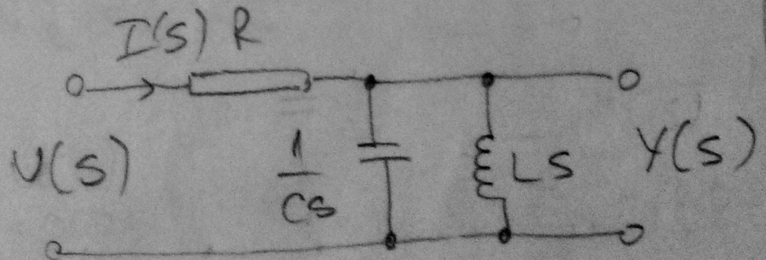
$\omega_0 = 6.277 \cdot 10^6 \rightarrow f_0 \sim 1\,MHz$.

·) FTB:



↓

Laplace



$$H(s) = \frac{Z}{R+Z} = \frac{\cancel{Z}}{\cancel{Z}\left(\frac{R}{Z}+1\right)} = \frac{1}{1 + R\cdot\left(\frac{1}{Ls} + Cs\right)} =$$

$$= \frac{1}{1 + \frac{R}{Ls} + RCs} = \frac{Ls}{Ls + R + RCsLs} = \frac{Ls}{s^2RLC + sL + R}.$$

equiv: $\frac{1}{Z_{eq}} = \frac{1}{\frac{1}{Cs}} + \frac{1}{Ls} = \frac{1}{Ls} + Cs$

$$Z_{eq} = \frac{1}{Cs + \frac{1}{Ls}} = \frac{Ls}{Cs^2 + Ls}$$

$U(s) = I(s)\,[R + Z_{eq})$

$Y(s) = I(s) \cdot Z_{eq}$

1 zero + 2 poli

$$H(s) = \frac{LS}{s^2RLC + sL + R}$$

zero $LS = 0 \Rightarrow s = 0$

$$s^2RLC + sL + R = 0.$$

$$\Delta = L^2 - 4R^2LC$$

$$p_{1,2} = \frac{-L \pm \sqrt{L^2 - 4R^2LC}}{2RLC} = -\frac{1}{2RC} \pm \frac{1}{2RC}\sqrt{\frac{\cancel{L^2}\left(1 - 4\frac{R^2C}{L}\right)}{\cancel{L^2}}} =$$

$$= -\frac{1}{2RC}\left[1 \pm \sqrt{1 - 4\frac{R^2C}{L}}\right]$$

$$|H(s)| = \frac{\sqrt{(L\sigma)^2 + (L\omega)^2}}{\sqrt{(\sigma - p_1)^2 + \omega^2} \cdot \sqrt{(\sigma - p_2)^2 + \omega^2}} = \frac{L\sqrt{\sigma^2 + \omega^2}}{\sqrt{(\sigma - p_1)^2 + \omega^2} \cdot \sqrt{(\sigma - p_2)^2 + \omega^2}}$$

$R = 220\ \Omega$

$C = 47\,pF$

$L = 54\,\mu H$

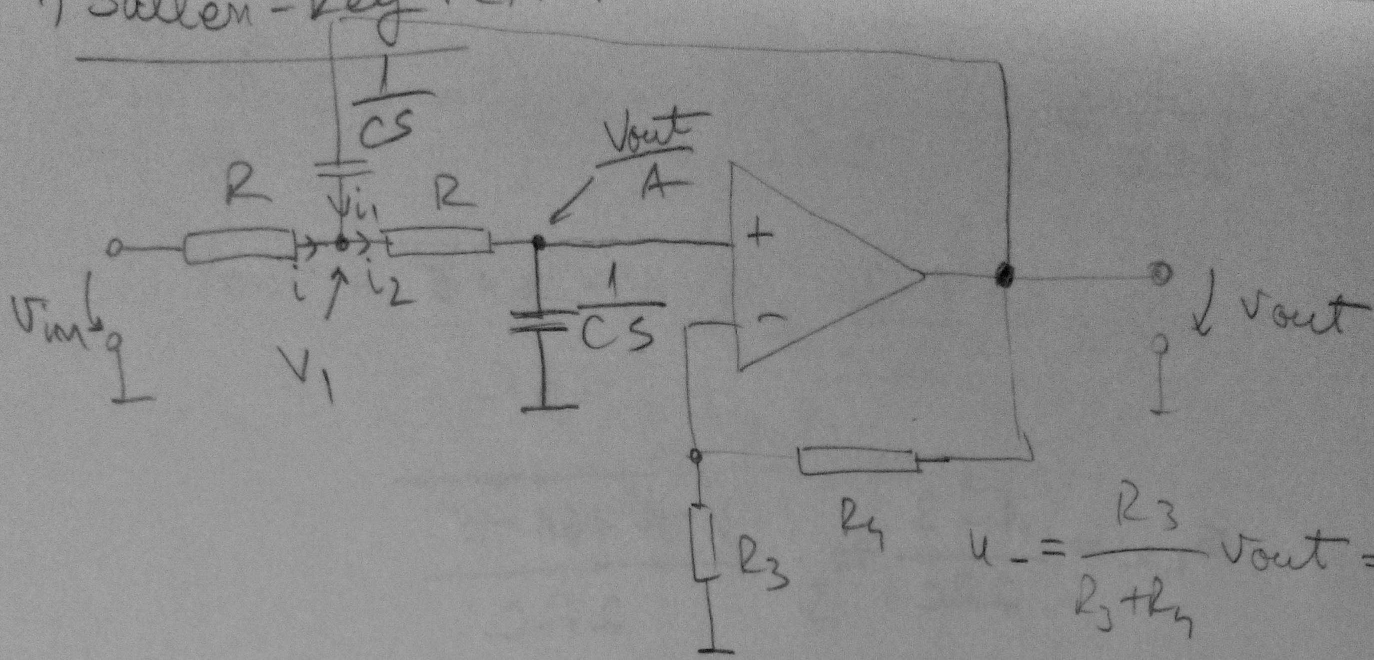
$$54 \times 10^{\cancel{12}} \cdot \frac{10^6}{10^{12}}.$$

$p_1 = -92449912 = -92,4 \cdot 10^6 \cdot$ $(-5000 = 0.005 \cdot 10^6)$

$p_2 = -4261886 = -4.2 \cdot 10^6 \cdot$ $(-5000 = 0.005 \cdot 10^6)$

$$|H(s)| = \frac{54^{\times 10^{-12}}\sqrt{x^2 + y^2}}{\sqrt{(x+93)^2 + y^2} \cdot \sqrt{(x+4.2)^2 + y^2}}$$

1) Sallen-Key. LPF.



$$u_- = \frac{R_3}{R_3+R_4} V_{out} =$$

$$= V_{out} \cdot \frac{1}{1+\frac{R_4}{R_3}} = \frac{V_{out}}{A}$$

$$V_1 = I_2 R + \frac{V_{out}}{A}.$$

$$\frac{I_2}{CS} = \frac{V_{out}}{A} \Rightarrow I_2 = \frac{V_{out}}{A} CS.$$

$$V_1 = \frac{V_{out}}{A} RCS + \frac{V_{out}}{A} = \boxed{\frac{V_{out}}{A}(1+RCS) = V_1}$$

$$I_2 = I_1 + I$$

$$\frac{I_1}{CS} = V_{out} - V_1. \quad I_1 = (V_{out} - V_1) CS.$$

$$IR = V_{in} - V_1 \quad I = \frac{V_{in} - V_1}{R}.$$

$$\frac{V_{out}}{A} CS = (V_{out} - V_1) CS + \frac{V_{in} - V_1}{R} \quad \Big| \cdot A \cdot R$$

$$\left.\begin{aligned} V_{out}\, RCS &= (V_{out} - V_1) ARCS + A (V_{in} - V_1) \\ V_1 &= \frac{V_{out}}{A} (1 + RCS) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow V_{out} RCS = V_{out} ARCS - V_{out} RCS (1 + RCS) +$$

$$+ A V_{in} - \cancel{A} \cdot \frac{V_{out}}{\cancel{A}} (1 + RCS)$$

$$V_{out} \left[RCS - ARCS + RCS(1 + RCS) + 1 + RCS \right] = A V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + 2RCS - ARCS + RCS + R^2C^2S^2}$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{S^2 \cdot R^2C^2 + S \cdot RC(3 - A) + 1}$$

$$S^2 \cdot R^2C^2 + S \cdot RC(3 - A) + 1 = 0$$

$$\Delta = R^2C^2 (3 - A)^2 - 4R^2C^2$$

$$s_{1,2} = \frac{\cancel{RC}(A - 3) \pm \sqrt{R^2C^2(3 - A)^2 - 4R^2C^2}}{2R^{\cancel{2}}C^{\cancel{2}}} =$$

$$= \frac{A - 3}{2RC} \pm \frac{\cancel{RC}\sqrt{(3 - A)^2 - 4}}{2R^{\cancel{2}}C^{\cancel{2}}}$$

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$$s_{1,2} = \frac{A-3}{2RC} \pm \frac{\sqrt{9-6A+A^2-4}}{2RC}$$

$$s_{1,2} = \frac{A-3}{2RC} \pm \frac{\sqrt{A^2-6A+5}}{2RC}$$

$$s_{1,2} = \frac{A-3}{2RC} \pm j \cdot \frac{\sqrt{-A^2+6A-5}}{2RC}$$

pt. $A = 1.$

$R = 1\,\Omega$

$C = 1\,F.$

$$s_{1,2} = -\frac{1}{RC} \pm j \cdot \cancel{\frac{\sqrt{-1+6-5}}{2RC}}^{\,0}$$

$$= -1.$$

pt. $A = 1.586$

$R = 1\,\Omega$

$C = 1\,F$

$$s_{1,2} = -0.707 \pm j \cdot 0.707$$

$$\sqrt{-2.515+9.516-5}$$

$$x^2+y^2 = r^2$$

$$1$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{1-x^2}$$

pt. $A = 2.5$.

$$s_{1,2} = -0.25 \pm j \cdot \frac{1,936}{2} = -0.25 \pm j \cdot 0.97.$$

Temă: Sallen-Key HPF.