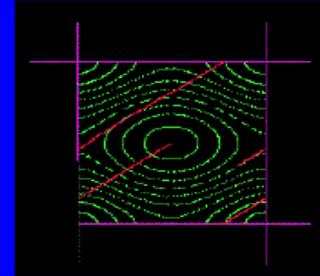
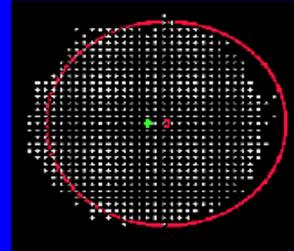
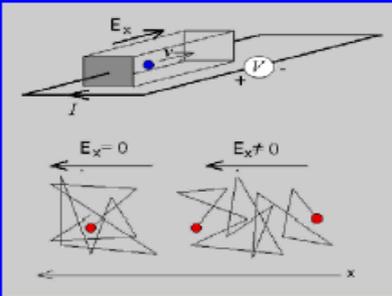


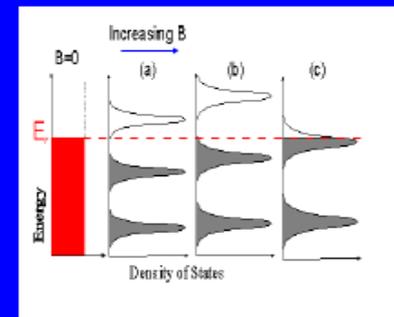
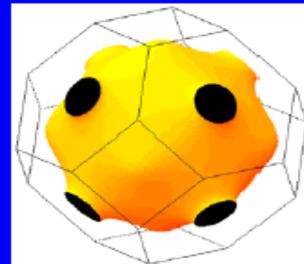
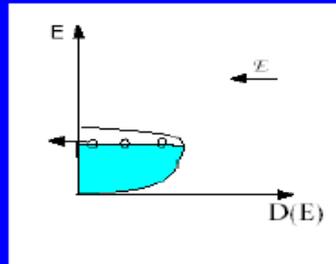
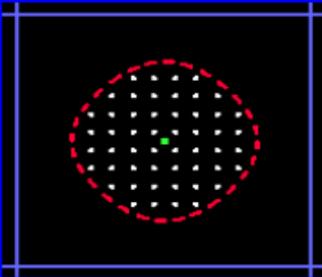
# Transport Phenomena in Solids

## Motions of electrons and transport phenomena



$$\sigma = \frac{ne^2\tau}{m}$$

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2} \sum_j \frac{\partial^2 E(\vec{k})}{\partial k_i \partial k_j}$$

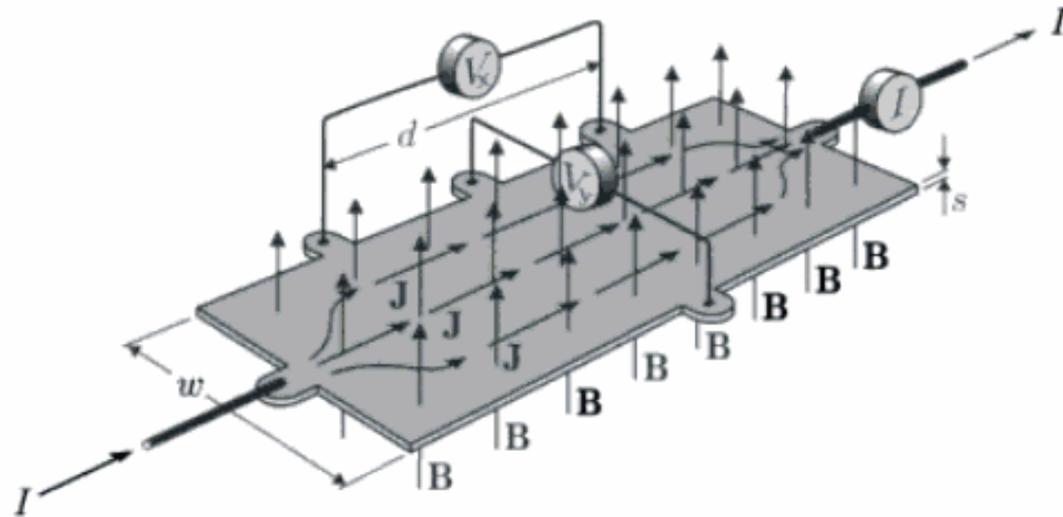


# Hall effect in magnetic metals

## The Basic Idea:

1. What happens when B fields are applied to metals?
2. What happens when those metals are magnetic?

# Non-magnetic metals

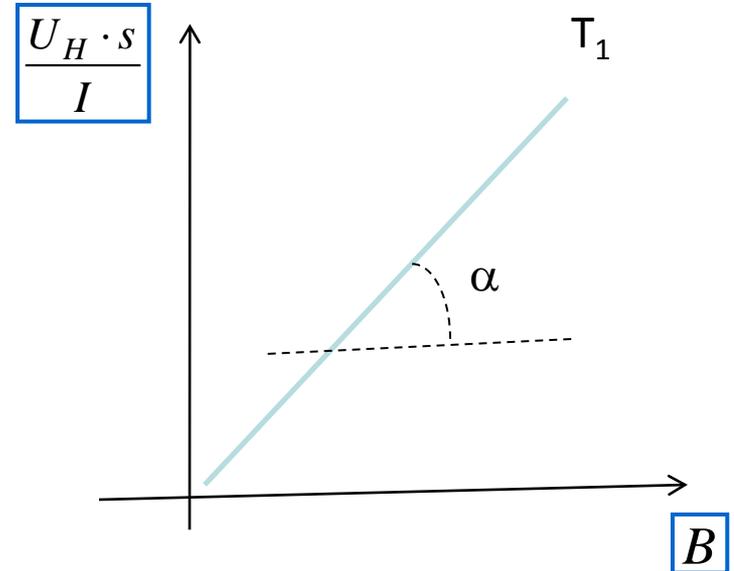


$$\rho_H = \frac{E_y}{J_x}$$

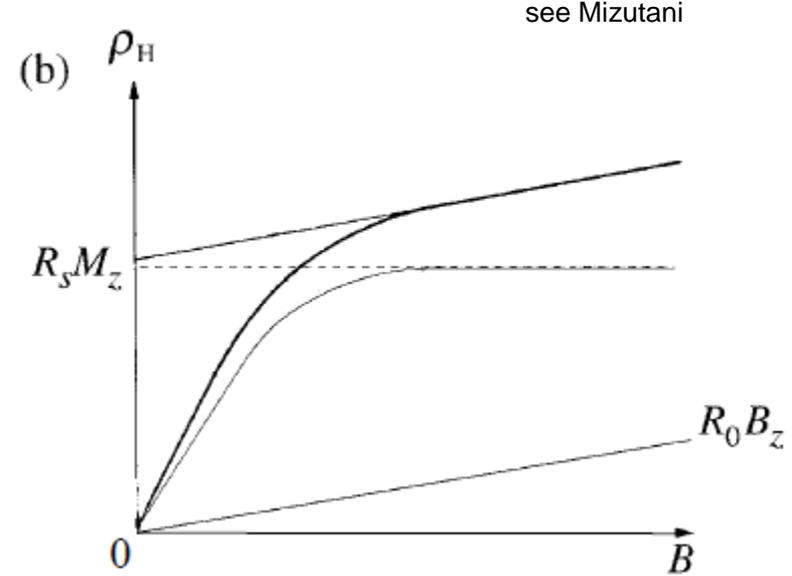
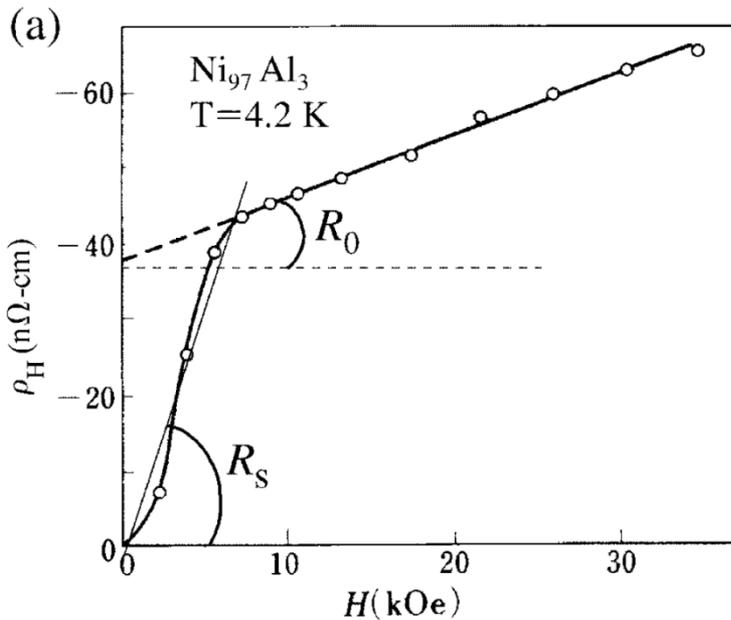
$$\mu = \frac{R_H}{\rho_x}$$

$$R_H = \frac{E_y}{j_x B} = \frac{\frac{U_H}{w}}{j_x B} = \frac{U_H}{j_x B w} = \frac{U_H}{\frac{I}{s \cdot w} B w}$$

$$\rho_H = \frac{U_H}{I} s = R_H \cdot B$$



This is, however, no longer true in magnetic metals because of the **additional contribution** arising from the **localized magnetic moment**.



see Mizutani

Magnetic field dependence of the Hall resistivity for  $\text{Ni}_{97}\text{Al}_3$  alloy at 4.2 K.

The Hall resistivity is decomposed into the normal and anomalous contributions.

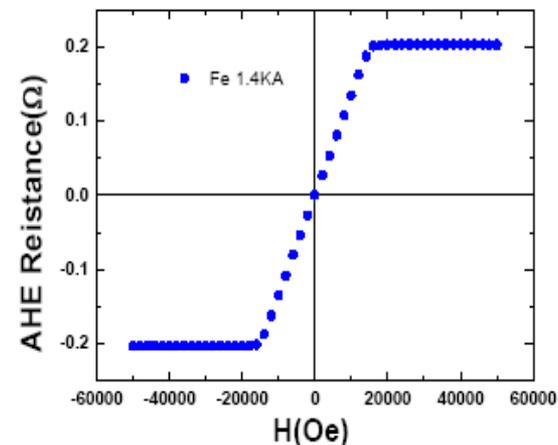
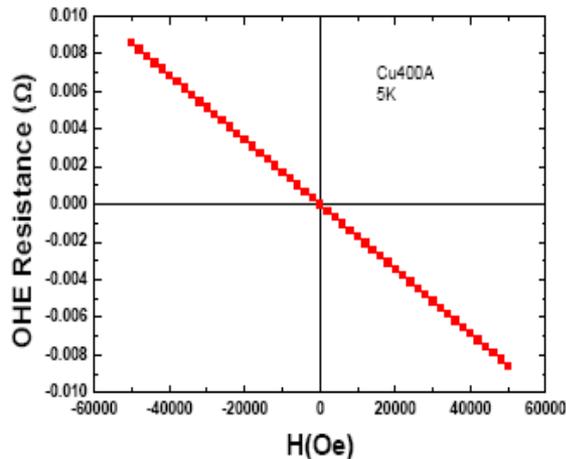
$$\rho_H = \frac{E_y}{J_x} = R_0 B_z + R_s M_z,$$

$R_s$  is called the anomalous Hall coefficient.

The second term can be present in a ferromagnetic domain even in the absence of an applied field.

The normal Hall coefficient is deduced from a slope of the  $\rho_H$ - $B$  curve at high magnetic fields while the anomalous Hall coefficient can be roughly estimated from the initial slope.

## Ordinary & Anomalous Hall Effect (AHE)



$$\rho_{xy} = \frac{1}{nq} H = R_o H$$

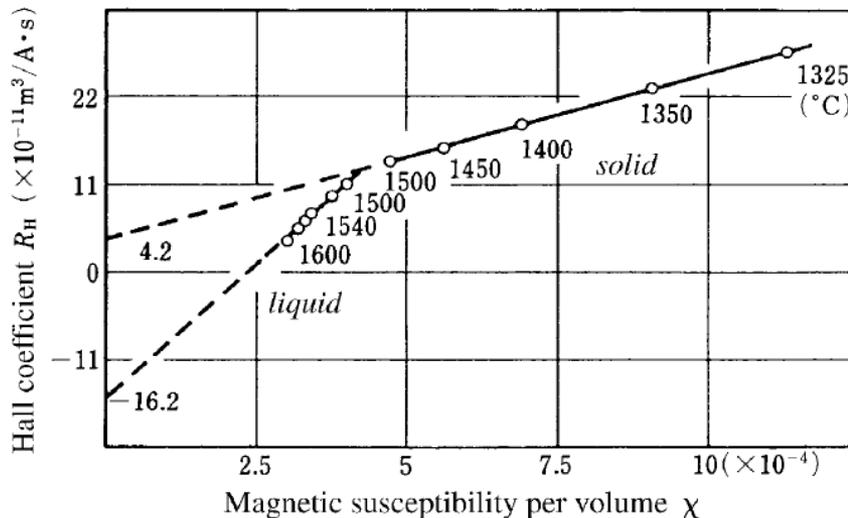
$$\rho_{xy} = R_o H + 4\pi R_S M$$

# Hall effect in the paramagnetic state, above the Curie temperature

$$M_z = \chi \mu_0 H$$

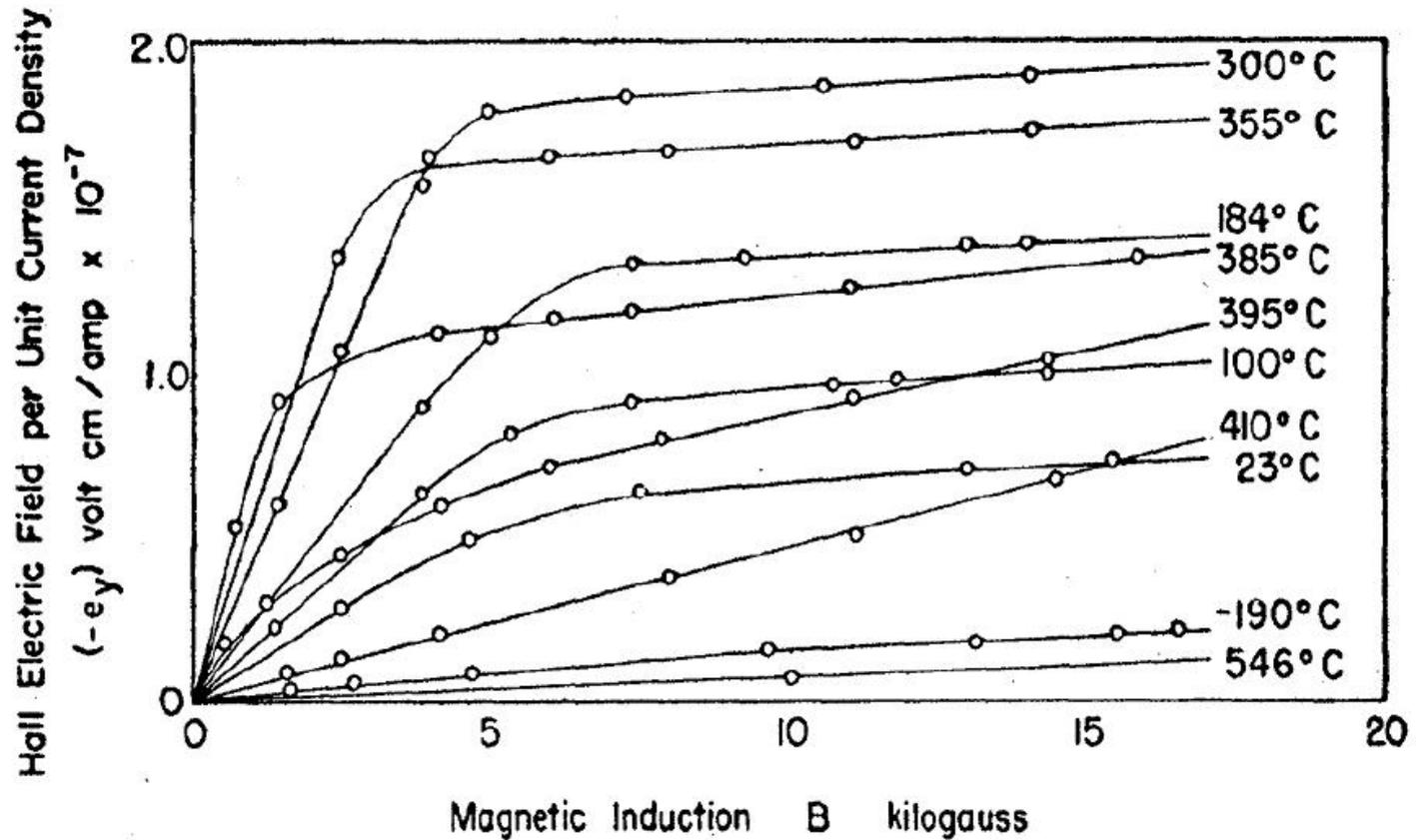
$$R_H \equiv \frac{E_y}{J_x B_z} = R_0 + \frac{\chi}{1 + \chi} R_s \approx R_0 + \chi R_s$$

$$B = \mu_0(1 + \chi)H$$



Hall coefficient versus magnetic susceptibility per volume for liquid and solid Co. In each state, the normal and anomalous Hall coefficients are derived from the intercept and slope, respectively, of the straight line drawn through the data points. [H.-J. Güntherodt *et al.*, *Liquid Metals*, (The Institute of Physics, 1977) p. 342]

from Mizutani



The Hall effect in Ni [data from A. W. Smith, Phys. Rev. 30, 1 (1910)]. [From Ref. Pugh and Rostoker, 1953.]

$R_s$  was found to depend subtly on a variety of material specific parameters and, in particular, on the longitudinal resistivity  $\rho_{xx} = \rho$ .

Early experiments to measure the relationship between  $\rho_{xy}$  and  $\rho$  generally assumed to be of the power law

$$\rho_{xy} \sim \rho^\beta$$

$$\beta \sim 2$$

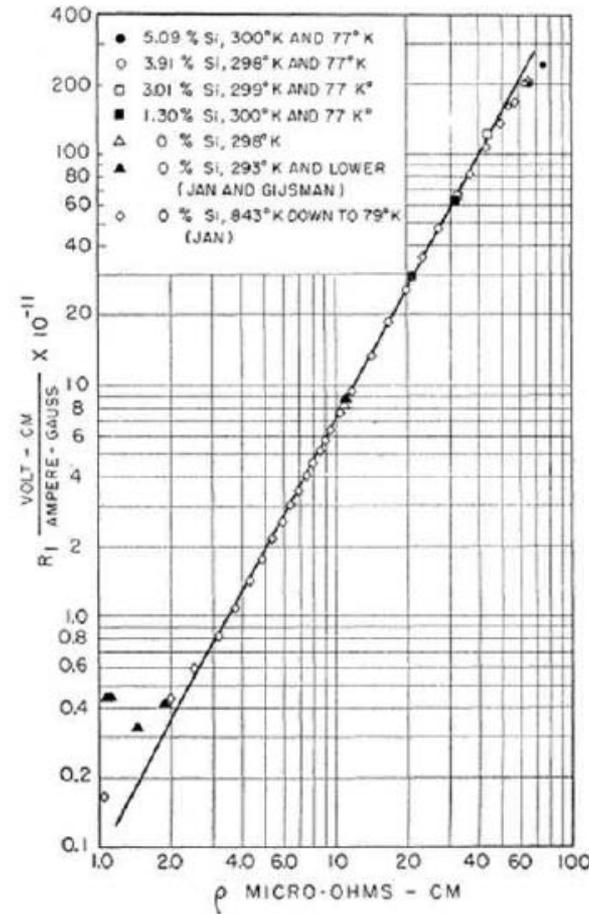


FIG. 2 Extraordinary Hall constant as a function of resistivity. The shown fit has the relation  $R_s \sim \rho^{1.9}$ . [From Ref. Kooi, 1954.]

## Anomalous Hall effect (AHE)

In conducting ferromagnets one generically observes a **Hall voltage** in the *absence* of an applied magnetic field

$$E_{\text{AH}} \sim j M$$

(**M**: magnetization) or  $E_{\text{AH}} \propto \mathbf{E} \times \mathbf{M}$

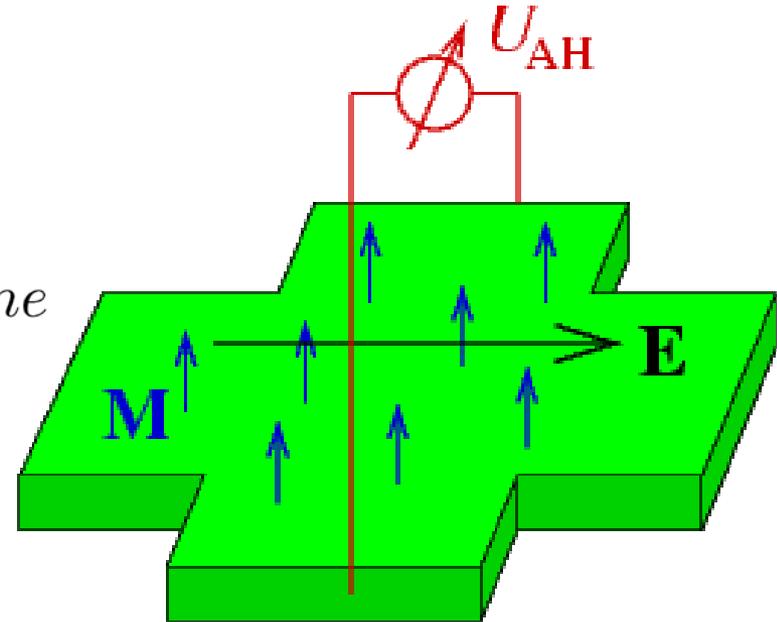
Compare normal Hall effect:  $E_H = -jB/ne$

How can the *orbital motion* feel the *spin magnetization*?

→ **Spin-orbit coupling**

Three mechanisms:

- skew scattering
- side-jump scattering
- intrinsic Berry-phase effect (no scattering)



Berry's phase is a quantum phase effect arising in systems that undergo a slow, cyclic evolution

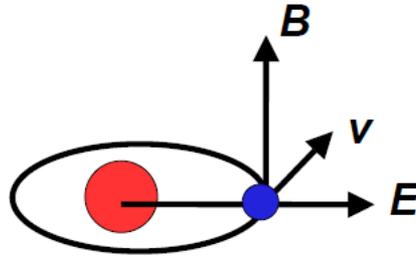
phase of the wave function

## Spin-orbit interaction

---

If an observer is moving with a velocity  $\mathbf{v}$  in an electric field  $\mathbf{E}$ , he sees a magnetic field  $\mathbf{B} = (1/c) (\mathbf{v} \times \mathbf{E})$ , where  $c$  is the speed of light.

Electron in an atom



So, the electron spin is subject to an effective magnetic field  $\mathbf{B}$  and has an energy  $+\mu B$  or  $-\mu B$ , depending on the direction of the electron spin.

- Consequences:**
- \* fine structure of atomic spectra,
  - \* values of  $g$ -factors different from 2,
  - \* spin asymmetry in scattering (Mott effect)

**Spin-orbit interaction is strongly enhanced for atoms with large  $Z$  !!**

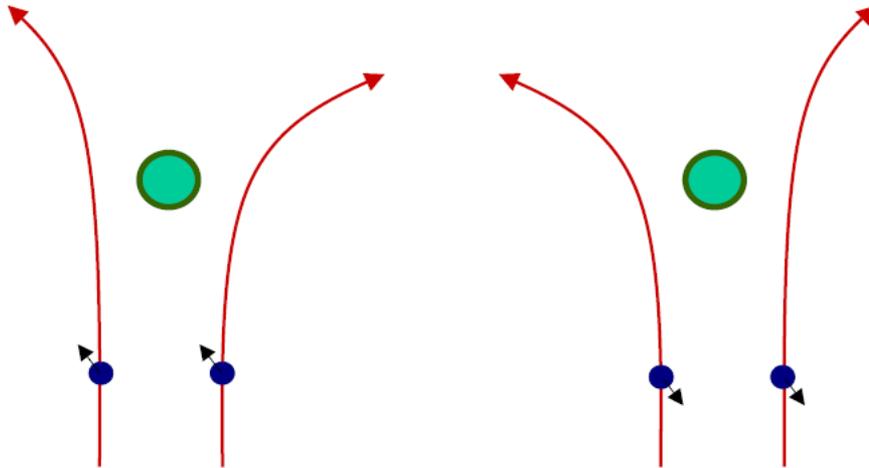
## Magnus effect and Mott scattering

---

### **Magnus effect**

A spinning tennis ball deviates from a straight path to the right or to the left, depending on the sense of rotation

### **Schematic illustration of spin-dependent asymmetry in scattering**



Skew scattering or the Mott effect (1929)



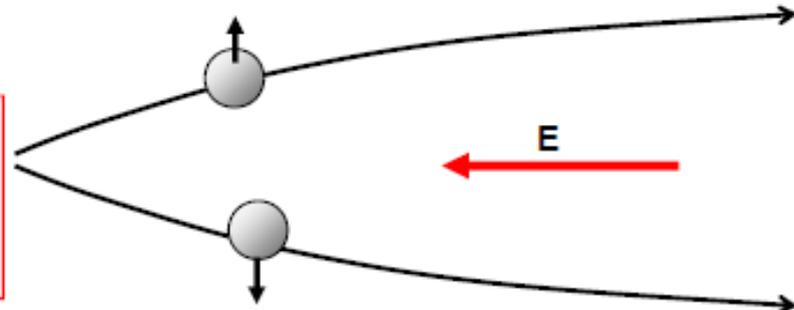
Sir Neville Mott

### a) Intrinsic deflection

Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

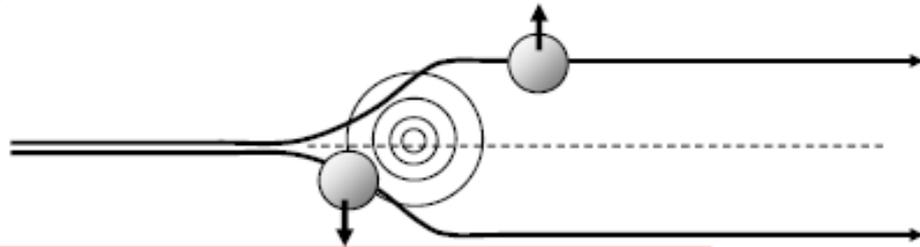
$$\frac{d\langle \vec{r} \rangle}{dt} = \frac{\partial E}{\hbar \partial \vec{k}} + \frac{e}{\hbar} \vec{E} \times \vec{b}_n$$

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature



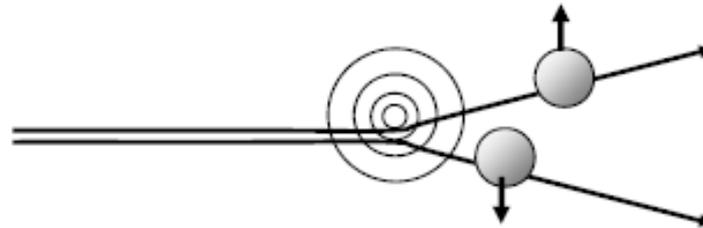
### b) Side jump

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.



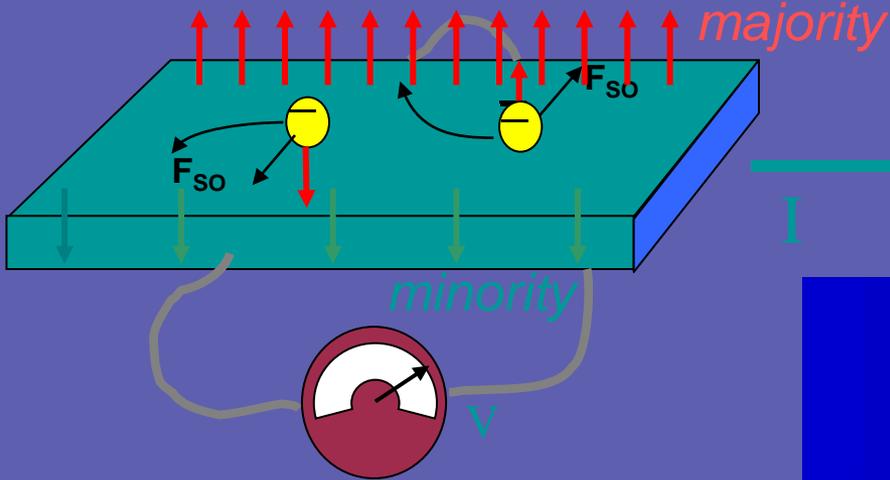
### c) Skew scattering

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.



# Anomalous Hall effect

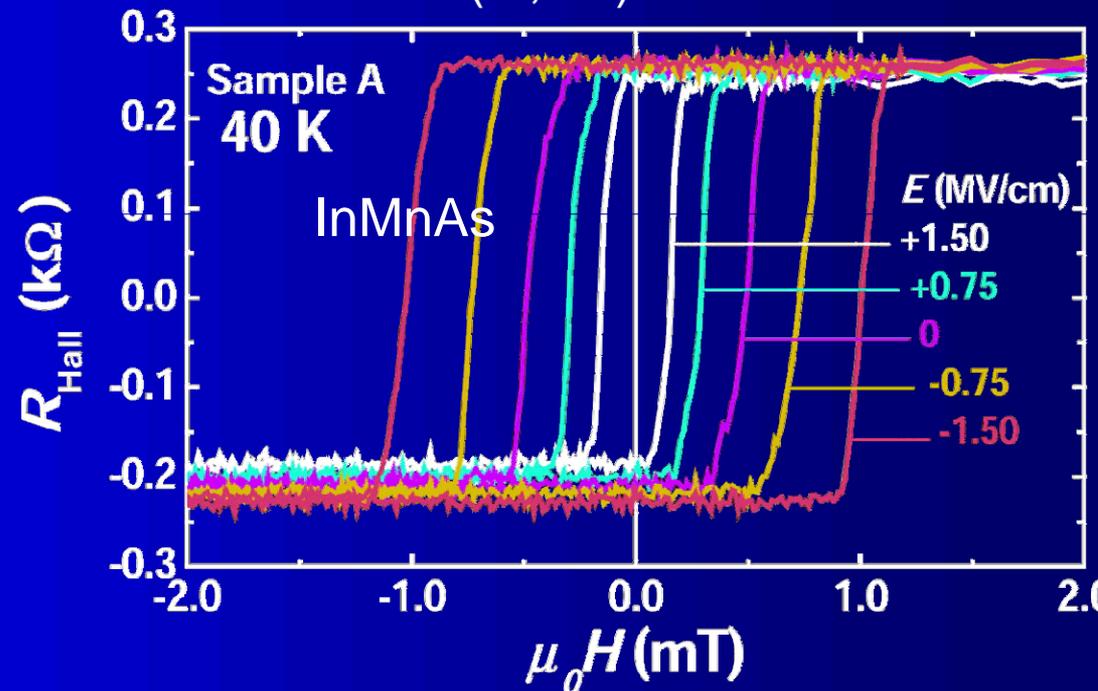
Spin-orbit coupling “force” deflects *like-spin* particles



$$\rho_H = R_0 B + 4\pi R_s M$$

Simple electrical measurement of magnetization

combined effect of exchange interaction and spin orbit coupling



D. Chiba *et al.*, Science **301**, 943 (2003)

- **Intrinsic**

Karplus and Luttinger: spin-orbit coupling in Bloch bands

Berry phase effects on Bloch electrons

- **Extrinsic:**

Imperfection scattering of electrons in  
ferromagnetic materials

1) Skew Scattering

2) Side jump

asymmetric (skew) scattering from impurities caused by the spin-orbit interaction (SOI) (Smit, 1955, 1958).

side-jump experienced by quasiparticles upon scattering from spin-orbit coupled impurities (Berger, 1970).

## AHE and longitudinal resistivity

Same origin: electron scattering by imperfections

$$R_S = a\rho_{xx} + b\rho_{xx}^2$$

Skew Scattering

Side jump

$$R_s \propto \rho_{xx}^n$$

- If  $n \approx 1$ , Skew Scattering
- If  $n \approx 2$ , Side jump

**If  $1 < n < 2$ , both contribute**

# summary

In ferromagnetic materials (and paramagnetic materials in a magnetic field), the Hall resistivity includes an additional contribution, known as the **anomalous Hall effect** (or the **extraordinary Hall effect**), which depends directly on the magnetization of the material, and is often much larger than the ordinary Hall effect. (Note that this effect is *not* due to the contribution of the magnetization to the total magnetic field.) Although a well-recognized phenomenon, there is still debate about its origins in the various materials. The anomalous Hall effect can be either an *extrinsic* (disorder-related) effect due to spin-dependent scattering of the charge carriers, or an *intrinsic* effect which can be described in terms of the Berry phase effect in the crystal momentum space ( $k$ -space).

- at its core, the AHE problem involves concepts based on topology and geometry that have been formulated only in recent times.

- In 1954, Karplus and Luttinger showed that when an external electric field is applied to a solid, electrons acquire an additional contribution to their group velocity. KL's *anomalous velocity* was perpendicular to the electric field and therefore could contribute to Hall effects.

Because this contribution depends only on the band structure and is largely independent of scattering, it has recently been referred to as the *intrinsic contribution* to the AHE.

Berry phase

# Anomalous Hall Effect

In a ferromagnet  $\vec{B} = \vec{H} + 4\pi\vec{M}$

**M** couples to **j** due to spin-orbit interaction.

Equations:

**B** → **H** (Ordinary)

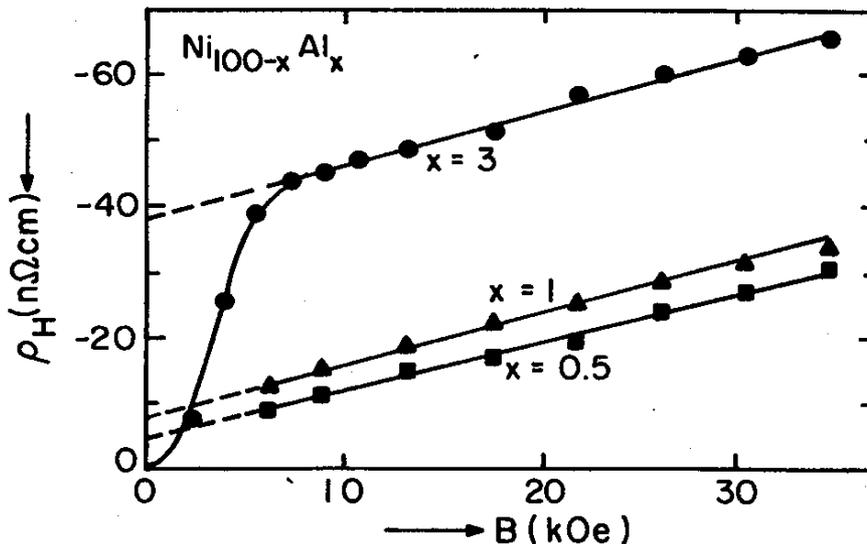
**B** → **M** (Anomalous)

# Anomalous Hall Effect

$$\text{Hall Resistivity: } \rho_H = \rho_{oH}(H) + \rho_{aH}(M)$$

Two types of scattering processes:

1. *Skew or s-d scattering (M)*
2. *Side-jump scattering (impurities)*



High-field slope due to OHE

High-field extrapolation to zero is  
AH resistivity

# Magnetoresistance in magnetic metals

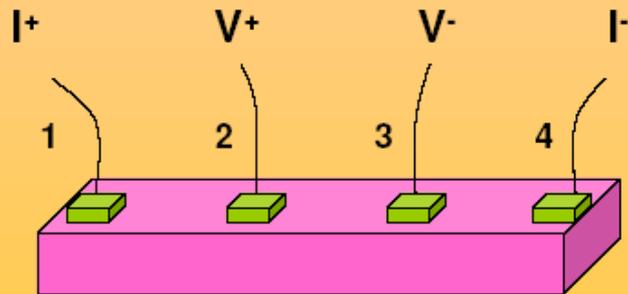
# GEOMETRIES FOR THE MEASUREMENT OF RESISTANCE

Bulk samples are normally measured in **bar-shaped geometry** and **four-point linear contacts**. Resistivity can be determined.

J.M. de Teresa,  
Universidad de Zaragoza,  
Spain, ESM  
2005 Constanta

$$\rho = F \frac{V_{2,3}}{I_{1,4}} \frac{S}{d}$$

(F can be approximated to 1 in most of the situations)



Typical size  
is millimetric

$\rho$  (ohm x cm)

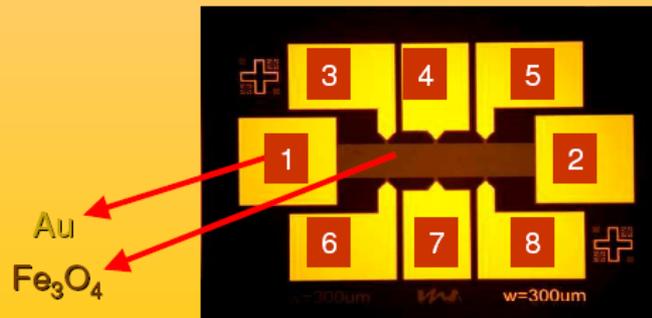
Relation between conductivity  
and resistivity  $\sigma=1/\rho$  (Siemens)

\*Four-contact measurements eliminate the contact and lead resistances. One should be careful regarding offset signals such as thermoelectric effects, electronic offsets, electromotive forces, which can be minimised by current inversion in d.c. measurements or using a.c. measurements:

Design for R, MR and Hall effect  
measurements of a thin film

MR: I(1,2); V(3,5)

Hall: I(1,2); V(4,7)



# Ordinary Magnetoresistance

Experimental observations:

$$R \propto B^2 \quad \text{For small fields}$$

$$R \propto B \quad \text{For very high fields}$$

Always an increase in resistance

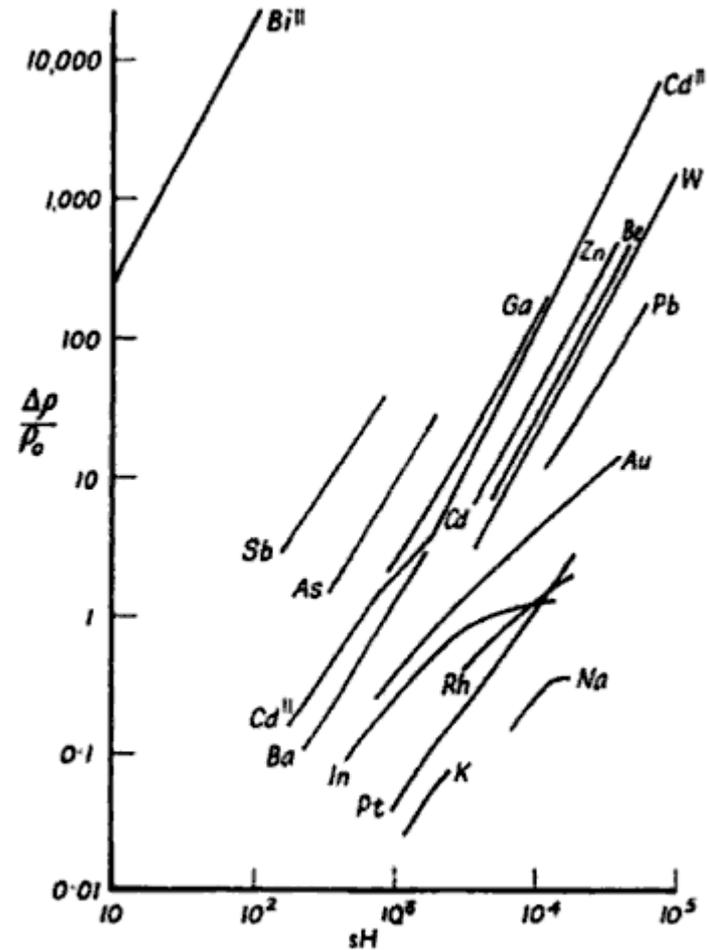


FIG. 158. Reduced Kohler diagram (Kohler, 1949 b).

# Ordinary Magnetoresistance

Why increase resistance?

Electrons orbit around B field until scattered, thus  
longer relaxation time = larger change in resistance

$$\frac{\Delta\rho}{\rho} = \frac{R(B) - R(0)}{R(0)} = f(B/\rho) \quad \text{Kohler's Rule}$$

What does this mean physically?

# times electron goes around orbit is proportional to  
mean free path divided by cyclotron radius

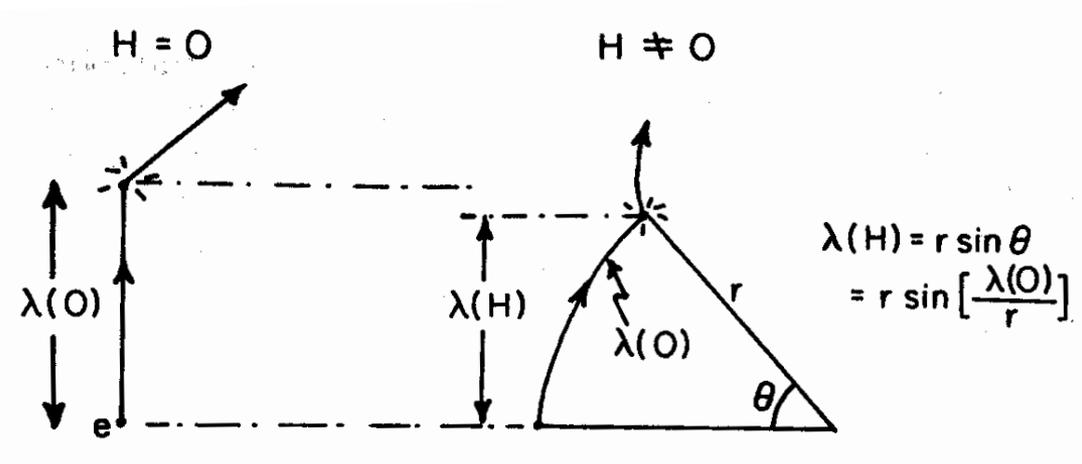
$$\lambda = \frac{mv}{e^2 n \rho} \quad r = \frac{mv}{eB} \quad \Rightarrow \quad \frac{\lambda}{r} \propto \frac{B}{\rho}$$

# Ordinary Magnetoresistance

## More on Kohler's Rule

Deflecting charges left or right gives an increase in resistance, thus

$$\frac{\Delta\rho}{\rho} \propto \left(\frac{B}{\rho}\right)^2 \quad \text{lowest order}$$



Taylor expand (weak field)

$$\lambda(H) \approx \lambda(0) \left[ 1 - \frac{\lambda(0)^2}{6r^2} \right]$$

$$\frac{\Delta\rho}{\rho} = a \left(\frac{B}{\rho}\right)^2$$

## RESISTIVITY OF NON-MAGNETIC METALS

$$\rho(T) = \rho_0 + \rho_P(T) + \rho_m(B, T)$$

(Matthiessen's rule)

caused by defects

caused by phonons

caused by Magnetism

### LORENTZ MR

$$\vec{E}_1 = \rho_{\perp}(B)\vec{J}$$

-DUE TO THE CURVING OF THE CARRIER TRAJECTORY BY THE LORENTZ FORCE (  $q\vec{v} \times \vec{B}$  )

-VERY SMALL IN MOST METALS EXCEPT AT LOW TEMPERATURES OR FOR CERTAIN ELEMENTS

-IT FOLLOWS THE DEPENDENCE  $\Delta\rho/\rho=f(B/\rho_0)$  (Kohler's RULE) AND AT LOW FIELDS  $\frac{\Delta\rho}{\rho} = \left(\frac{1}{\rho}\right)\left(\frac{1}{ne}\right)B^2$

**$\Rightarrow$  The fundamental quantity for LMR is  $\omega_c\tau$ , the mean angle turned along the helical path between collisions, where  $\omega_c$  is the cyclotron frequency ( $\omega_c=eB/m^*c$ )**

# Resistivity of a ferromagnetic metal :

$$\rho(T) = \rho_i + \rho_{\text{ph}}(T) + \rho_{e-e}(T) + \rho_m(B, T)$$

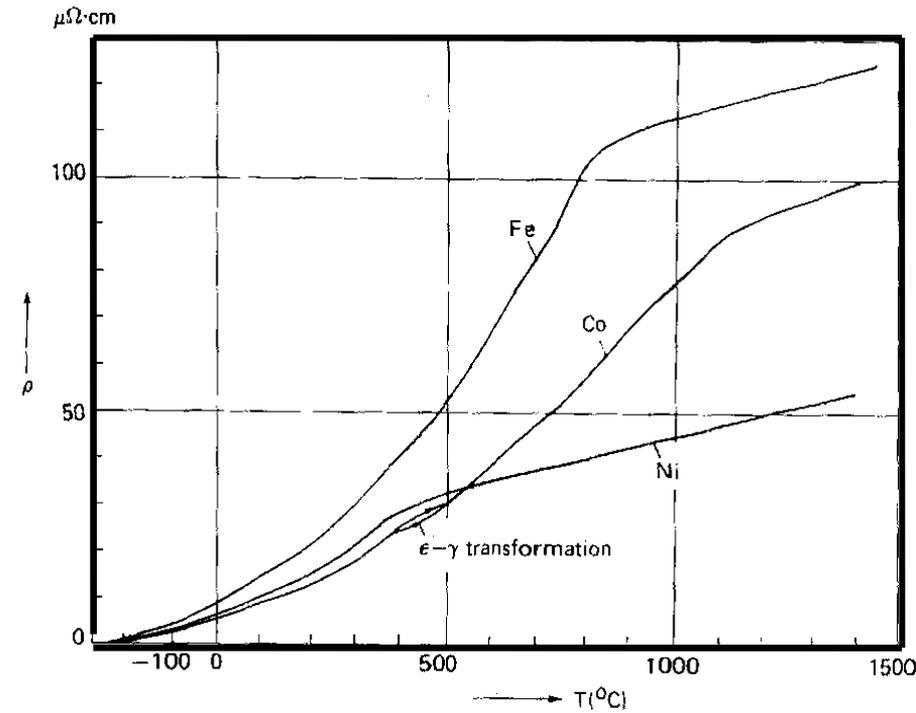
Impurities  
= constant

Phonons  
in  $T^5$  then  
 $T$

Interactions  
 $e^-e^-$ ,  $T^2$  at  
LT

Magnons  
in  $T^2$

Experimental measurements  
in 3d metals :



## Temperature dependence of Resistivity in Metals

$$\rho(T) = \rho_0 + \alpha T$$

- Anomaly for ferromagnet near a magnetic transition
- In paramagnetic state scattering is from
  - Disorder in spin system
  - Lattice vibrations

$$\rho(T) \propto \begin{cases} \text{constant} & \text{lowest } T \\ T^5 & \text{if } T \ll \theta_D \\ T^3 & \text{if } T < \theta_D \\ T & \text{if } T \gg \theta_D \end{cases}$$

Resistivities of Ni and Pd normalized to their values at  $T_c$  of Ni, 631K, versus temperature.

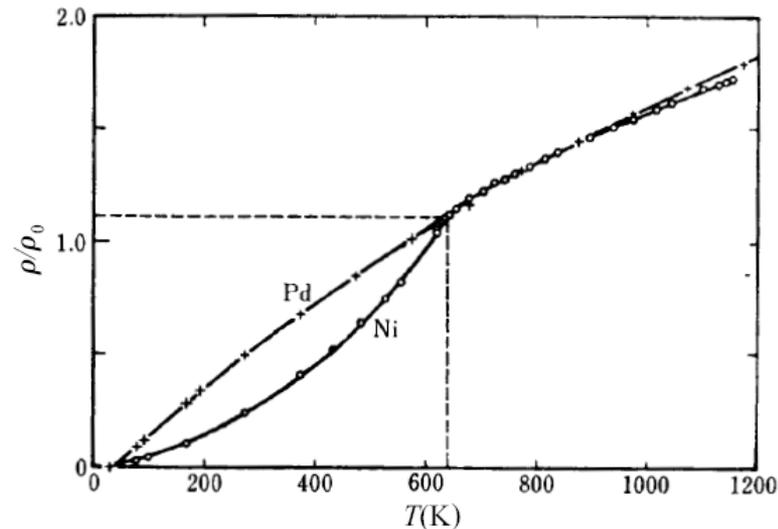


Figure 13.9. Temperature dependence of the normalized electrical resistivity in Ni and Pd. The data are shown such that both sets of data coincide with each other at the Curie point of pure Ni. [J. M. Ziman, *Electrons and Phonons* (Clarendon Press, Oxford 1962) p. 380]

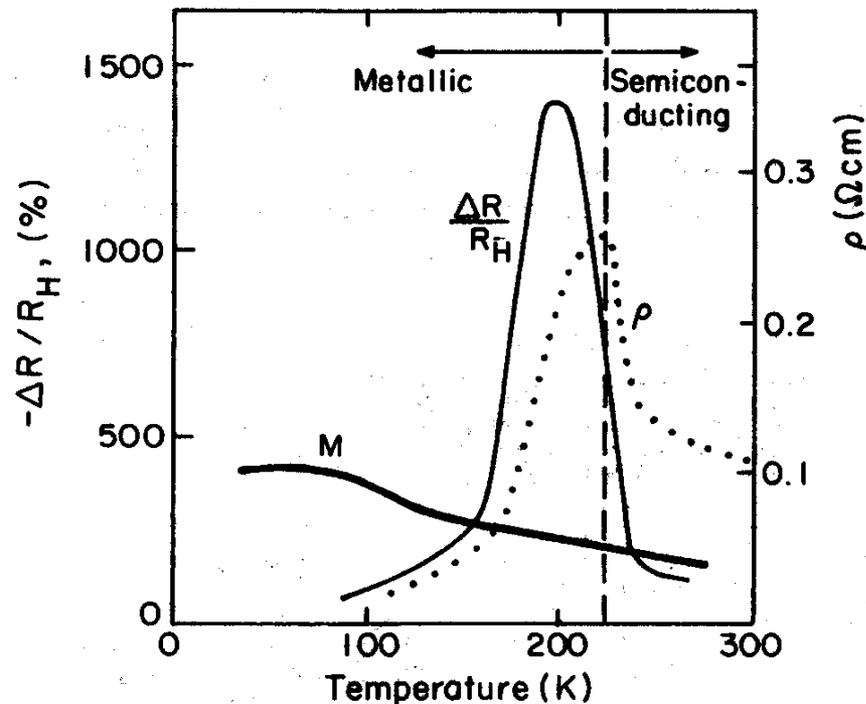
## Temperature dependence of Resistivity in Oxides

Jin, S. et al. *Science* **264**, 413 (1994).

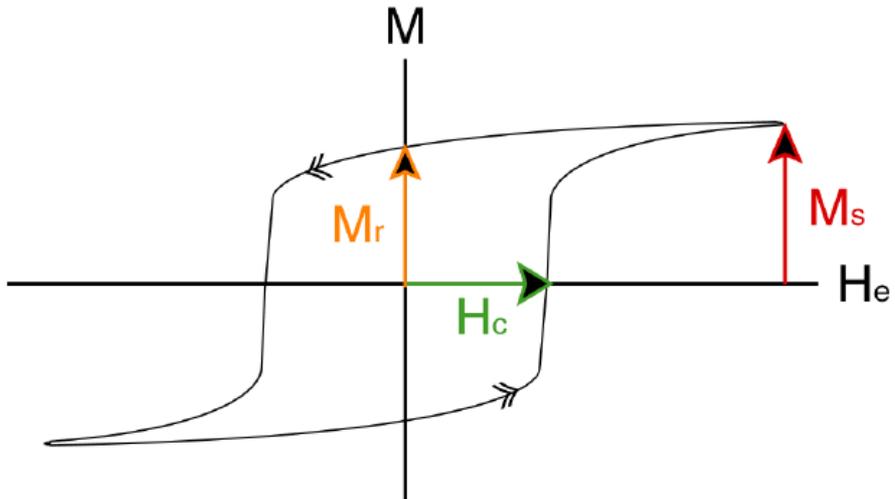
$$\sigma = \sigma_0 \exp[-(2E_g)/(k_B T)]$$

Resistivity peaks at metal-insulator transition but the field induced resistance change  $\Delta R/R(H)$ , peaks about 25° below this transition.

Variation of MR, resistivity, and magnetization in  $(La_{2/3}Ca_{1/3})MnO_3$  films.

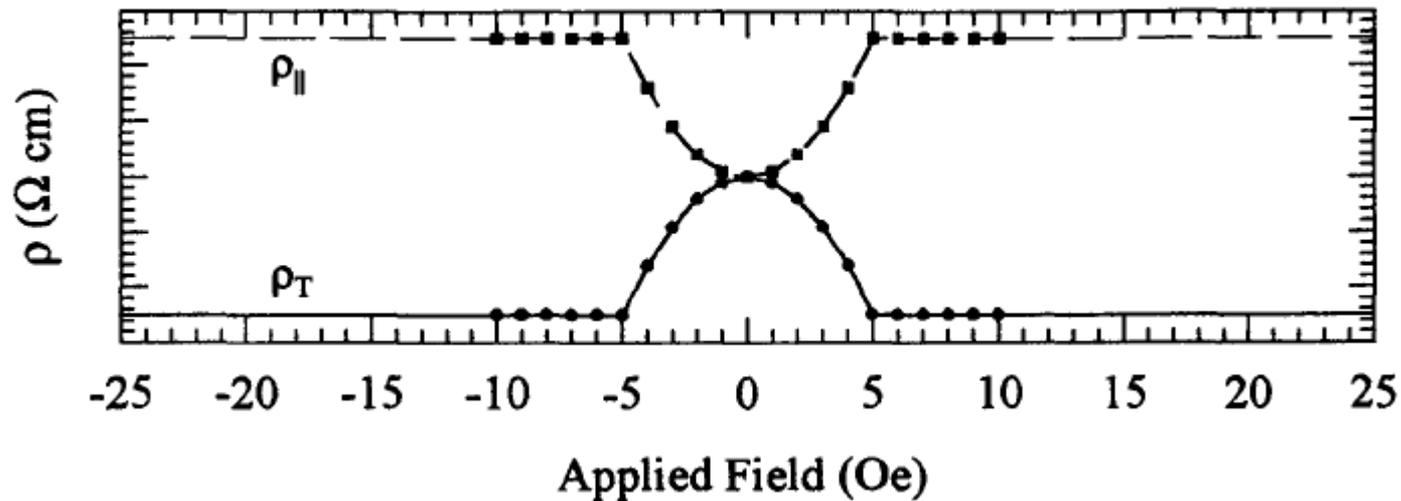


# Anisotropic Magnetoresistance



1.  $R(M(T))$  - Resistance changes due to indirect manipulation of magnetization through thermal changes
2.  $R(M)$  - Resistance changes due to direct manipulation of magnetization
3.  $R(\theta_{M,I})$  - Resistance changes due to the angle between magnetization and current

a dependence of electrical resistance on the angle between the direction of electric current and orientation of magnetic field



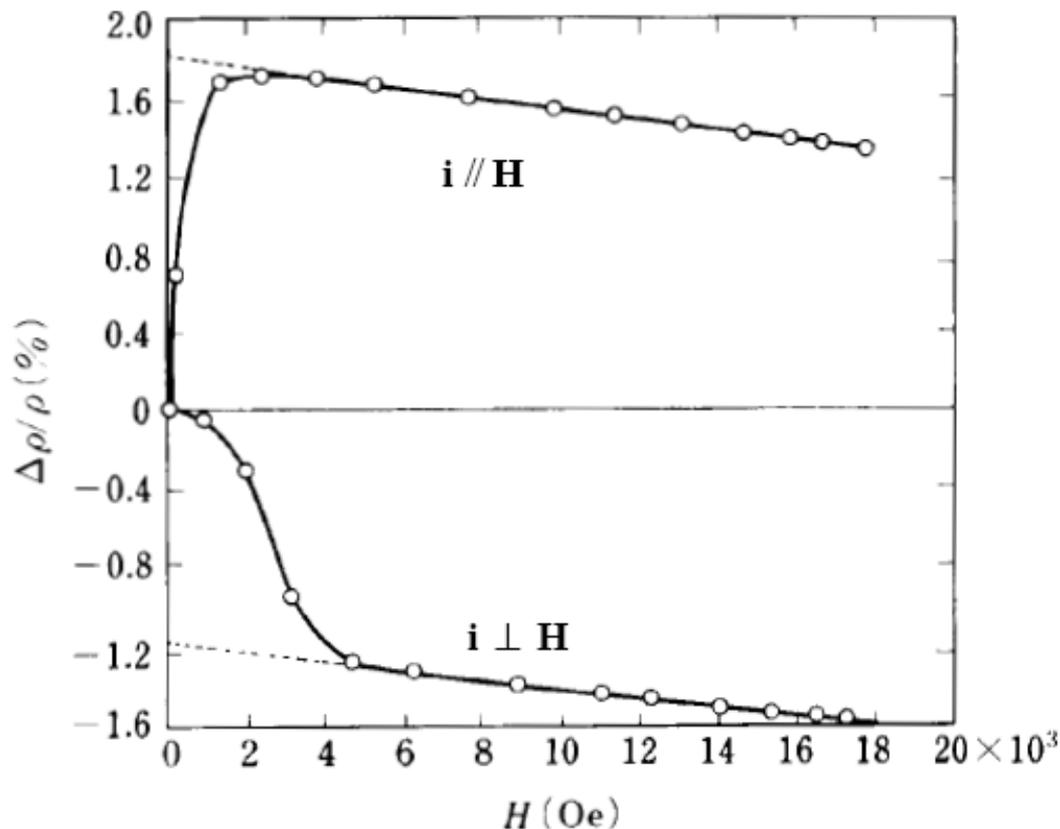
**Schematic representation of anisotropic magnetoresistance in permalloy for field applied parallel ( $\rho_{||}$ ) and transverse ( $\rho_T$ ) to the current direction.**

Magnetoresistance Overview Janice Nickel Computer Peripherals  
Laboratory HPL-95-60 June, 1995

**Permalloy** is a [nickel-iron](#) magnetic [alloy](#), with about 20% iron and 80% nickel content. It is notable for its very high [magnetic permeability](#), which makes it useful as a [magnetic core](#) material in electrical and electronic equipment, and also in [magnetic shielding](#) to block [magnetic fields](#). Commercial permalloy alloys typically have [relative permeability](#) of around 100,000, compared to several thousand for ordinary steel.

Fe–Co and Fe–Ni ferromagnetic alloys are used as magnetic field sensors because they possess a large magnetoresistance.

The magnetoresistance can be longitudinal or transverse, depending on whether the magnetic field is applied in a direction parallel to or perpendicular to the direction of the electrical current, respectively.



An initial large change in resistivity is accompanied by growth of magnetic domains parallel to the direction of the magnetic field.

Once it is saturated, the resistivity changes more or less linearly with increasing magnetic field.

Longitudinal and transverse magnetoresistance of pure Ni.  
[E. Englert, *Ann. Phys.* **14** (1932) 589]

Now the anisotropy in magnetoresistance is defined as

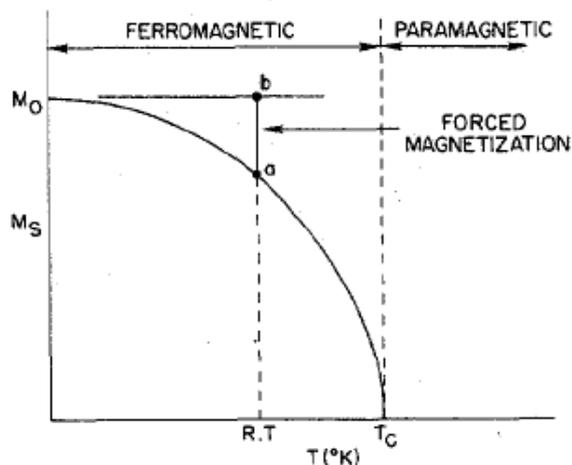
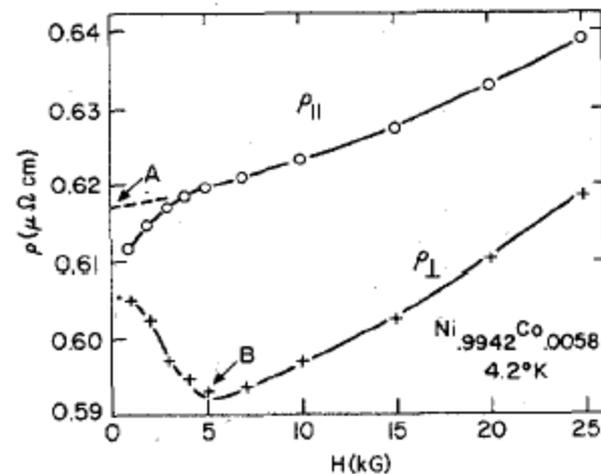
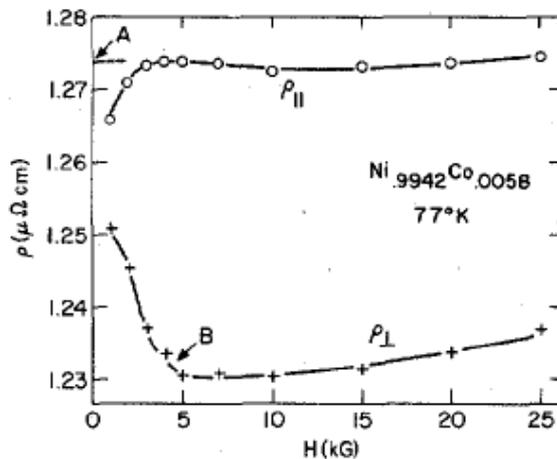
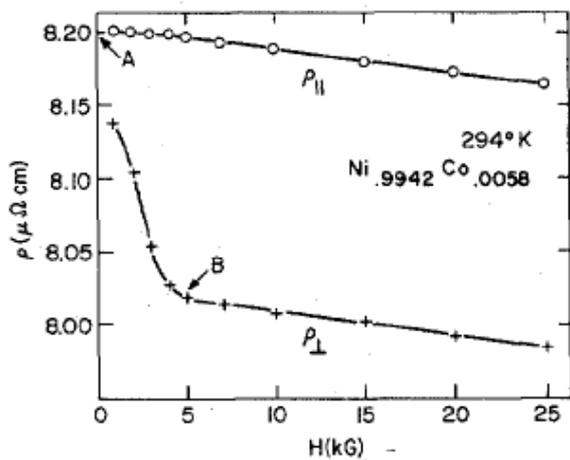
$$\frac{\Delta\rho}{\rho_{||}} = (\rho_{||} - \rho_{\perp})/\rho_{||}$$

ferromagnetic anisotropy of resistivity → FAR

- The largest value of FAR at room temperature so far reported in the literature is 6.5% for the Ni<sub>70</sub>Co<sub>30</sub> alloy.
- exceeds 10% at the liquid nitrogen temperature of 77K.

# Anisotropic Magnetoresistance

## Experimental Observations

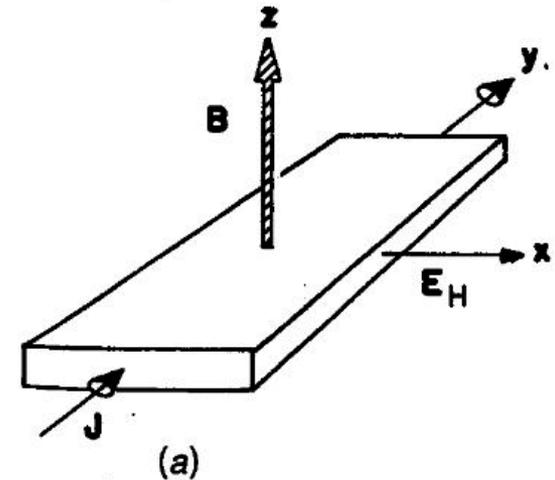


# AMR

Using Ohm's Law, we can get results that match observation in many systems:

$$\vec{E} = \rho \vec{j}$$

$$\rho = \begin{pmatrix} \rho_{\perp}(H) & -\rho_H(H) & 0 \\ \rho_H(H) & \rho_{\perp}(H) & 0 \\ 0 & 0 & \rho_{\parallel}(H) \end{pmatrix}$$



Then rotate field to y direction to measure the resistivity, and subtract:  $\Delta\rho = \rho_{\parallel} - \rho_{\perp}$

## Ohm's Law

$$\vec{E} = \hat{H} \left( \vec{j} \cdot \hat{H} \right) \Delta\rho + \rho_{\perp} \vec{j} + \rho_H \hat{H} \times \vec{j}$$

Since resistivity is measured along current direction  $\rho = \frac{\vec{E} \cdot \vec{j}}{|\vec{j}|^2}$

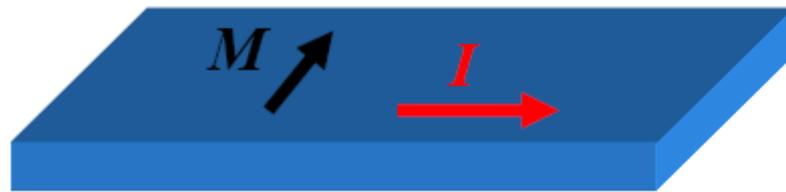
Plug in E

$$\Rightarrow \rho = \rho_{\perp} + \Delta\rho \cos^2(\theta)$$

Define:  $\rho_{avg} \equiv \frac{1}{3} \rho_{\parallel} + \frac{2}{3} \rho_{\perp} \approx \rho_o$       Then  $\frac{\rho(H) - \rho_{avg}}{\rho_{avg}} = \frac{\Delta\rho}{\rho_{avg}} \left( \cos^2(\theta) - \frac{1}{3} \right)$

This result matches observations in many systems

# Anisotropic Magneto-resistance (AMR)

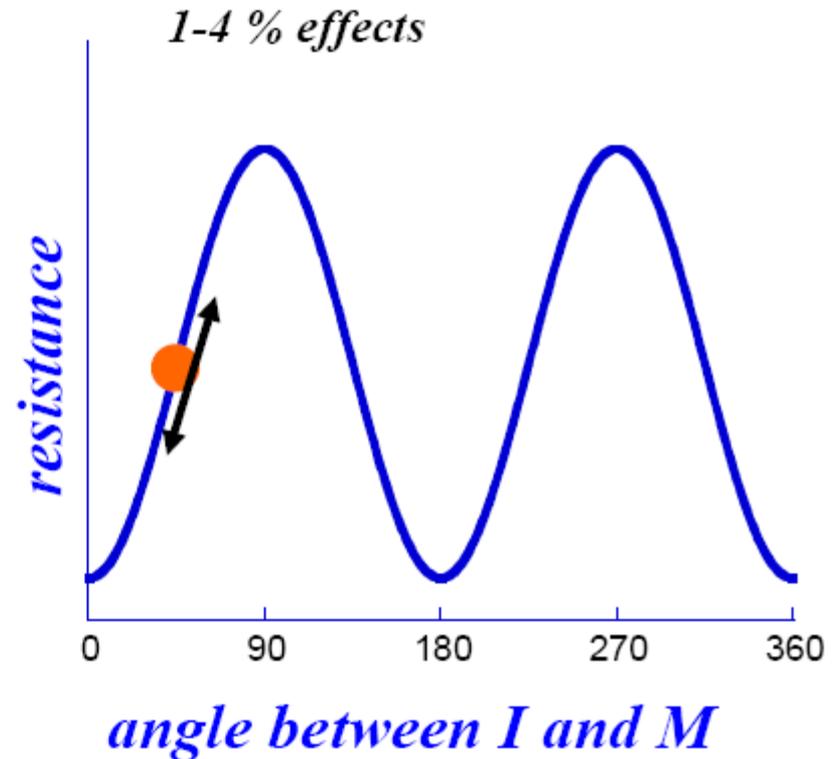


*high resistance*



*low resistance*

*Bulk property of magnetic materials*



$$R = R_0 + \Delta R \cos^2 \theta_{i,M}$$

# What's actually going on in there...?

*Things to consider:*

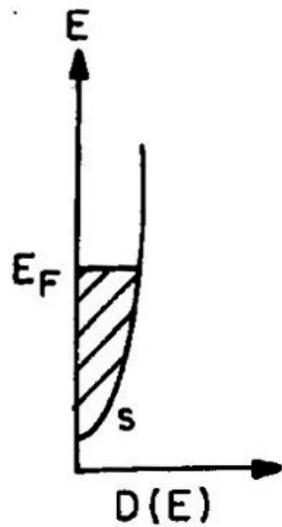
How are the different spins behaving in the FM?

What is the main scattering mechanism?

How does the magnetization affect the current?

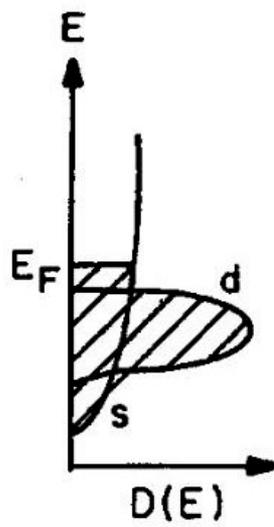
Alkali metals

Na  
Cs



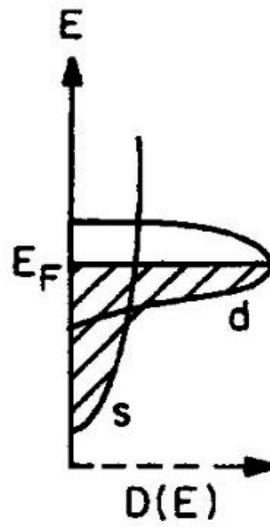
Noble metals

Cu  
Ag



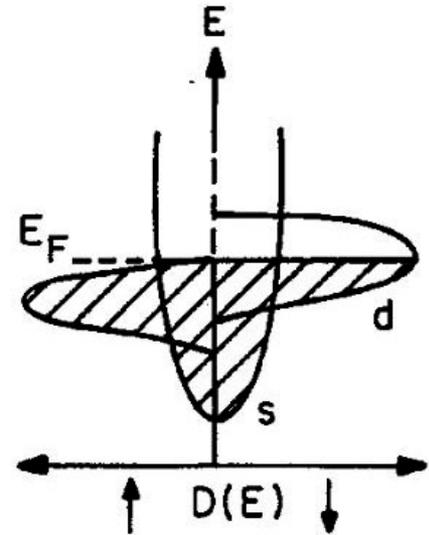
Nonferromagnetic

V  
Zr



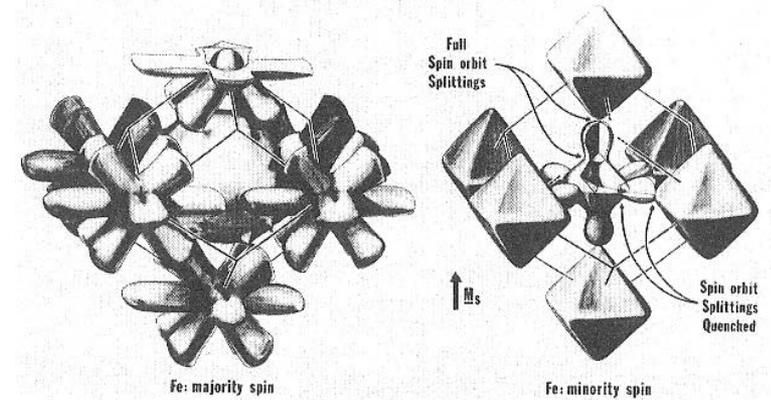
Ferromagnetic transition metals

Fe  
Ni

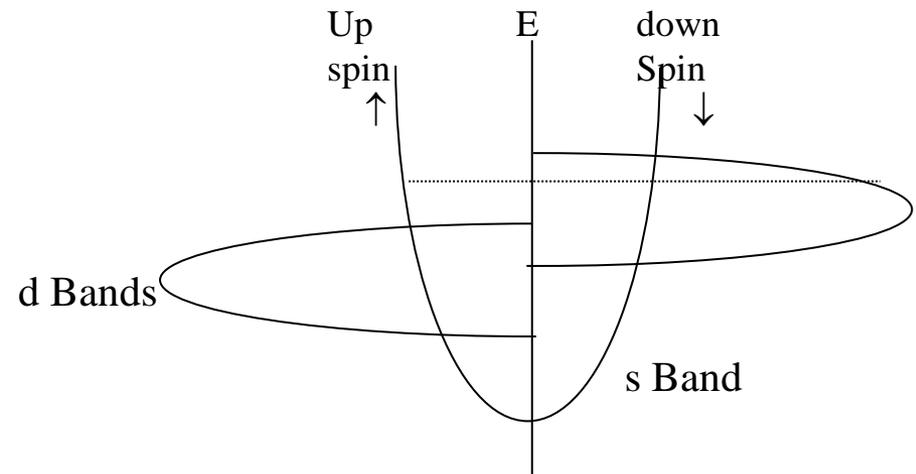


# Spin dependent electrical transport in ferromagnetic metals

Different Fermi surface for up and down spins :



Schematical DOS :



- s electrons : low density of states + high mobility
- d electrons : large density of states + low mobility
- Transport is dominated by s electrons scattered into d bands
- d bands split by the exchange energy
- diffusion is spin dependent
- Two current model

# Scattering in a FM

## Assumptions:

- current carried mostly by s electrons (d electrons have much higher effective masses)

$$\sigma \propto \frac{1}{m_{eff}}$$

- s-d scattering more likely than s-s scattering because

$$D_d(\epsilon_F) \gg D_s(\epsilon_F)$$

# Transition Rates

Drude

Relaxation time:  $\tau = \frac{1}{R}$       remember       $\rho = \frac{m}{ne^2\tau}$

where  $R$  is the transition rate between two states

From quantum mechanics:

$$R \propto D_{final}(\epsilon_F) |f|^2 \quad \text{and} \quad f \propto \langle n', k' | V | n, k \rangle \quad \text{in our case} \quad V = V_{sd}$$

So for s-d scattering, 
$$R_{sd} = \frac{\pi}{\hbar} D_d(\epsilon_F) \left| \int \psi_s V_{sd} \psi_d^* d\omega \right|^2$$

But we're missing something important...

# Spin-Orbit Interaction

The d electrons' (*responsible for the magnetization*) spin couples to their angular motion.

The d states experience an energy shift according to the Hamiltonian

$$H = -\vec{\mu} \cdot \vec{B} = K\vec{L} \cdot \vec{S}$$

Use first order perturbation theory  $\psi_d \rightarrow \psi_d^1$

SOI allows spin-mixing and creates d band holes so that majority spin electrons can scatter into empty d states

# Anisotropic magnetoresistance

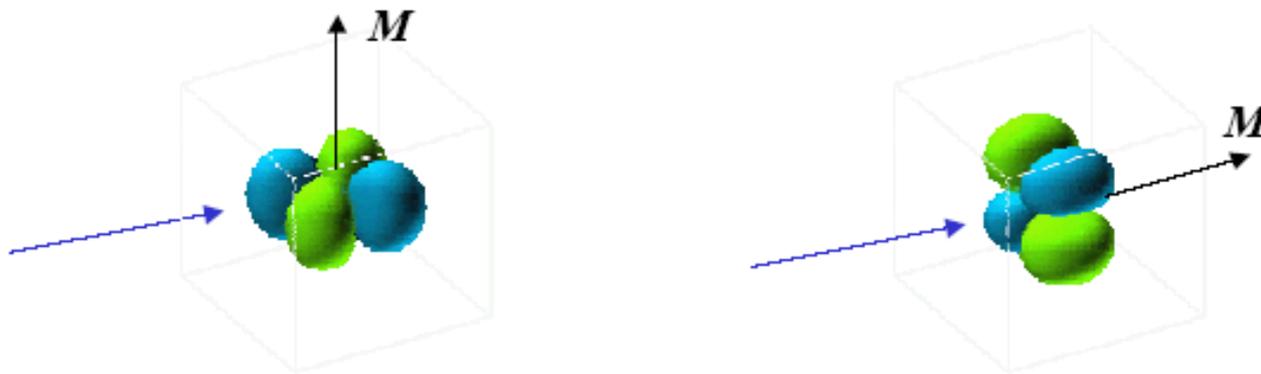
Physical origin: spin-orbit coupling leads to spin-dependent scattering of conduction electrons.

Conduction in (for ex.) Ni due to  $4s$  and  $3d$  electrons.

Crudely, the  $3d$  orbitals are affected by  $\mathbf{M}$ , and are mixed (slightly reoriented) so that they present a larger scattering cross-section to electrons moving *parallel* to  $\mathbf{M}$ .

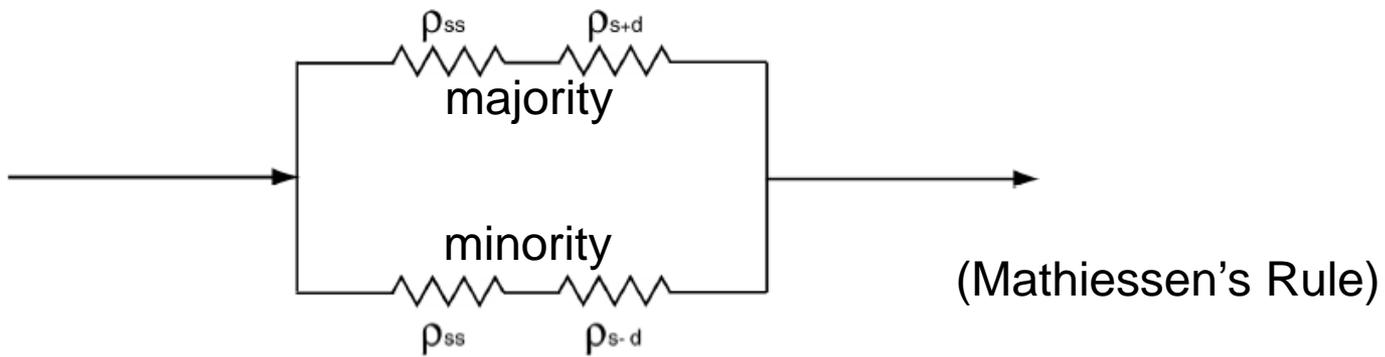
More scattering = higher resistance.

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2005 Constanta



# Recipe for resistivity

1. Pick unperturbed d electron wavefunctions
2. Choose the magnetization
3. Perturb the d states (using SOI)
4. Calculate scattering rate
5. Use rate to get relaxation time
6. Sum over all atoms
7. Repeat for other scattering mechanisms (if you want)
8. Use Mathiessen's rule to get total resistivity



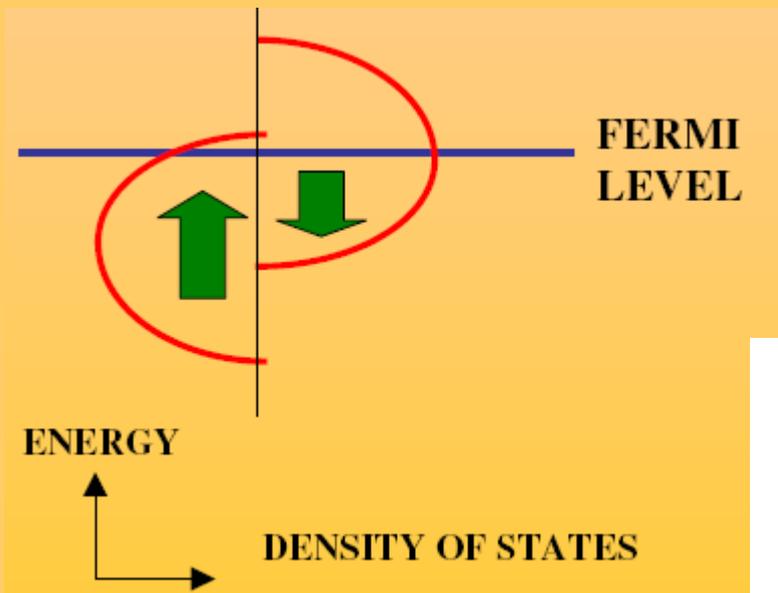
## two-current model

The FAR effect has been discussed in terms of the two-current model originally proposed by Mott . Magnetization in ferromagnetic metals arises as a result of the splitting of the **spin-up** or majority-spin band relative to the **spin-down** or minority-spin band,

Mott suggested that conduction electrons in ferromagnetic metals can propagate by **repeating scattering** events **without changing spin orientations** at temperatures well below the Curie temperature

the spin-up and spin-down conduction electrons can be treated independently

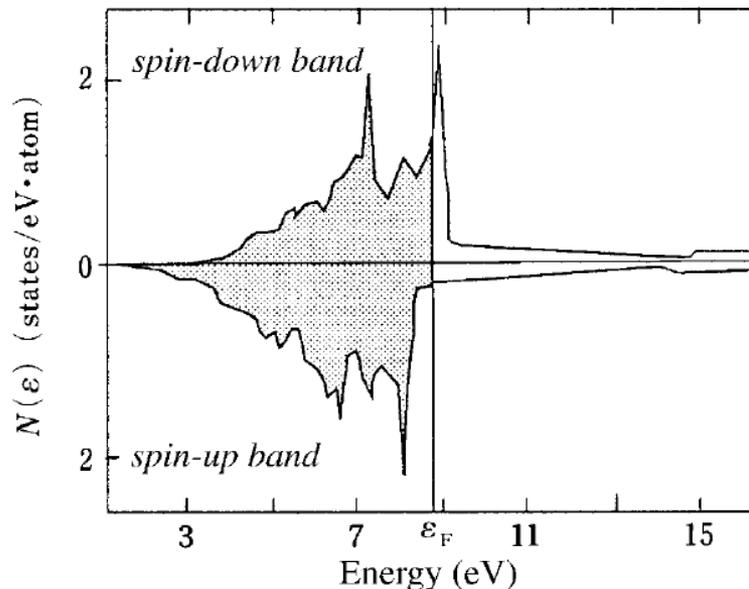
the spin-up and spin-down conduction electrons would possess different relaxation times.



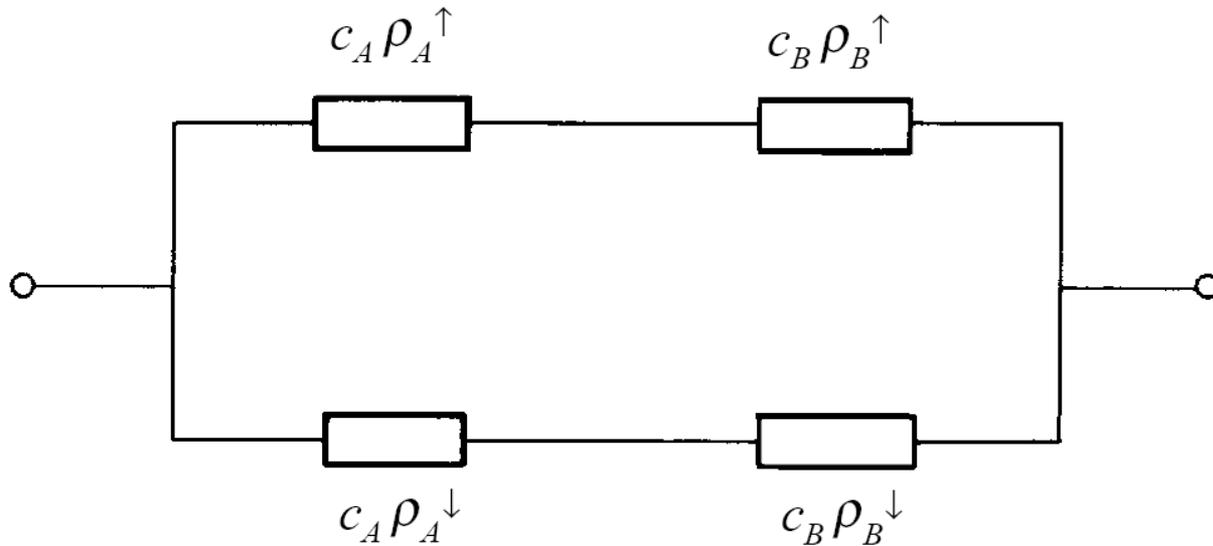
## Magnetization

$$M = N \uparrow - N \downarrow$$

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Valence band in the ferromagnetic state of pure Ni. The spin-up band is shifted to lower binding energies relative to the spin-down band due to the exchange energy.



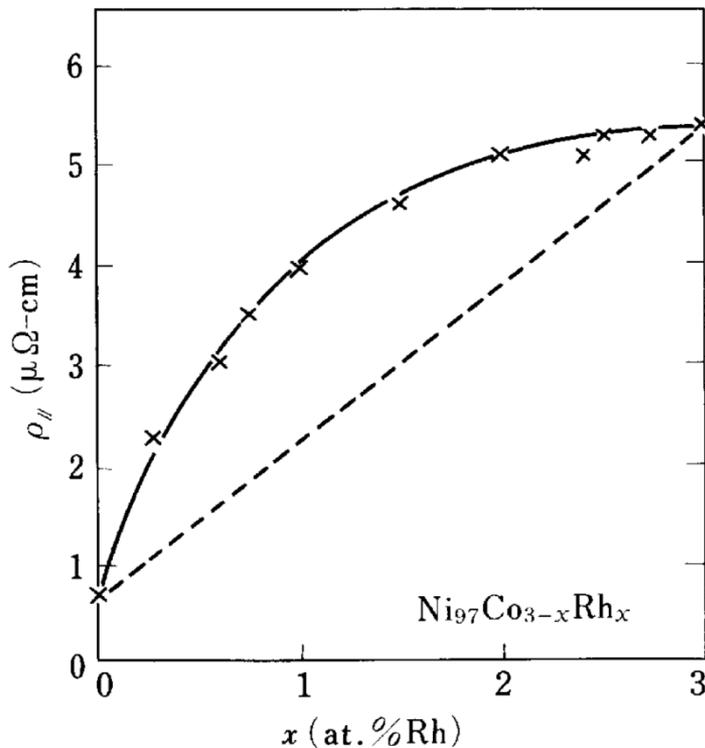
Equivalent circuit based on the two-current model of the residual resistivity at 4.2 K for a ferromagnetic metal containing  $c_A$  % of A atoms and  $c_B$  % of B atoms. [J. W. F. Dorleijn and A. R. Miedema, *J. Phys. F: Metal Phys.* **5** (1975) 487]

see Mizutani

to discuss the electron transport on the basis of the two-current model, one must find a way to separate the spin-up electron conduction from the spin-down one by measuring the resistivity for pseudo-binary dilute alloys.

e.g.  $\text{Ni}_{97}\text{A}_{3-x}\text{B}_x$

$\text{Ni}_{97}\text{Co}_{3-x}\text{Rh}_x$  alloys



Longitudinal resistivity at 4.2 K for  $\text{Ni}_{97}\text{Co}_{3-x}\text{Rh}_x$  alloys.  
[J. W. F. Dorleijn and A. R. Miedema, *J. Phys. F: Metal Phys.* **5** (1975) 1543]

$$\rho = \frac{(c_A \rho_A^\uparrow + c_B \rho_B^\uparrow)(c_A \rho_A^\downarrow + c_B \rho_B^\downarrow)}{c_A \rho_A^\uparrow + c_B \rho_B^\uparrow + c_A \rho_A^\downarrow + c_B \rho_B^\downarrow},$$

$$c_A + c_B = 3$$

$\rho_A^\uparrow$  and  $\rho_B^\uparrow$

$\rho_A^\downarrow$  and  $\rho_B^\downarrow$

represent the residual resistivity caused by the scattering of the spin-up (spin-down) electrons by  $A$  and  $B$  atoms

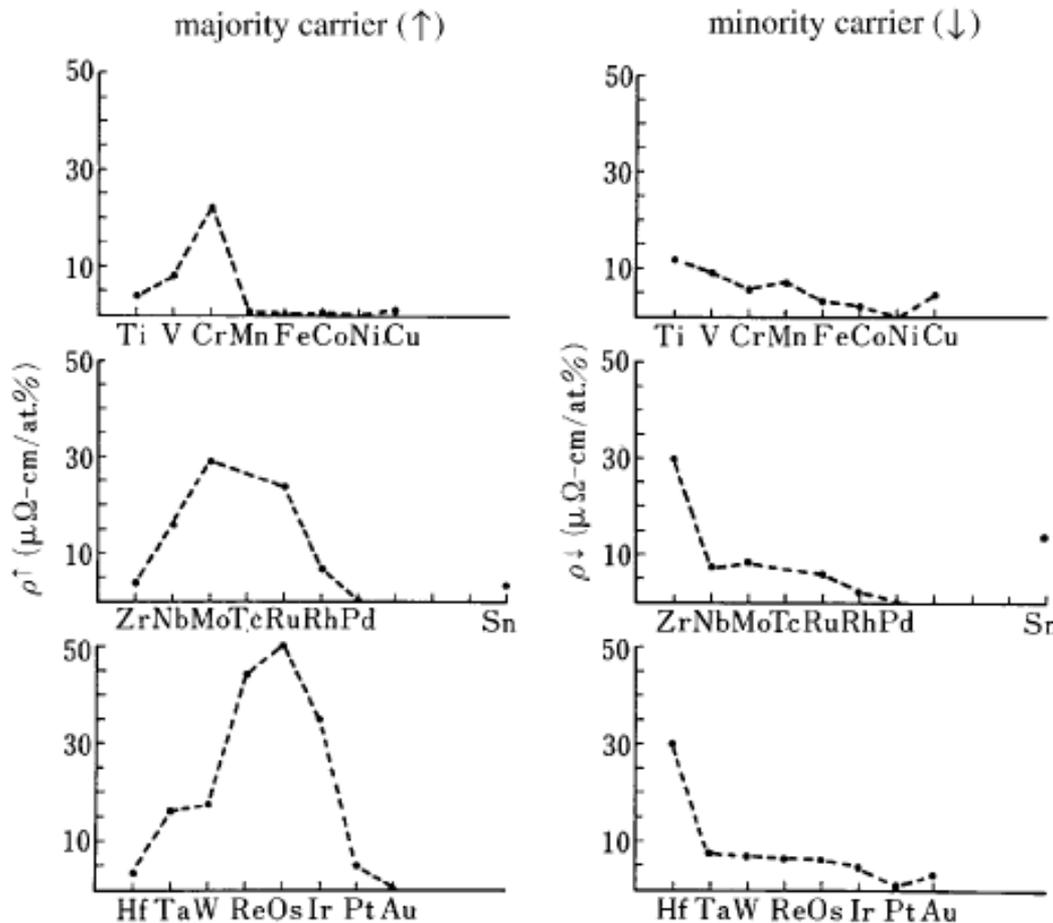
by measuring the resistivities for more than four samples with different concentrations.

$\text{Ni}_{97}\text{Cr}_{3-x}\text{M}_x$  alloys with  $M = \text{Al}, \text{Fe}, \text{Mn}, \text{Ti}$

$\rho_{\text{Cr}}^{\uparrow}$  and  $\rho_{\text{Cr}}^{\downarrow}$  are deduced

from  
Mizutani

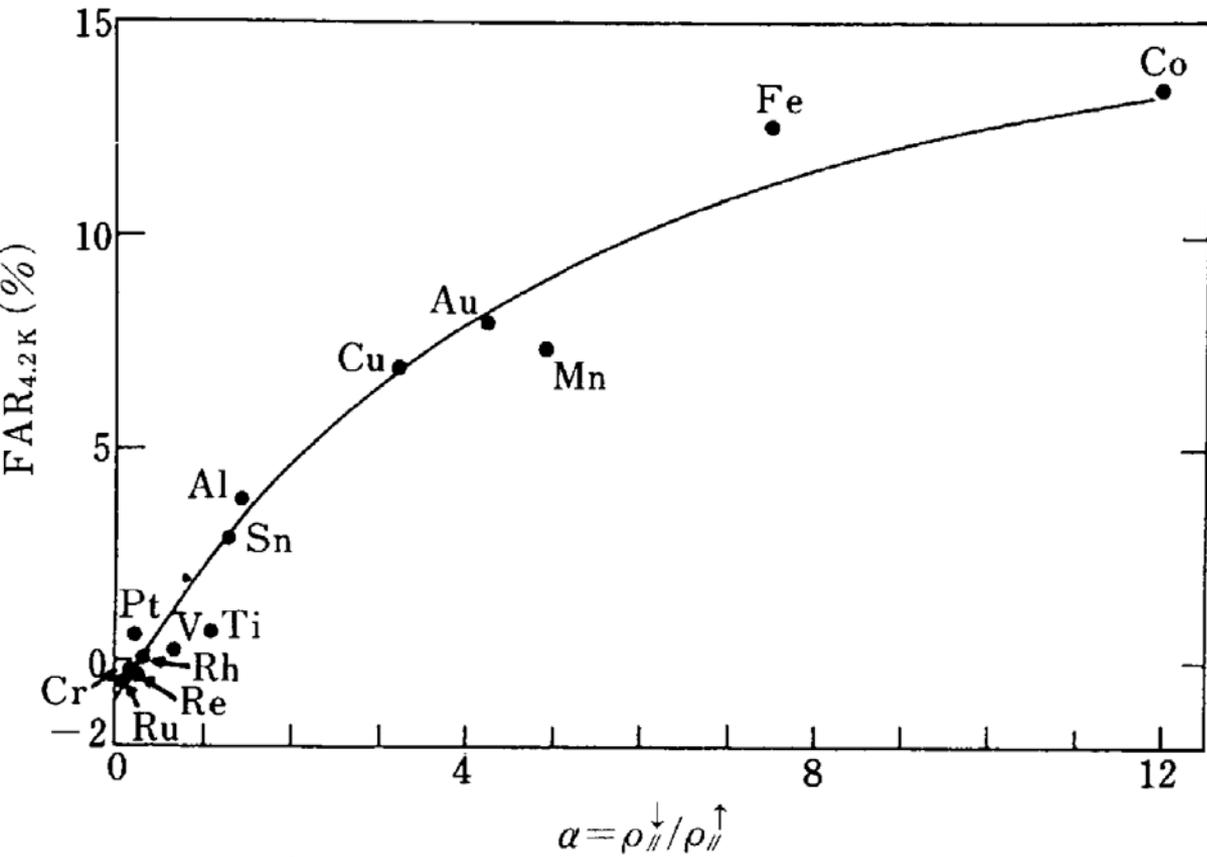
the resulting value is independent of the atomic species  $M$ .



Residual resistivity due to spin-up and spin-down electrons for Ni containing various transition metals. [J.W. F. Dorleijn and A. R. Miedema, *J. Phys. F: Metal Phys.* **5** (1975) 487]

the resistivity increment  $\rho^{\downarrow}$  due to the spin-down electron is always larger for the lighter elements Ti, Zr and Hf and decreases with increasing atomic number in the respective series.

$$\frac{\Delta\rho}{\rho_{||}} = \left(\frac{\alpha}{1+\alpha}\right) \left(\frac{\Delta\rho}{\rho_{||}}\right)^{\uparrow} + \left(\frac{1}{1+\alpha}\right) \left(\frac{\Delta\rho}{\rho_{||}}\right)^{\downarrow} \quad \alpha = \rho_{||}^{\downarrow} / \rho_{||}^{\uparrow}$$



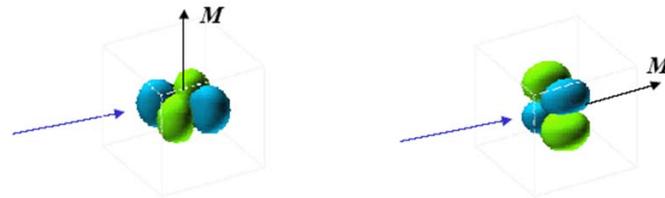
$\alpha$  dependence of FAR at 4.2 K for Ni-based alloys.

[J. W. F. Dorleijn and A. R. Miedema, *J. Phys. F: Metal Phys.* **5** (1975) 1543]

from Mizutani

$$\left(\frac{\Delta\rho}{\rho_{||}}\right)^{\uparrow} = +10\% \text{ and } \left(\frac{\Delta\rho}{\rho_{||}}\right)^{\downarrow} = -2\%$$

Ni possess a larger scattering cross section in the longitudinal configuration than in the transverse configuration whereas the anisotropy is small for the spin-down electrons.



the magnitude of FAR in a dilute alloy is mainly decided by the resistivity ratio  $\alpha$  of the spin-down conduction electrons over the spin-up electrons.

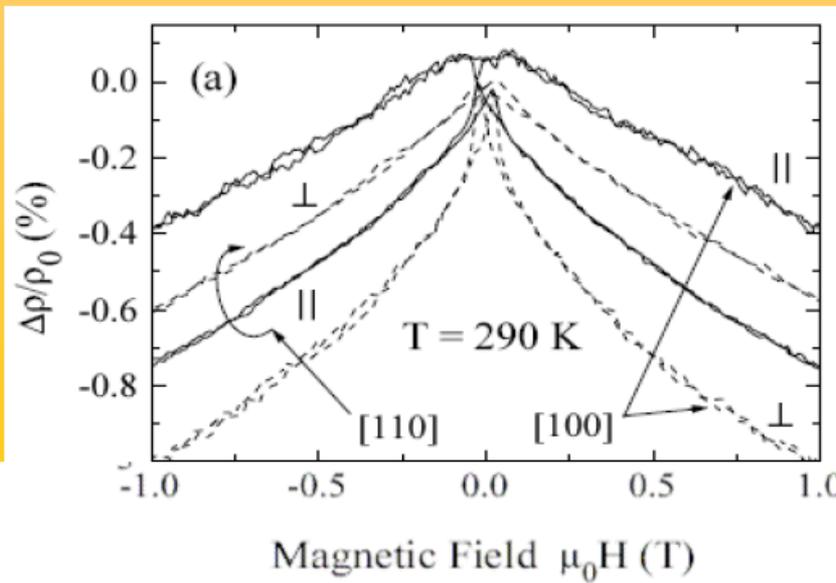
the largest FAR has been obtained at 30 at.%Co in the Ni–Co system and the second largest at 15 at.%Fe in the Ni–Fe system.

Magnetic field sensors have been commercially manufactured using these alloys.

Generally, in transition-metal-based compounds, it is normally very small (because the **orbital moment** is almost **quenched**) except in some particular cases such as Ni-Co and Ni-Fe alloys (AMR up to 6% at 300 K). Thin films based on this kind of alloys were used for the first MR read heads. It has been found for the spontaneous AMR:

$$\Delta\rho / \rho = \gamma(\alpha - 1) \quad (\text{with } \gamma = \text{spin-orbit constant and } \alpha = \rho_{\uparrow} / \rho_{\downarrow})$$

In single-crystals, the AMR depends on the direction of the current with respect to the crystallographic axis



### Fe<sub>3</sub>O<sub>4</sub> THIN FILMS

$$\text{For } I // [100] \longrightarrow \frac{\Delta\rho}{\rho} = \frac{\rho_{\parallel} - \rho_{\perp}}{\rho_0} > 0$$

$$\text{For } I // [110] \longrightarrow \frac{\Delta\rho}{\rho} = \frac{\rho_{\parallel} - \rho_{\perp}}{\rho_0} < 0$$