CURS 8 SIS

Metode de îmbunătățire a raportului semnal - zgomot:

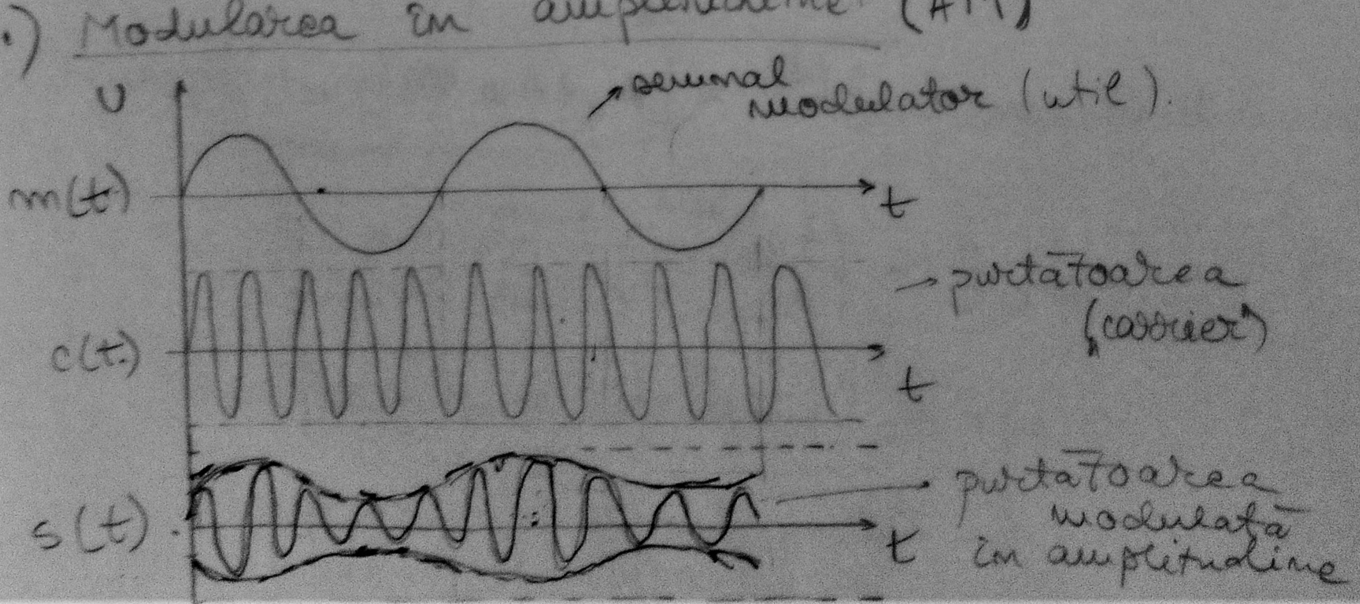
- filtre analogice
- modularea semnalelor (AM, FM)
- detecția sensibilă la fază (lock-in).
- filtre digitale

Modularea semnalelor:

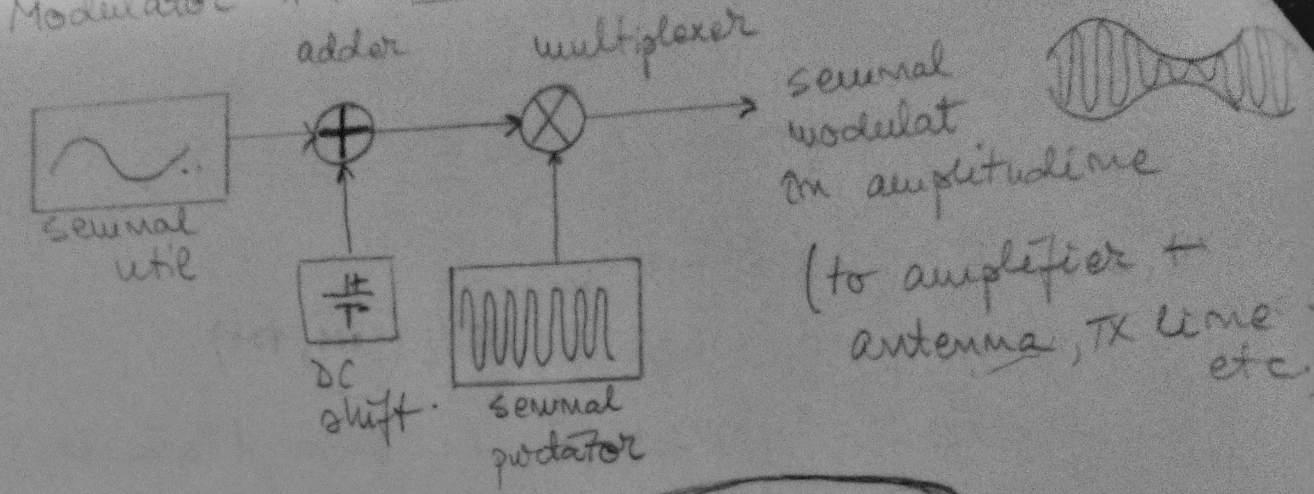
Modularea (modulația). → variația parametrilor unui semnal periodic (purtător) folosind un alt semnal (modulator, semnal util, informația de transmis).

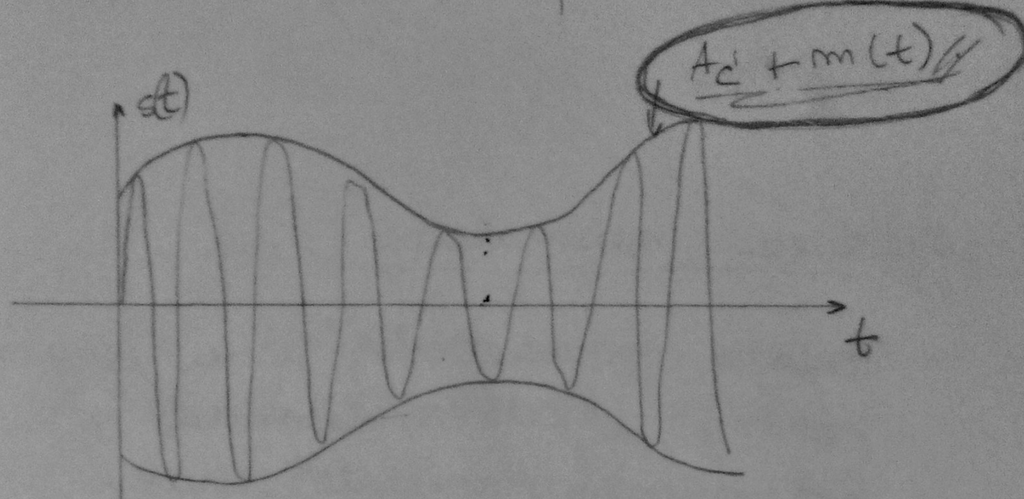
Modulare → analogică (AM, FM, PM) → amplitude modulation; FM, PM → angle modulation
Modulare → digitală

•) Modularea în amplitudine (AM)



Modulator AM - schema bloc:





Semnalul modulator este:

$$m(t) = A_m \cos(\omega_m t).$$

$(\omega = 2\pi f$, f - frecv$)$

Semnalul purtator este:

$f_c \gg f_m$.

$$c(t) = A_c \cos(\omega_c t)$$

Semnalul modulat: $\boxed{s(t) = [A_c + A_m \cos(\omega_m t)] \cos(\omega_c t)}$

$$s(t) = A_c \left[1 + \left(\frac{A_m}{A_c}\right) \cos(\omega_m t)\right] \cos(\omega_c t).$$

$$s(t) = A_c [1 + \mu \cos(\omega_m t)] \cos(\omega_c t).$$

$\boxed{\mu = \frac{A_m}{A_c}}$ - factorul de modulație (modulation index / depth).

Notăm: A_{max} - amplit. maximă a semnalului modulat

A_{min} - amplit. minimă a semnalului modulat

→ A_{max} se obține atunci când $\cos(\omega_m t) = 1$

$A_{max} = A_c + A_m$ | $A_{min} = A_c - A_m$ =>

→ $A_{min} \Leftarrow \cos(\omega_m t) = -1$

$$\Rightarrow A_{max} + A_{min} = A_c + \cancel{A_m} + A_c - \cancel{A_m} = 2A_c \Rightarrow$$

$$\Rightarrow A_c = \frac{A_{max} + A_{min}}{2}.$$

$$A_{max} - A_{min} = \cancel{A_c} + A_m - \cancel{A_c} + A_m = 2A_m$$

$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$\mu = \frac{A_m}{A_c} = \frac{\frac{A_{max} - A_{min}}{\cancel{2}}}{\frac{A_{max} + A_{min}}{\cancel{2}}}$$

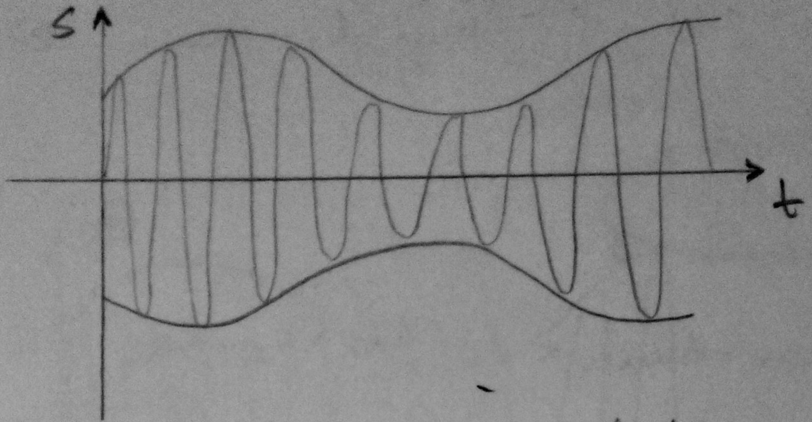
$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

Modulație perfectă $\mu = 1$.

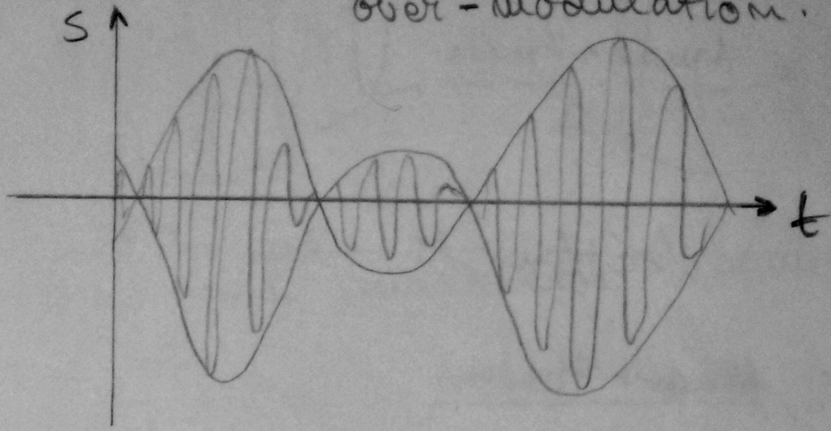
$\mu < 1 \rightarrow$ undă submodulată (undermodulated wave)

$\mu > 1 \rightarrow$ undă supramodulată (overmodulated wave).

under-modulation



over-modulation.



•) Lărgimea de bandă a semnalului AM:

$$BW = f_{max} - f_{min} \quad [Hz]$$

$$s(t) = A_c [1 + \mu \cos(\omega_m t)] \cos(\omega_c t) =$$

$$= A_c \cos(\omega_c t) + \mu A_c \cos(\omega_c t) \cos(\omega_m t)$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$\Rightarrow$

$$2 \cos a \cos b = \cos(a+b) + \cos(a-b)$$

$$\cos a \cos b = \frac{\cos(a+b) + \cos(a-b)}{2}$$

$$s(t) = A_c \cos(\omega_c t) + \frac{\mu A_c}{2}\cos[(\omega_c+\omega_m)t] + \frac{\mu A_c}{2}\cos[(\omega_c-\omega_m)t].$$

superioară   inferioară

3 frecvențe: f_c, $\underbrace{f_c+f_m}_{BLS}$; $\underbrace{f_c-f_m}_{BLi}$

(benzi laterale)

(upper sideband) USB   (lower side band) LSB.

$f_{max} = f_c + f_m$

$f_{min} = f_c - f_m$

$BW = f_{max} - f_{min}$

$\Rightarrow BW = \cancel{f_c} + f_m - \cancel{f_c} + f_m$

$$\boxed{BW = 2 f_m}$$

i) Puterea semnalului modulat AM:

$$s(t) = A_c \cos(\omega_c t) + \frac{\mu A_c}{2}\cos[(\omega_c+\omega_m)t] + \frac{\mu A_c}{2}\cos[(\omega_c-\omega_m)t]$$

$$P_T = P_c + P_{BLS} + P_{BLi}$$

$$P_{RMS} = \frac{V_{RMS}^2}{R} = \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{R}$$

$V = A\cos\omega t$

- pt. purtătoare: ($R_{antene} \simeq 50\,\Omega$).

$$P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

$$P_{BLS} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}$$

$$P_{BLi} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} = \frac{\mu^2 A_c^2}{8R}$$

$$P_T = \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$P_T = \frac{A_c^2}{R}\left(1+\frac{\mu^2}{4}+\frac{\mu^2}{4}\right) = \frac{A_c^2}{R}\left(1+\frac{\mu^2}{2}\right)$$

$$P_T = P_c\left(1+\frac{\mu^2}{2}\right)$$

Ex: $\mu = 1 \Rightarrow \underline{P_T = 1.5 P_c}$.

Exemple:

(1) $m(t) = 10\cos(2\pi\cdot 10^3 t)$ $\quad f_m = 1\,kHz$

$c(t) = 50\cos(2\pi\cdot 10^5 t)$ $\quad f_c = 100\,kHz$

Determinați μ, P_c, P_T $(R = 50\,\Omega)$

$\mu = \frac{A_m}{A_c}$

$A_m = 10V$

$A_c = 50V$

$\Rightarrow \mu = \frac{1}{5} = 0.2$

(20% modulație).

$$P_c = \frac{A_c^2}{2R} = \frac{50^{\cancel{2}}}{2\cdot\cancel{50}} = 25\,W.$$

$$P_T = P_c\left(1+\frac{\mu^2}{2}\right) = P_c\left(1+\frac{\cancel{4}^{\,2}\cdot 10^{-2}}{\cancel{2}}\right) =$$

$$= P_c(1+0.02) = 1.02\,P_c = 25.5\,W.$$

2.) Se dă $s(t) = \underbrace{20}_{A_c}[1 + \underbrace{0.8}_{\mu}\cos(2\pi \cdot 10^3 t)]\cos(4\pi \cdot 10^5 t)$.

Determinați P_c, $\underbrace{P_{BLS} + P_{BLi}}_{P_{BL}, P_{SB}}$, BW. $(R = 50\ \Omega)$.

$A_c = 20V.$ $f_m = 1\ KHz.$

$\mu = 0.8.$ $f_c = 200\ KHz.$

$$P_c = \frac{A_c^2}{2R} = \frac{20 \cdot 2\cancel{0}}{2 \cdot 5\cancel{0}} = 4W.$$

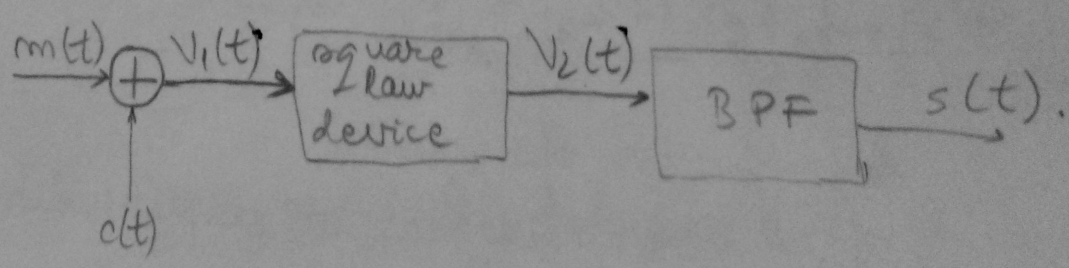
(sideband power) $P_{BL} = P_{BLi} + P_{BLS} = P_c \cdot \frac{\mu^2}{2} = \frac{4}{\cancel{2}} \cdot \cancel{8}^{4} \cdot 10^{-2} =$

$= 16 \cdot 10^{-2} = 0.16\ W.$

$BW = 2 f_m = 2 \cdot 1\ KHz = 2\ KHz.$

Modulatoare AM → square law
Modulatoare AM → în comutație (switching)

1) <u>Square law modulator</u>:



square law device ≡ dispozitiv neliniar (diodă, tranzistor etc).

- semnalul modulator $m(t)$.
- purtătoarea $c(t)=A_c\cos(\omega_c t)$.

La ieșirea sumatorului:

$$V_1(t)=m(t)+A_c\cos(\omega_c t).$$

La ieșirea dispozitivului neliniar

$$V_2(t)=k_1V_1(t)+k_2V_1^2(t).$$

k_1, k_2 - constante.

$$V_2(t)=k_1\left[m(t)+A_c\cos(\omega_c t)\right]+k_2\left[m(t)+A_c\cos(\omega_c t)\right]^2=$$

$$=k_1m(t)+k_1A_c\cos(\omega_c t)+k_2m^2(t)+k_2A_c^2\cos^2(\omega_c t)+$$

$$+2k_2m(t)A_c\cos(\omega_c t)$$

$$V_2(t)=k_1m(t)+k_2m^2(t)+k_2A_c^2\cos^2(\omega_c t)+$$

$$+k_1A_c\left[1+\frac{2k_2}{k_1}m(t)\right]\cos(\omega_c t).$$

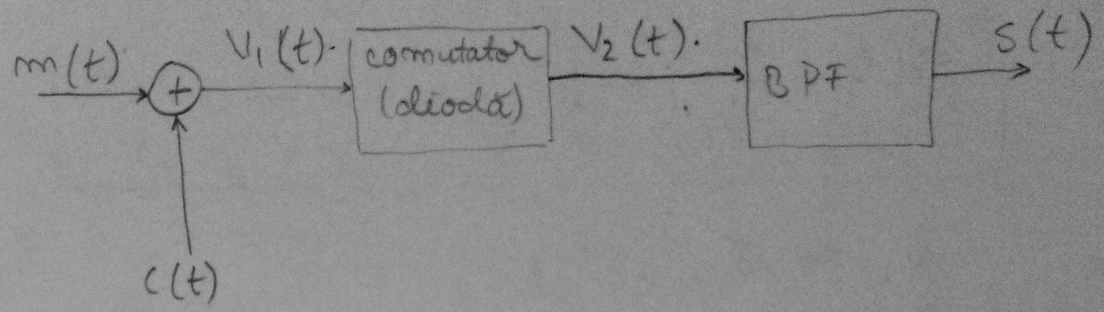
folosind BPF (FTB) păstrăm doar acest termen.

$$\boxed{s(t)=k_1A_c\left[1+k_a\cdot m(t)\right]\cos(\omega_c t).}$$

k_1 - factor de scală

k_a - sensibilitatea în amplitudine.

·) switching modulator :



- semnalul modulator $m(t)$
- semnalul purtător $c(t) = A_c \cos(\omega_c t)$.

$$V_1(t) = m(t) + c(t) = m(t) + A_c \cos(\omega_c t).$$

- presupunând că $A_m << A_c$: $\qquad (\omega_c = 2\pi f_c)$

$$V_2(t) = \begin{cases} V_1(t) & \text{dacă } c(t) > 0 \\ 0 & \text{dacă } c(t) < 0 \end{cases}$$

→ Putem scrie $\boxed{V_2(t) = V_1(t) \cdot x(t)}$, unde $x(t)$ e un tren de pulsuri periodice cu perioada $T = \frac{1}{f_c}$.

→ $x(t)$ se poate dezvolta în serie Fourier:

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n-1}}{2n-1} \cos\left[(2n-1)\omega_c t\right] \right\}.$$

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \cos(\omega_c t) - \frac{2}{3\pi} \cos(3\omega_c t) + \dots$$

$$V_2 = \left[m(t) + A_c \cos(\omega_c t)\right]\left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_c t) - \frac{2}{3\pi}\cos(3\omega_c t) + \dots\right]$$

$$= \frac{A_c}{2}\left[1 + \left(\frac{4}{\pi A_c}\right) m(t)\right] \cos(\omega_c t) + \frac{m(t)}{2} + \frac{2A_c}{\pi}\cos^2(\omega_c t) - \dots$$

Folosind BPF putem păstra numai primul termen:

$$\boxed{s(t) = \frac{A_c}{2} \cdot \left[1 + \left(\frac{4}{\pi A_c}\right) m(t)\right] \cos(\omega_c t)},$$

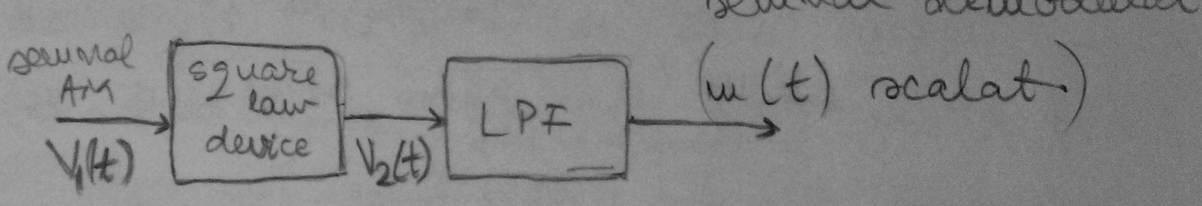
$$k_1 = 0.5$$

$$k_a = \frac{4}{\pi A_c}$$

Demodulația AM:

- square law
- envelope detector.

i) Demodulator square law:



semnalul modulat AM.

$$V_1(t) = A_c\left[1 + k_a m(t)\right]\cos(\omega_c t).$$

square law: $V_2(t) = k_1 V_1(t) + k_2 V_1^2(t)$

$$V_2(t) = k_1 A_c\left[1 + k_a m(t)\right]\cos(\omega_c t) + $$
$$+ k_2 A_c^2\left[1 + k_a m(t)\right]^2\cos^2(\omega_c t)$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1 \Rightarrow$$

$$\Rightarrow \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$\Rightarrow$

$$\Rightarrow V_2(t) = k_1 A_c\left[1 + k_a m(t)\right]\cos(\omega_c t) + k_2 A_c^2\left[1 + k_a m(t)\right]^2\left[\frac{1+\cos(2\omega_c t)}{2}\right]$$

$$V_2(t) = k_1 A_c\left[1 + k_a m(t)\right]\cos(\omega_c t) + \underbrace{\frac{k_2 A_c^2}{2}}_{\text{DC component}} + \frac{k_2 k_a^2 A_c^2 m^2(t)}{2} + \underbrace{k_2 k_a A_c^2 m(t)}_{\text{semnalul util}} + k_2 A_c^2\left[1 + k_a m(t)\right]^2\frac{\cos(2\omega_c t)}{2}.$$

- Folosind LPF:

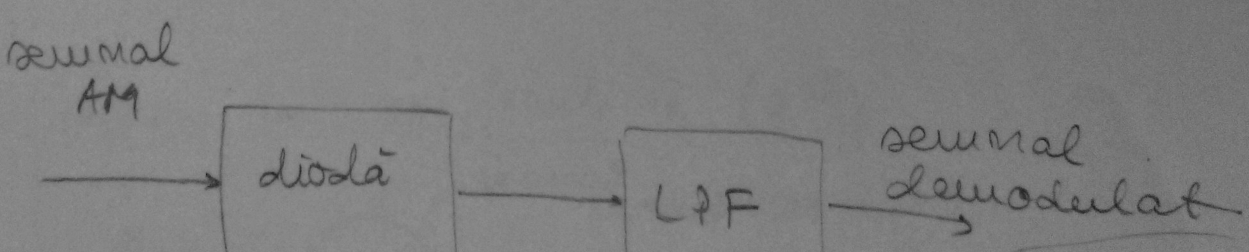
$$V_2(t) = \frac{k_2 A_c^2}{2} + k_2 k_a A_c^2 m(t).$$

- Folosind condensator de cuplaj

$$V_2(t) = k_2 k_a A_c^2 m(t)$$

↳ $m(t)$ scalat.

·) Envelope detector:



$\tau = RC$; $\frac{1}{f_c} < \tau < \frac{1}{f_m}$