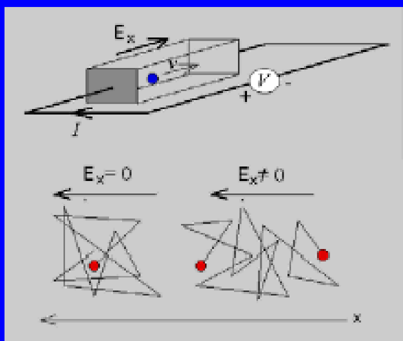
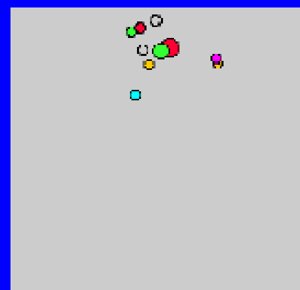
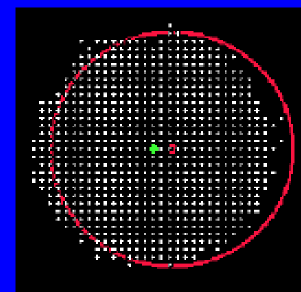
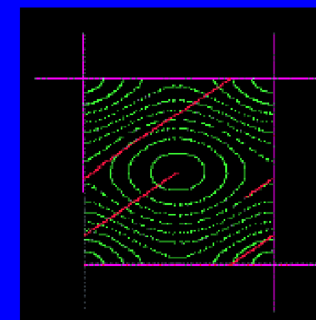
Transport Phenomena in Solids

Motions of electrons and transport phenomena



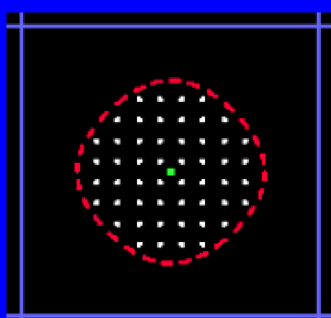


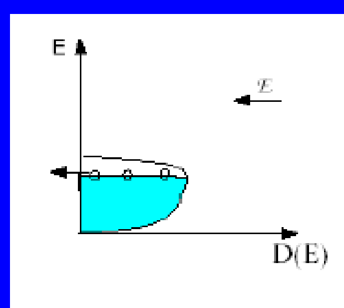


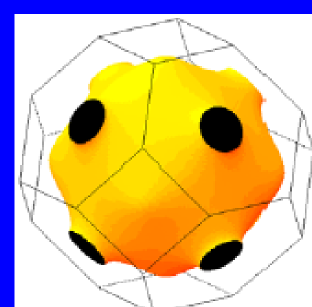


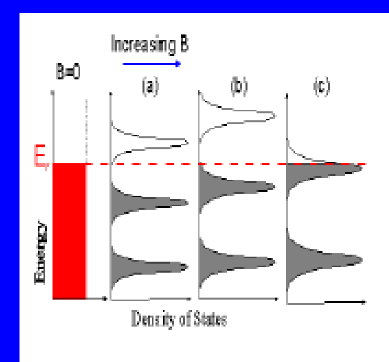
$$\sigma = \frac{ne^2\tau}{m}$$

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2}\sum_j \frac{\partial^2 E(\vec{k})}{\partial k_i \partial k_j}$$







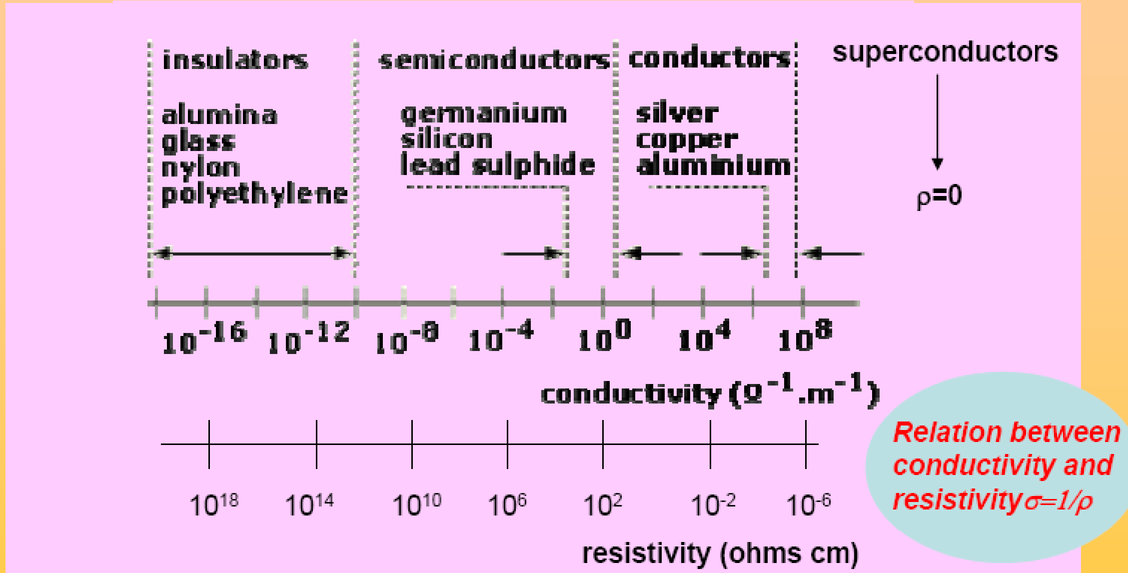


Some pictures are taken from the UvA-VU Master Course: Advanced Solid State Physics by Anne de Visser (University of Amsterdam), Solid State Course by Mark Jarrel (Cincinnati University), from Ibach and Lüth, from Ashcroft and Mermin and from several sources on the web.

MAGNETORESISTANCE AND HALL EFFECT

J.M. de Teresa,
Universidad de
Zaragoza,
Spain, ESM
2005 Constanta

TYPES OF MATERIALS IN TERMS OF CONDUCTION BEHAVIOUR



⇒***All kinds of these materials (in terms of conductivity properties) have found applications in different technological domains***

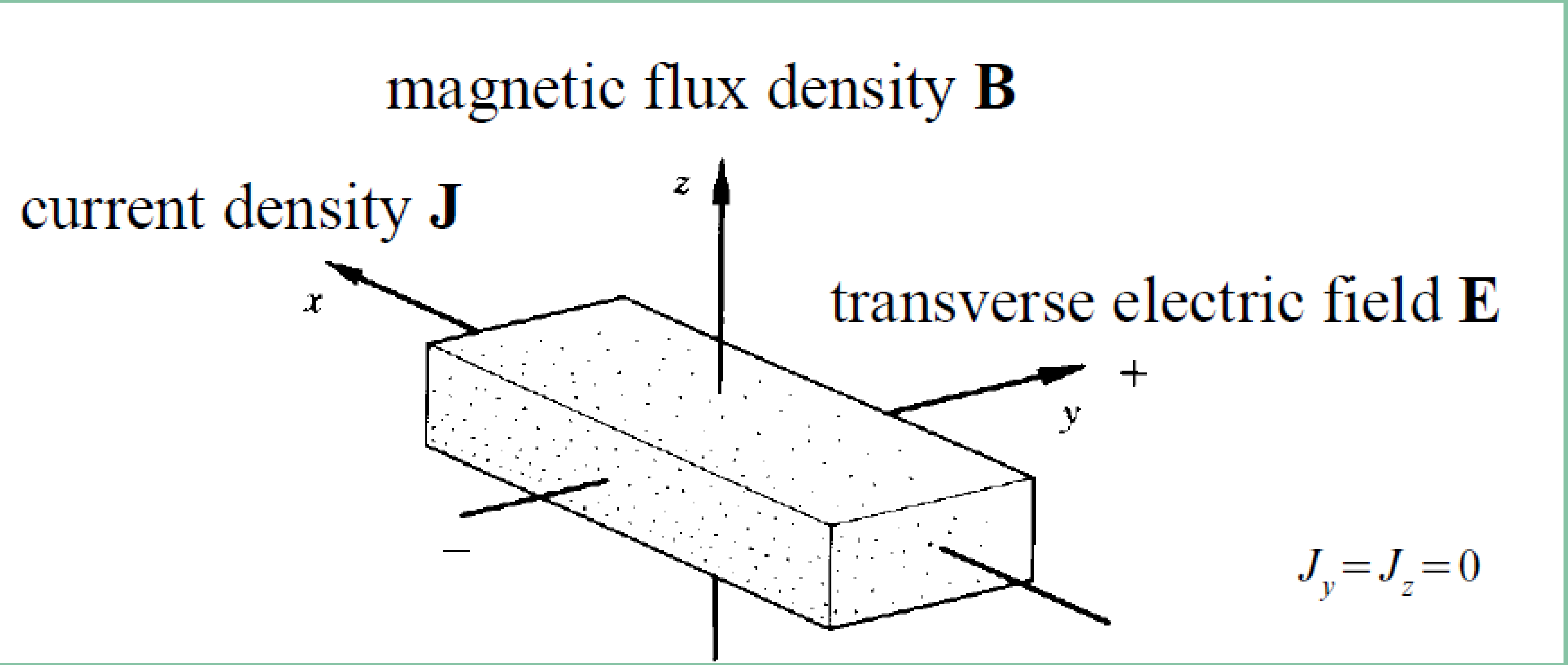
⇒***From a basic point of view, the electrical properties indirectly inform the researcher on the band structure, phase transitions, ground state, magnetic effects, impurities in the sample, etc. The dependence of the resistivity under magnetic field gives additional and important information on all these aspects***

APPLICATIONS IN:

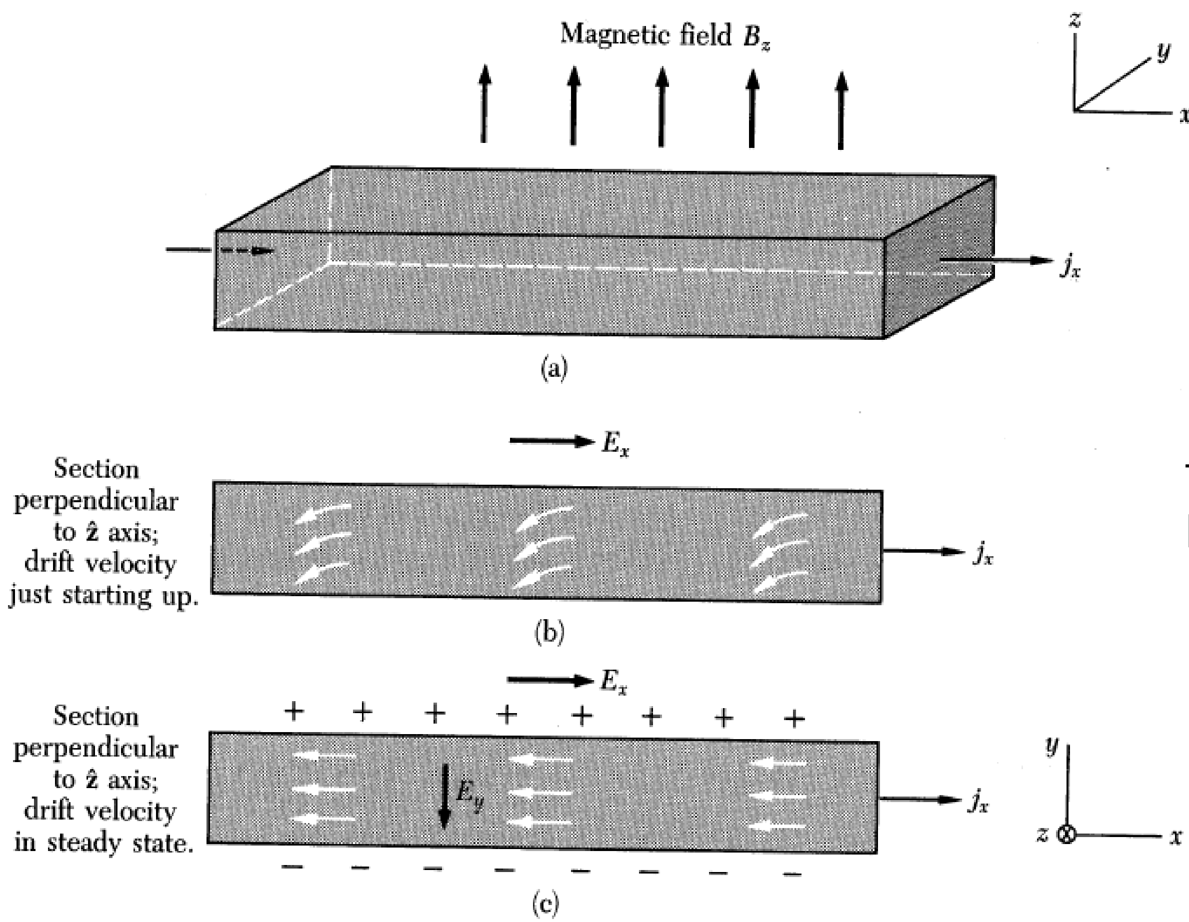
Magnetic read heads, position sensors, earth magnetic field sensing, non-contact potentiometers, non-volatile memories, detection of biological activity, spintronics,...

http://magnetism.eu/esm/2005-constanta/slides/deteresa-slides.pdf

The Hall effect is a phenomenon observed in the presence of both electric and magnetic fields



The geometry for the Hall effect measurement



from Kittel

Figure 7.1: The standard geometry for the Hall effect: a specimen of rectangular cross-section is placed in a magnetic field B_z as in (a). An electric field E_x applied across the end electrodes causes an electric current density j_x to flow down the bar. The drift velocity of the electrons immediately after the electric field is applied is shown in (b). The deflection in the y direction is caused by the magnetic field. Electrons accumulate on one face of the bar and a positive ion excess is established on the opposite face until, as in (c), the transverse electric field (Hall field) just cancels the force due to the magnetic field.

The Hall effect can be treated by incorporating two external forces $\boxed{(-e)\mathbf{E}}$ and $\boxed{(-e)\mathbf{v}_{\mathrm{k}}\times\mathbf{B}}$ into the linearized Boltzmann transport equation.

Mizutani

$$\boxed{-\mathbf{v}_{\mathbf{k}}\cdot\nabla f(\mathbf{r},\mathbf{k})-\frac{(-e)}{\hbar}(\mathbf{E}+\mathbf{v}_{\mathbf{k}}\times\mathbf{B})\cdot\frac{\partial f_{\mathbf{k}}}{\partial\mathbf{k}}=-\left(\frac{\partial f}{\partial t}\right)_{\mathrm{scatter}}.}$$

$$\phi(\mathbf{r},\mathbf{k})=f(\mathbf{r},\mathbf{k})-f_0(\varepsilon_{\mathbf{k}},T),$$

$$f_0(\varepsilon_{\mathbf{k}},T)=1/\{\exp[(\varepsilon_{\mathbf{k}}-\zeta)/k_{\mathrm{B}}T]+1\}$$

$$\frac{\partial f_0}{\partial T}=-\left(\frac{\partial f_0}{\partial\varepsilon}\right)\left[\left(\frac{\varepsilon-\zeta}{T}\right)+\frac{\partial\zeta}{\partial T}\right],$$

$$\boxed{\begin{aligned}&\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\mathbf{v}_{\mathbf{k}}\cdot\left[-\left(\frac{\varepsilon(\mathbf{k})-\zeta}{T}\right)\nabla T+(-e)\left(\mathbf{E}-\frac{\nabla\zeta}{(-e)}\right)\right]\\&=-\left(\frac{\partial f}{\partial t}\right)_{\mathrm{scatter}}+\mathbf{v}_{\mathbf{k}}\cdot\frac{\partial\phi}{\partial\mathbf{r}}+\frac{(-e)}{\hbar}(\mathbf{v}_{\mathbf{k}}\times\mathbf{B})\cdot\frac{\partial\phi}{\partial\mathbf{k}}.\end{aligned}}$$

This is the linearized Boltzmann transport equation.

$$\boxed{-\left(\frac{\partial f}{\partial t}\right)_{\text{scatter}}=\frac{f(\mathbf{r},\mathbf{k})-f_0(\varepsilon_{\mathbf{k}},T)}{\tau}=\frac{\phi(\mathbf{r},\mathbf{k})}{\tau}}$$

Relaxation time approximation

$$\mathbf{E}=(E,\ 0,\ 0)\ \text{and}\ \mathbf{B}=(0,\ 0,\ B)$$

Electrical conductivity formula

$$-\left(\frac{\partial f}{\partial t}\right)_{\text{scatter}}=\left(-\frac{\partial f_0}{\partial \varepsilon}\right)\mathbf{v}_{\mathbf{k}}\cdot(-e)\mathbf{E}.$$

$$\frac{\phi(\mathbf{k})}{\tau}=\left(-\frac{\partial f_0}{\partial \varepsilon}\right)\mathbf{v}_{\mathbf{k}}\cdot(-e)\mathbf{E}.$$

$$\boxed{\begin{aligned}\nabla T&=0\\ \nabla\zeta&=0\end{aligned}}$$

in homogeneous samples

Hall effect

$$\boxed{(-e)E\mathrm{v}_x\left(-\frac{\partial f_0}{\partial \varepsilon}\right)=\frac{\phi}{\tau}-\frac{(-e)}{\hbar}B\left(\mathrm{v}_x\frac{\partial \phi}{\partial k_y}-\mathrm{v}_y\frac{\partial \phi}{\partial k_x}\right)}$$

Since the magnetic field affects only the *x- and y-components of the wave vectors, the* function ϕ may be assumed to have a form:

$$\boxed{\phi = ak_x + bk_y}$$

$v_x = \hbar k_x/m$ and $v_y = \hbar k_y/m$

The resulting conductivity tensor defined as $\mathbf{J} = \boldsymbol{\sigma}\mathbf{E}$ for electrons under the condition $\mathbf{B} = (0, 0, B)$ is explicitly written as

$$\boxed{\sigma_{ij} = \frac{n(-e)^2\tau}{m}\begin{pmatrix} \frac{1}{1+\alpha^2} & \frac{-\alpha}{1+\alpha^2} & 0 \\ \frac{\alpha}{1+\alpha^2} & \frac{1}{1+\alpha^2} & 0 \\ 0 & 0 & 1 \end{pmatrix},}$$

$$\rho_{ij} = \frac{m}{n(-e)^2\tau}\begin{pmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

where $\alpha = \omega_c\tau$ and $\omega_c = (+e)B/m$.

from Mizutani p.558

The x- and y-components of the current density are given by

$$J_x = \frac{(-e)}{4\pi^3}\int\int\int \mathrm{v}_x \phi(\mathbf{k})dk_x dk_y dk_z = \frac{(-e)\hbar}{4\pi^3 m}\int\int\int k_x(ak_x + bk_y)dk_x dk_y dk_z$$

$$= \frac{(-e)\hbar}{4\pi^3 m}\int\int\int (ak_x^2 + bk_x k_y)dk_x dk_y dk_z,$$

$$J_y = \frac{(-e)}{4\pi^3}\int\int\int \mathrm{v}_y \phi(\mathbf{k})dk_x dk_y dk_z = \frac{(-e)\hbar}{4\pi^3 m}\int\int\int k_y(ak_x + bk_y)dk_x dk_y dk_z$$

$$= \frac{(-e)\hbar}{4\pi^3 m}\int\int\int (ak_y k_x + bk_y^2)dk_x dk_y dk_z,$$

the x-component

$$-(-e)E\mathrm{v}_x\frac{\partial f_0}{\partial \varepsilon}=\frac{\phi}{\tau}-\frac{(-e)}{\hbar}B\left(\frac{\hbar}{m}\right)(bk_x-ak_y)$$

and is rewritten as

$$\boxed{\omega_c=(+e)B/m}$$

$$\boxed{-(-e)\tau E\mathrm{v}_x\frac{\partial f_0}{\partial \varepsilon}=(a-b\omega_c\tau)k_x+(b+a\omega_c\tau)k_y}$$

X k_x

Similarly, we obtain the following relation from the *y- and z-components*

$$0=(a-b\omega_c\tau)k_x+(b+a\omega_c\tau)k_y$$

should hold for any k_x and k_y $\Rightarrow$

$$\boxed{\begin{aligned} a&=\omega_c\tau b\\ b&=-\omega_c\tau a\end{aligned}}$$

$$\boxed{[(-e)/4\pi^3]\iiint dk_x dk_y dk_z} \quad \text{X} \quad \boxed{-(-e)\tau E\left(\frac{m}{\hbar}\right)\mathrm{v}_x\mathrm{v}_x\frac{\partial f_0}{\partial \varepsilon}=(a-b\omega_c\tau)k_x^2+(b+a\omega_c\tau)k_xk_y}$$

$$-\left(\frac{(-e)^2}{4\pi^3}\right)\left(\frac{m}{\hbar}\right)\tau E\int\int\int \mathrm{v}_x \mathrm{v}_x \frac{\partial f_0}{\partial \varepsilon} dk_x dk_y dk_z = \frac{(-e)}{4\pi^3}\int\int\int (ak_x^2 + bk_x k_y)\, dk_x dk_y dk_z$$

$$-\frac{(-e)}{4\pi^3}\omega_c \tau \int\int\int (bk_x^2 - bk_x k_y)\, dk_x dk_y dk_z.$$

We use:

$$\int\int\int d\mathbf{k} = \int\int dS \int dk_\perp = \int\int dS \int \frac{d\varepsilon}{|\partial\varepsilon/\partial k_\perp|} = \int\int dS \int \frac{d\varepsilon}{|\nabla_{k_\perp}\varepsilon|},$$

And we get

as we know from lecture 2:

$$\int dS_E \frac{\tau(\mathbf{k}) v_x^2(\mathbf{k})}{v(\mathbf{k})} = \frac{4\pi}{3} k_F^2 \tau(E_F) v(E_F)$$
$$= \frac{4\pi}{3} k_F^2 \tau(E_F) \frac{\hbar k_F}{m^*}$$

$$\left[\frac{(-e)^2\, \tau \mathrm{v}_F S_F}{12\pi^3 \hbar}\right] E = \frac{(-e)}{4\pi^3}\left(\frac{\hbar}{m}\right)\int\int\int (ak_x^2 + bk_y k_x) dk_x dk_y dk_z$$

$$-\frac{(-e)}{4\pi^3}\omega_c \tau \left(\frac{\hbar}{m}\right)\int\int\int (bk_x^2 - bk_y k_x)\, dk_x dk_y dk_z.$$

with $\sigma = \frac{e^2}{4\pi^3}\int \frac{\tau v_i^2 dS_F}{\hbar v_{k_\perp}} = \frac{e^2\tau}{4\pi^3\hbar v_F}\cdot\frac{v_F^2}{3}\int dS_F = \frac{e^2\tau v_F S_F}{12\pi^3\hbar}$ $\qquad v_F^2 = \Sigma_{i=x,y,z} v_i^2 = 3v_i^2$

Drude formula $\quad$ $\boxed{n = \frac{S_F v_F m}{12\pi^3\hbar}}$

$$\left[\frac{n(-e)^2\tau}{m}\right]E = J_x - \frac{(-e)}{4\pi^3}\omega_c\tau\left(\frac{\hbar}{m}\right)\int\int\int(bk_x^2 - ak_xk_y)\,dk_xdk_ydk_z$$

$$= J_x - \frac{(-e)}{4\pi^3}(\omega_c\tau)^2\left(\frac{\hbar}{m}\right)\int\int\int\frac{(bk_x^2 - ak_yk_x)}{\omega_c\tau}dk_xdk_ydk_z$$

An insertion of the relations $a = \omega_c\tau b$ and $b = -\omega_c\tau a$ yields

$$\left[\frac{n(-e)^2\tau}{m}\right]E = J_x + \frac{(-e)}{4\pi^3}(\omega_c\tau)^2\left(\frac{\hbar}{m}\right)\int\int\int(ak_x^2 + bk_yk_x)\,dk_xdk_ydk_z$$

$$\boxed{\left[\frac{n(-e)^2\tau}{m}\right]E = J_x + (\omega_c\tau)^2 J_x.}$$

The ratio of J_x/E_x is the xx-component of the conductivity tensor.

The ratio of J_x/E_x is the xx-component of the conductivity tensor.

$$\sigma_{xx} = [n(-e)^2\tau/m]\ \{1/[1+(\omega_c\tau)^2]\}$$

Similarly, the xy-component σ_{xy} of the conductivity tensor can be calculated

The resulting conductivity tensor defined as $\mathbf{J} = \boldsymbol{\sigma}\mathbf{E}$ for electrons under the condition $\mathbf{B} = (0, 0, B)$ is explicitly written as

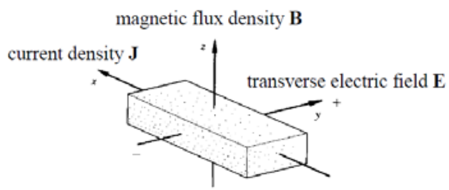
$$\boxed{\sigma_{ij} = \frac{n(-e)^2\tau}{m}\begin{pmatrix} \frac{1}{1+\alpha^2} & \frac{-\alpha}{1+\alpha^2} & 0 \\ \frac{\alpha}{1+\alpha^2} & \frac{1}{1+\alpha^2} & 0 \\ 0 & 0 & 1 \end{pmatrix},}$$

from Mizutani

where $\alpha = \omega_c\tau$ and $\omega_c = (+e)B/m$.

The resistivity tensor defined as $\mathbf{E} = \boldsymbol{\rho}\mathbf{J}$ for electrons under the condition $\mathbf{B} = (0, 0, B)$ is obtained by inversion of equation

$$\rho_{ij} = \frac{m}{n(-e)^2\tau}\begin{pmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$



the current density $\mathbf{J} = (J_x, 0, 0)$

gives rise to a *y-component of the electric field E_y, which is perpendicular to* the directions of both current and magnetic field.

$$E_y = -\alpha\left(\frac{m}{n(-e)^2\tau}\right)J_x = -\left(\frac{(+e)\tau}{m}\right)\cdot\left(\frac{m}{n(-e)^2\tau}\right)B_zJ_x = \frac{1}{n(-e)}B_zJ_x,$$

where $\alpha = (+e)B_z\tau/m$. The Hall coefficient R_H is defined as the coefficient of B_zJ_x in the transverse electric field E_y:

$$R_H = \frac{1}{n(-e)}.$$

in the free-electron model is independent of the magnetic field and depends only on the number of electrons per unit volume.

The Hall effect of either electrons or holes may be discussed in a simpler but less rigorous way by using the Lorentz force (as we did in the basic solid state physics course):

Kittel/Aschroft

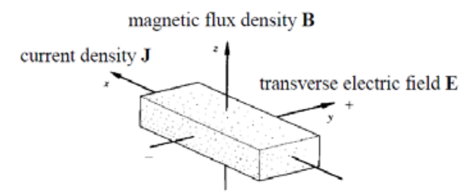
$$m^*\left(\frac{d\mathbf{v}_\mathrm{D}}{dt}+\frac{\mathbf{v}_\mathrm{D}}{\tau}\right)=(\mp e)(\mathbf{E}+\mathbf{v}_\mathrm{D}\times\mathbf{B}) \quad m^*\left(\frac{d\mathbf{v}_\mathrm{D}}{dt}+\frac{\mathbf{v}_\mathrm{D}}{\tau}\right)=(\mp e)\left(\mathbf{E}+\frac{1}{c}\mathbf{v}_\mathrm{D}\times\mathbf{H}\right)[\mathrm{CGS}],$$

where $\mp$ refers to the sign of the charge of the respective carriers. Since the condition $d\mathbf{v}_\mathrm{D}/dt=0$ holds in the steady state, we get

$$\boxed{\mathbf{v}_\mathrm{D}=\left(\frac{(\mp e)\tau}{m^*}\right)(\mathbf{E}+\mathbf{v}_\mathrm{D}\times\mathbf{B}).}$$

$$\mathbf{J}=n(\mp e)\mathbf{v}_\mathrm{D}$$

$$\omega_c=(+e)B_z/m^*$$



$$J_x=\frac{n(\mp e)^2\tau}{m^*(1+\omega_c^2\tau^2)}(E_x\mp\omega_c\tau E_y)$$

$$J_y=\frac{n(\mp e)^2\tau}{m^*(1+\omega_c^2\tau^2)}(\omega_c\tau E_x\pm E_y)$$

$$J_z=\frac{n(\mp e)^2\tau}{m^*}E_z.$$

Since no current flows along the *y- and z-directions*

the condition J_y=J_z=0 *holds.*

$$\boxed{E_y=\mp\omega_c\tau E_x}$$

The Hall coefficient for either the electron or hole is expressed as

$$\boxed{R_H = \frac{1}{nq} \qquad R_H = \frac{1}{nqc}\ [\mathrm{CGS}],}$$

$\omega_c = (+e)H/m^*c$ in CGS units, where c is the speed of light

The Hall coefficient is negative and temperature independent in the free electron model.

The density of the conduction electron *n is related to the Fermi* radius k_F *in the free-electron model*

$$k_F = \left[3\pi^2\left(\frac{N_0}{V}\right)\right]^{1/3}.$$

$$\boxed{2k_F = 1.139\times 10^{-3}(|R_H|)^{-1/3},}$$

where the Hall coefficient and the Fermi diameter are in the units of $m^3/A\cdot s$ and $(\text{Å})^{-1}$, respectively.

in the free-electron model

$$\left|R_H^{free}\right| = 1.036 \times 10^{-11}\left[\frac{A}{d \cdot (e/a)}\right],$$

where A is the atomic weight in g, d is the mass density in (g/cm^3) and e/a is the number of carriers per atom.

Hall coefficients in pure elements

at room temperature in low magnetic fields

metal	e/a	Hall coefficient at 300 K $R_H^{300\,K}$ ($\times 10^{-11}$ m^3/A·s)	Hall coefficient from the free-electron model R_H^{free} ($\times 10^{-11}$ m^3/A·s)	$R_H^{300\,K}/R_H^{free}$
Li	1	−15	−13.4	1.12
Na	1	−25.8	−24.6	1.05
K	1	−35	−47.1	0.74
Cu	1	−5.07	−7.35	0.69
Ag	1	−8.8	−10.6	0.83
Au	1	−7.08	−10.6	0.67
Mg	2	−8.3	−7.23	1.14
Ca	2	−17.8	−13.5	1.31
Zn	2	5.5	−4.74	−1.16
Cd[a]	2	3.9 (13.9)	−6.74	−0.57 (−2.0)
Al	3	−3.44	−3.45	0.99
In	3	−0.216	−5.43	0.04
Sn	4	−0.22	−4.21	0.05
Pb	4	0.098	−4.59	−0.02
As		450		
Sb		−198		
Bi		−54 000		

Note:
[a] The values of 3.9 and (13.9) are obtained with the magnetic field parallel to and perpendicular to the c-axis, respectively.

From Mizutani

A close agreement with the free-electron value is observed only in limited number of metals, for example Na.

The sign of the Hall coefficient in the divalent metals Zn and Cd is positive and a significant deviation from the free-electron model is apparent.

both electron and hole Fermi surfaces coexist in polyvalent metals like Zn, Al and Pb

when

$$\mathbf{J} \perp \mathbf{B} \text{ and } \mathbf{E} \perp \mathbf{B}$$

$$\boxed{\mathbf{J}_i = \frac{\sigma_i}{1+\beta_i^2 B^2}\mathbf{E} - \frac{\sigma_i \beta_i}{1+\beta_i^2 B^2}\mathbf{B}\times\mathbf{E},}$$

where σ_i is the conductivity of the i-th carrier and $\beta_i = q_i \tau_i / m_i$

The total current:

for 2 types of carriers

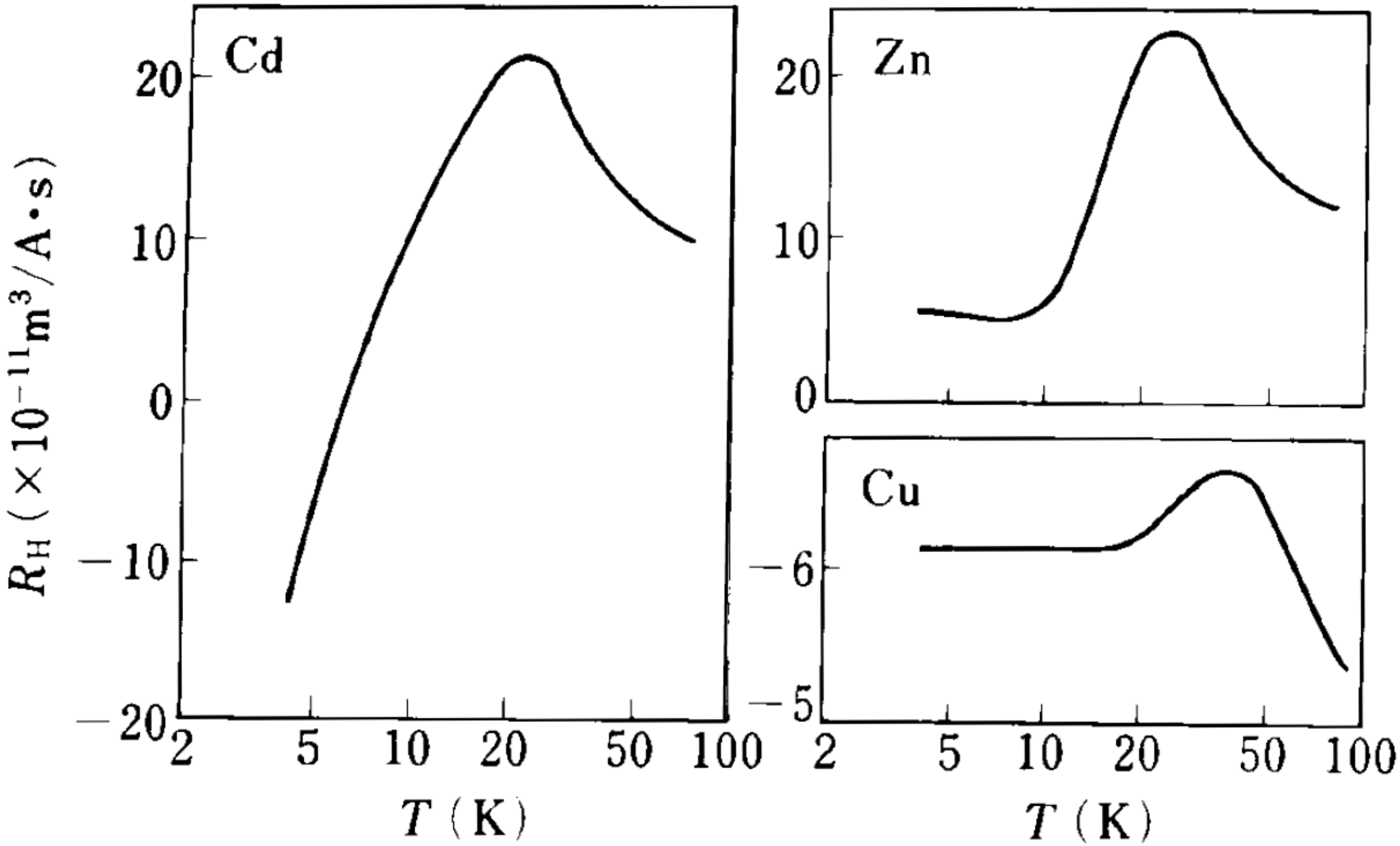
$$\boxed{\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 = \left(\frac{\sigma_1}{1+\beta_1^2 B^2} + \frac{\sigma_2}{1+\beta_2^2 B^2}\right)\mathbf{E} - \left(\frac{\sigma_1\beta_1}{1+\beta_1^2 B^2} + \frac{\sigma_2\beta_2}{1+\beta_2^2 B^2}\right)\mathbf{B}\times\mathbf{E}.}$$

The Hall coefficient at low magnetic fields is then deduced to be

$$\boxed{R = \frac{\sigma_1^2 R_1 + \sigma_2^2 R_2}{(\sigma_1 + \sigma_2)^2}}$$

the sign of the Hall coefficient is determined by the balance between the electrons and holes having *$R_1<0$ and $R_2>0$.*

A positive Hall coefficient in divalent metals like Zn and Cd means that the hole contribution dominates over the electron contribution.



Temperature dependence of the Hall coefficient of pure metals Cd, Zn and Cu. [K. E. Saeger and R. Lück, *Phys. Kondens. Materie* ***9 (1963) 91]***

In two-carrier metals, the Hall coefficient.

⇒

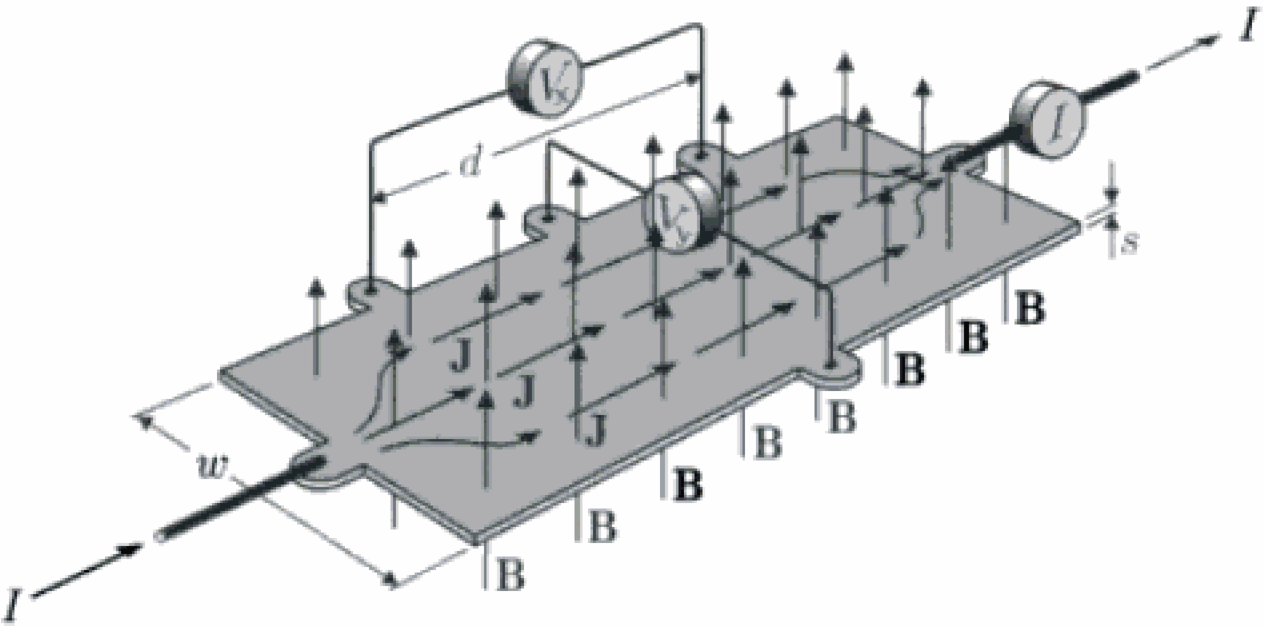
a strong temperature dependence

•In contrast to a single-carrier metal, the relaxation time is involved in the Hall coefficient through σ_i *and its temperature dependence is responsible for that* of the Hall coefficient.

•The presence of the Brillouin zone in crystal metals yields electrons and holes having different effective masses and relaxation times.

•The Hall coefficient reflects the anisotropy of the <u>electronic structure</u> and exhibits substantial deviation from the free-electron behavior.

• the Hall coefficient in amorphous alloys is essentially temperature independent because of the <u>lack of anisotropy of the Fermi surface</u>



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Olof Lindberg, PROCEEDINGS OF THE I.R.E. 1954

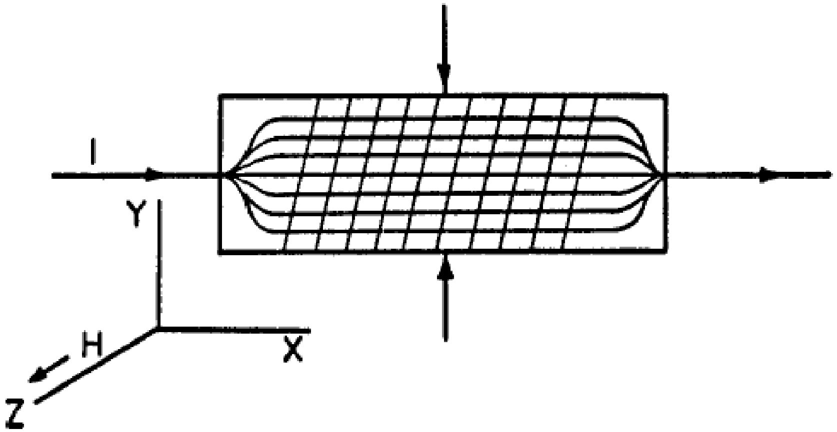
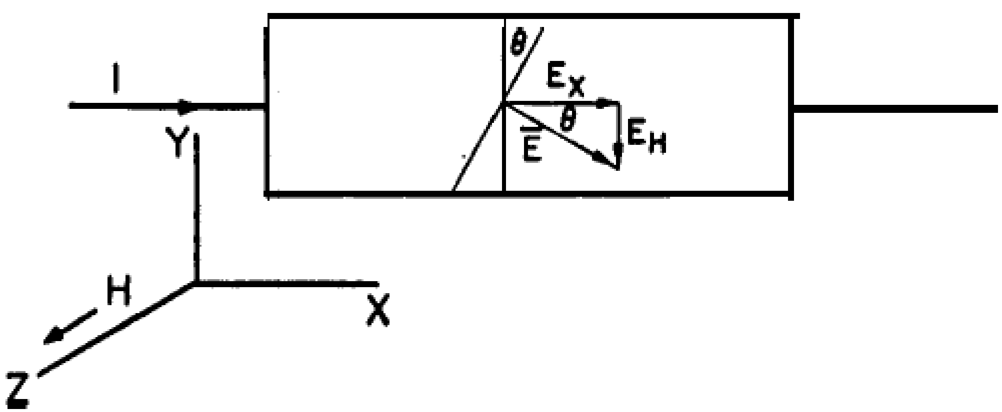


Fig. 1-Rotation of equipotentials by Hall effect, The Hall effect rotates the equipotentials so that they are no longer normal to the current flow.

$$\boxed{\tan\theta_H = \frac{E_H}{E_x}}$$



$$E_H = R_H \cdot J \cdot H$$

$$E_x = \frac{J}{\sigma}$$

$$\boxed{\tan\theta_H = R_H H \sigma}$$

Fig. 2-Vector diagram for the Hall effect. The Hall angle θ is the angle of rotation of equipotentials. E_H is the Hall field if the carriers are electrons.

MAGNETORESISTANCE

The term "magnetoresistance", meaning the change in resistance or resistivity upon the application of a magnetic field, is expressed in different ways:

(1) $\Delta R=R(H)-R(0)$;

(2) $\Delta R/R(0)$;

(3) $R(H)/R(0)$;

(4) $\Delta\rho=\rho(H)-\rho(0)$;

(5) $\Delta\rho/\rho(0)$; *and*

(6) $\rho(H)/\rho(0)$,

where R(H) and $\rho(H)$ *are the resistance and resistivity, respectively,* in an applied magnetic field *H.*

Ordinary Magnetoresistance (OMR)

The magnetoresistance is defined in terms of the diagonal components of the magnetoresistivity tensor

see Ashcroft

$$\rho_{ij} = \frac{m}{n(-e)^2\tau}\begin{pmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\overleftrightarrow{\rho}_B = [\overleftrightarrow{\sigma}_B]^{-1} = \frac{m^*}{ne^2\tau}\begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $\alpha = \omega_c\tau$ and $\omega_c = (+e)B/m$.

and, in general, depends on $(\omega_c\tau)^2$ or on B^2. Since $\omega_c\tau = (e\tau/m^*c)B = \mu B/c$, the magnetoresistance provides information on the carrier mobility.

CGS units

$$\sigma_{ij} = \frac{n(-e)^2\tau}{m}\begin{pmatrix} \frac{1}{1+\alpha^2} & \frac{-\alpha}{1+\alpha^2} & 0 \\ \frac{\alpha}{1+\alpha^2} & \frac{1}{1+\alpha^2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

The longitudinal magnetoresistivity $\Delta\rho_{zz}/\rho_{zz}$ is measured with the electric field parallel to the magnetic field.

On the basis of a spherical Fermi surface one carrier model, we have

$$\left.\begin{aligned} E_z &= j_z/\sigma_0 \\ \sigma_0 &= ne^2\tau/m^* \end{aligned}\right\} \Rightarrow$$

there is no longitudinal magnetoresistivity in this case

many semiconductors do exhibit longitudinal magnetoresistivity experimentally, and this effect arises from the non-spherical shape of their constant energy surfaces.

The transverse magnetoresistivity $\Delta\rho_{xx}/\rho_{xx}$ is measured with the current flowing perpendicular to the magnetic field.

$$\begin{aligned} j_y &= 0 \\ E_y &= (\omega_c\tau)E_x \end{aligned} \Rightarrow$$

$$j_x = \sigma_0\left[\frac{E_x}{1+(\omega_c\tau)^2} + \frac{(\omega_c\tau)^2 E_x}{1+(\omega_c\tau)^2}\right] = \sigma_0 E_x$$

there is no transverse magnetoresistance for a material with a single carrier type having a spherical Fermi surface.

In a two-band carriers model

The transverse magnetoresistance:

$$\frac{\Delta\rho_{xx}}{\rho_{xx}(0)} \equiv \frac{\rho_{xx}(B)-\rho_{xx}(0)}{\rho_{xx}(0)} = \frac{\sigma_1\sigma_2(\beta_1-\beta_2)^2 B^2}{(\sigma_1+\sigma_2)^2 + B^2(\beta_1\sigma_1+\beta_2\sigma_2)^2}.$$

where σ_i is the conductivity of the i-th carrier and $\beta_i = q_i\tau_i/m_i$

it is always POSITIVE but disappears when $\beta_1 = \beta_2$ and that it is proportional to B^2, as long as the magnetic field is low.

eventually $R \propto B$ in very large (~ 1 T) fields

Or saturates.

Magnetoresistance can give precious information about the shape of the Fermi surface

In a constant magnetic field (no electric field), the electron will move on a constant energy surface in k space in an orbit perpendicular to the magnetic field

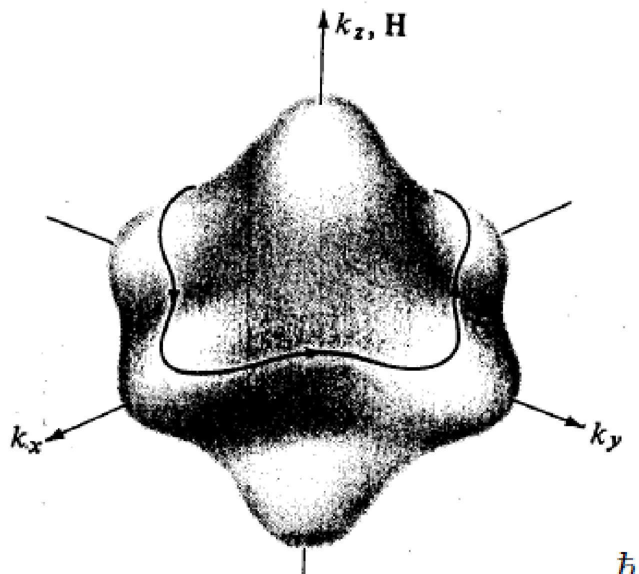
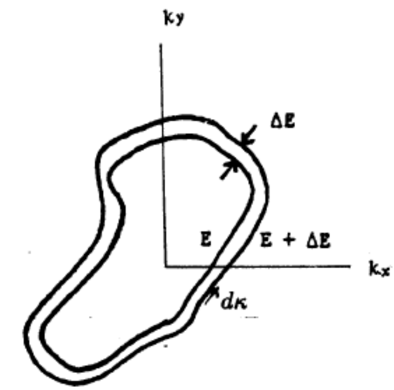


Figure 7.4: Schematic diagram of the motion of an electron along a constant energy (and constant k_z) trajectory in the presence of a magnetic field in the z-direction.



ΔA is the area of the strip

Transport Properties of Solids
M. S. Dresselhaus, MIT

$$m_c^* = \frac{\hbar^2}{2\pi}\frac{1}{\Delta E}\oint (\Delta k) d\kappa = \frac{\hbar^2}{2\pi}\frac{\Delta A}{\Delta E}$$

the cyclotron frequency

$$\boxed{\omega_c = eB/(m_c^* c)}$$

cyclotron effective mass

$$\boxed{m_c^* = \frac{1}{2\pi}\oint \frac{\hbar d\kappa}{|v|} = \frac{\hbar^2}{2\pi}\oint \frac{d\kappa}{|\partial E/\partial k|}}$$

from Transport Properties of Solids
M. S. Dresselhaus, MIT

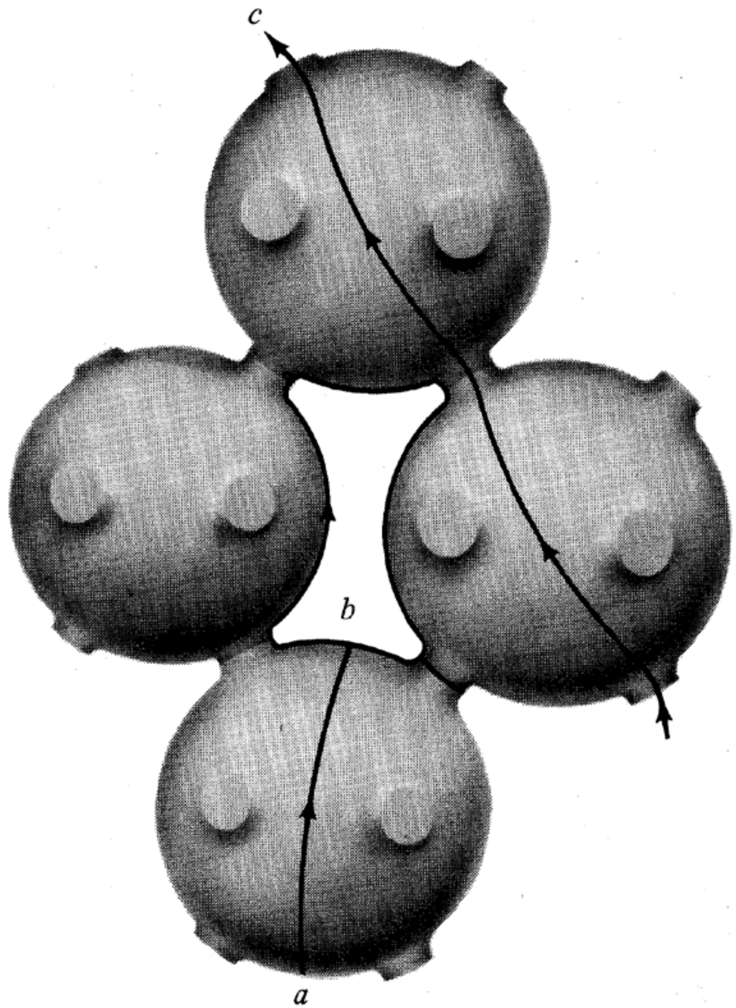


Figure 7.5: This diagram for the electron orbits in metallic copper indicates only a few of the many types of orbits an electron can pursue in k-space when a uniform magnetic field is applied to a noble metal. (Recall that the orbits are given by slicing the Fermi surface with planes perpendicular to the field.) The figure displays (a) a closed electron orbit; (b) a closed hole orbit; (c) an open hole orbit, which continues in the same general direction indefinitely in the repeated-zone scheme.

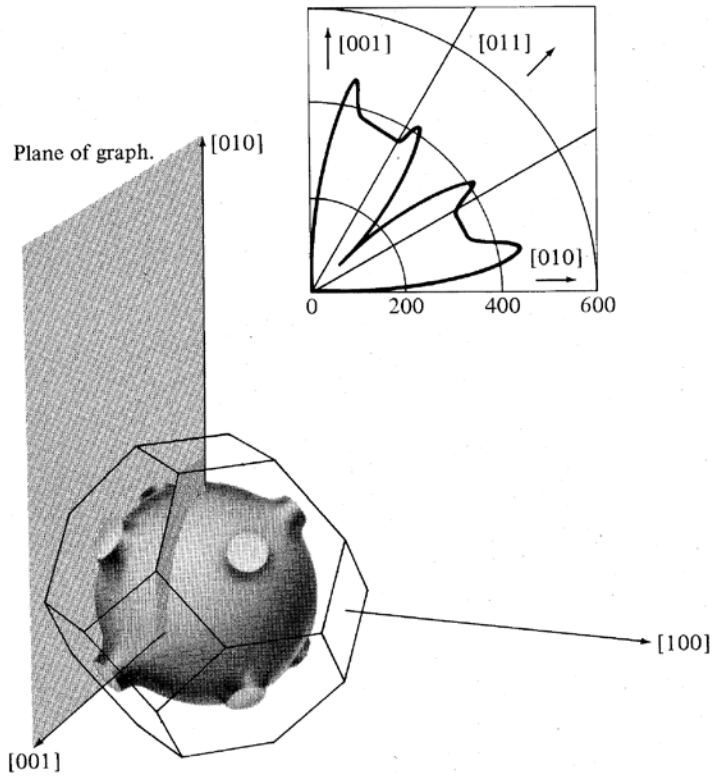


Figure 7.6: The spectacular directional dependence of the high-field magnetoresistance in copper is that characteristic of a Fermi surface supporting open orbits. The [001] and [010] directions of the copper crystal are as indicated in the figure, and the current flows in the [100] direction perpendicular to the graph. The magnetic field is in the plane of the graph. Its magnitude is fixed at 18 kilogauss, and its direction is varied continuously from [001] to [010]. The graph is a polar plot of the transverse magnetoresistance $[\rho(H) - \rho(0)]/\rho(0)$ vs. orientation of the field.

Transport Properties of Solids
M. S. Dresselhaus, MIT

- closed orbits produce a ρ_{xx} which saturates as $B \rightarrow \infty$
- open orbits produce a ρ_{xx} proportional to B^2 as $B \rightarrow \infty$

For non-magnetic metals, MR effects at low fields are very small, although the effect can become quite large for high fields. The change in resistivity, $\Delta\rho$, is positive for both magnetic field parallel ($\Delta\rho_{II}$) and transverse ($\Delta\rho_T$) to the current direction with $\Delta\rho_T > \Delta\rho_{II}$

•There are three distinct cases of ordinary magnetoresistance, depending on the structure of the electron orbitals at the Fermi surface:

i) In metals with closed Fermi surfaces, the electrons are constrained to their orbit in k space and the effect of the magnetic field is to increase the cyclotron frequency of the electron in its closed orbit. Cyclotron frequencies as large as $\omega_c t = 100$ have been achieved. In this case the resistance saturates at very large magnetic fields. Metals In, Al, Na and Li (see Figure 1a)

ii) For metals with equal numbers of electrons and holes, the magnetoresistance increases with H up to the highest fields measured, and is independent of crystallographic orientation. Examples of metals displaying this behavior are Bi, Sb, W, and Mo.

iii) Metals that contain Fermi surfaces with open orbits in some crystallographic directions will exhibit large magnetoresistance for fields applied in those directions, whereas the resistance will saturate in other directions, where the orbits are closed. This behavior will be found in Cu, Ag, Au, Mg, Zn, Cd, Ga, Tl, Sn, Pb, and Pt (see Figure lb).

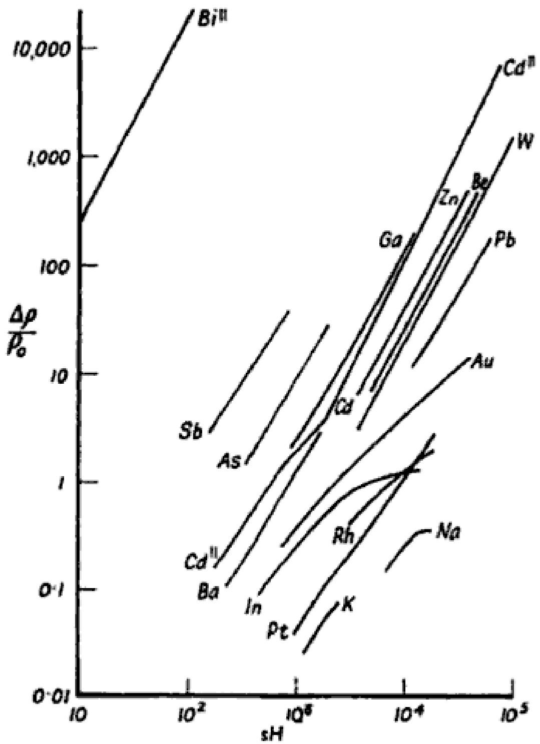


FIG. 158. Reduced Kohler diagram (Kohler, 1949 b).

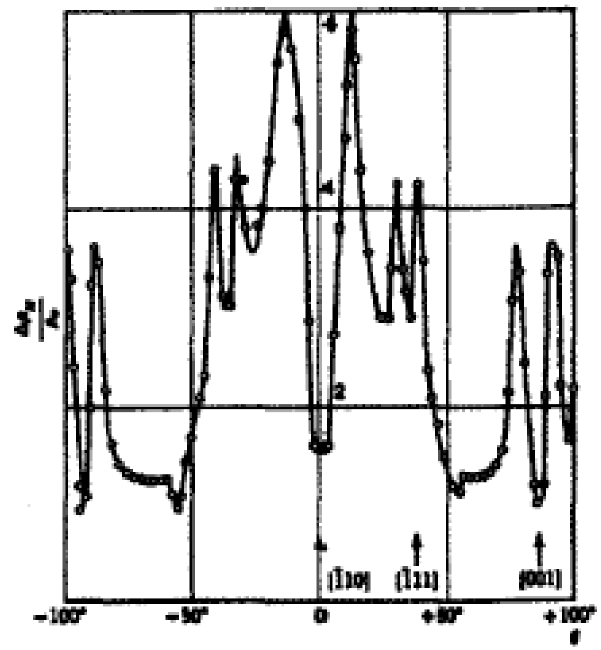


Figure 1: a) reduced Kohler plot giving OMR data for selected metals. $\rho(0)$ = resistance in zero field; ρ_θ = resistance at the Debye Temperature; and B is magnetic field in kOe. [Ref. 1]. b) Variation of transverse MR with field direction for single crystal Au. Current parallel to [110]; B=23.5 kOe. [Ref. 2].

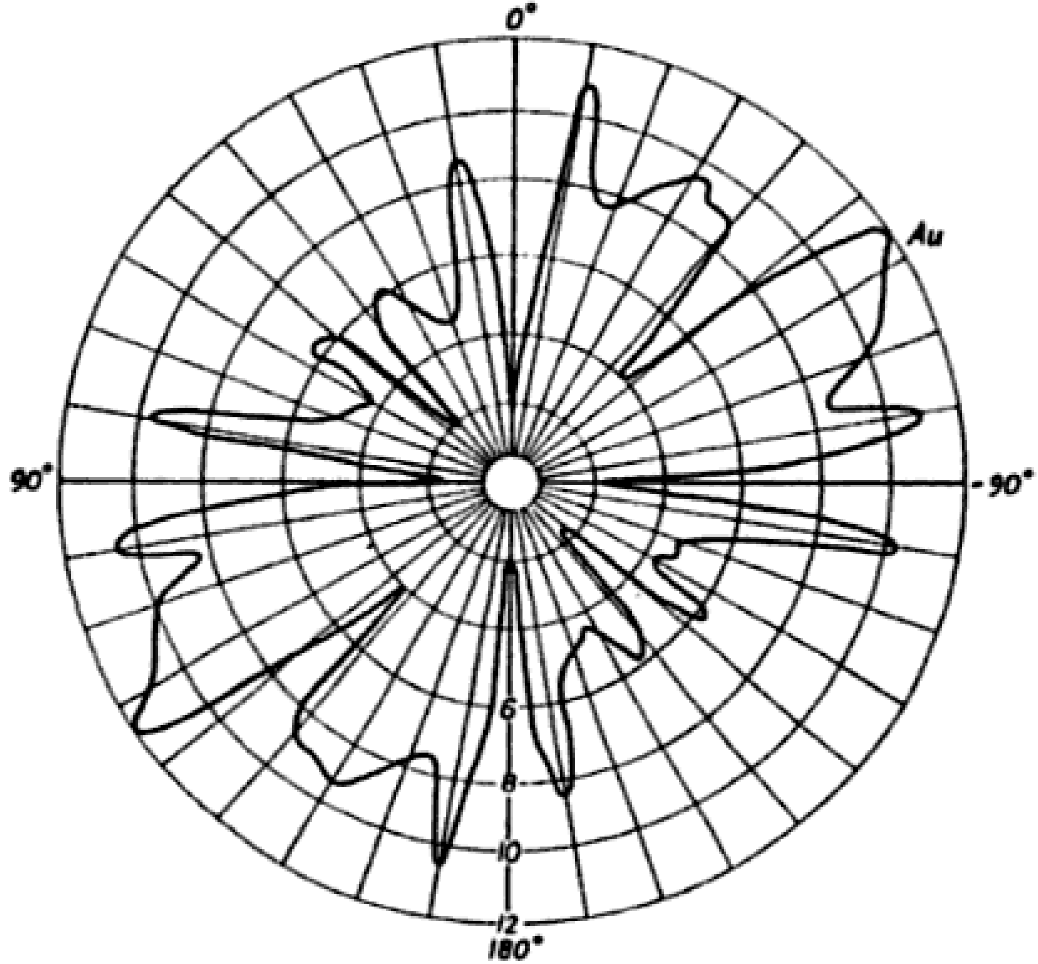


Fig. 159. Magnetoresistance of Au single crystal rotated about the magnetic field (Justi & Scheffers, 1938).

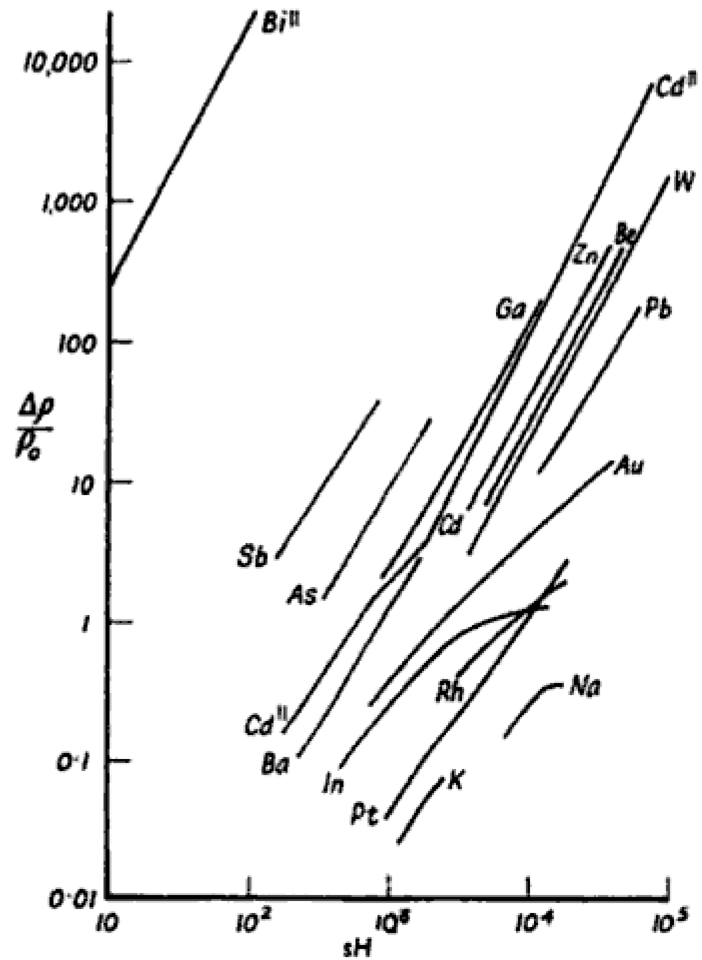


Fig. 158. Reduced Kohler diagram (Kohler, 1949 b).

Kohler's Rule

M. Kohler, Ann. Phys. 32 (1938) 211.

N. Luo, G.H. Miley / Physica C 371 (2002) 259–269

If there is only one relaxation rates in the transport process of a certain conductor, the relative magnetoresistance $\Delta\rho/\rho_0$ in a magnetic field H, can then be represented in the form

$$\boxed{\Delta\rho/\rho_0 = F(H\tau)}$$

where ρ_0 is the resistivity at $H = 0$ and F is a function given only by the intrinsic electronic structure and external geometry of the conductor.

relaxation rate actually changes with some parameters, temperature T for example, but as long as there is a single T dependence of τ, we call it one relaxation rate.

The steady-state Boltzmann equation in the presence of an electric field E and a magnetic field H is then

$$\frac{e}{\hbar}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}_{\mathbf{k}}\times\mathbf{H}\right)\cdot\frac{\partial f_{\mathbf{k}}}{\partial\mathbf{k}}=\left.\frac{\partial f_{\mathbf{k}}}{\partial t}\right|_{\text{scatt}},$$

CGS

or alternatively

$$\frac{e}{\hbar}\mathbf{E}\cdot\frac{\partial f_{\mathbf{k}}}{\partial\mathbf{k}}=\left.\frac{\partial f_{\mathbf{k}}}{\partial t}\right|_{\text{scatt}}-\frac{e}{\hbar c}(\mathbf{v}_{\mathbf{k}}\times\mathbf{H})\cdot\frac{\partial f_{\mathbf{k}}}{\partial\mathbf{k}}$$

the distribution function $f_{\mathbf{k}}=f_{\mathbf{k}}^{0}+g_{\mathbf{k}}$

$$\frac{e}{\hbar}\mathbf{E}\cdot\frac{\partial f_{\mathbf{k}}}{\partial\mathbf{k}}=\left.\frac{\partial f_{\mathbf{k}}}{\partial t}\right|_{\text{scatt}}-\frac{e}{\hbar c}(\mathbf{v}_{\mathbf{k}}\times\mathbf{H})\cdot\frac{\partial(f_{\mathbf{k}}^{0}+g_{\mathbf{k}})}{\partial\mathbf{k}}$$

$$(\mathbf{v_k} \times \mathbf{H}) \cdot \frac{\partial f_{\mathbf{k}}^0}{\partial \mathbf{k}} = (\mathbf{v_k} \times \mathbf{H}) \cdot \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} \frac{\partial f_{\mathbf{k}}^0}{\partial \epsilon_{\mathbf{k}}}$$

$$= (\mathbf{v_k} \times \mathbf{H}) \cdot \hbar \mathbf{v_k} \frac{\partial f_{\mathbf{k}}^0}{\partial \epsilon_{\mathbf{k}}} = 0$$

where $\epsilon_{\mathbf{k}}$ is the energy for state $\mathbf{k}$.

This is not a surprise because all current density vectors are rotated by H without any change in their magnitude.

$$\frac{e}{\hbar} \mathbf{E} \cdot \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} = \left. \frac{\partial f_{\mathbf{k}}}{\partial t} \right|_{\text{scatt}} - \frac{e}{\hbar c} (\mathbf{v_k} \times \mathbf{H}) \cdot \frac{\partial g_{\mathbf{k}}}{\partial \mathbf{k}}$$

•the external field is small so that the deviation from equilibrium is too small in a steady state, and as the consequence $\boxed{f_{\mathbf{k}} = f_{\mathbf{k}}^0 + g_{\mathbf{k}}}$ can be replaced by f^0_k on the left hand side.

•the relaxation-time approximation which hypothesizes that the scattering term is proportional to the deviation g_k and the relaxation rate $1/\tau$

$$\boxed{\left. \frac{\partial f_{\mathbf{k}}}{\partial t} \right|_{\text{scatt}} = -\frac{g_{\mathbf{k}}}{\tau}}$$

$$-e\mathbf{E}\cdot\frac{\partial f_{\mathbf{k}}^{0}}{\partial\mathbf{k}}=\left(\frac{\hbar}{\tau}+\frac{e}{c}\mathbf{v}_{\mathbf{k}}\times\mathbf{H}\cdot\frac{\partial}{\partial\mathbf{k}}\right)g_{\mathbf{k}}$$

which can be solved symbolically

$$g_{\mathbf{k}}=\left[1+(H\tau)\frac{e}{\hbar c}\mathbf{v}_{\mathbf{k}}\times\widehat{\mathbf{H}}\cdot\frac{\partial}{\partial\mathbf{k}}\right]^{-1}\left(-\frac{\tau e}{\hbar}\mathbf{E}\cdot\frac{\partial f_{\mathbf{k}}^{0}}{\partial\mathbf{k}}\right)$$

$$=\left[1+(H\tau)\frac{e}{\hbar c}\mathbf{v}_{\mathbf{k}}\times\widehat{\mathbf{H}}\cdot\frac{\partial}{\partial\mathbf{k}}\right]^{-1}g_{\mathbf{k}}^{0}$$

where

$$g_{\mathbf{k}}^{0}=-\frac{\tau e}{\hbar}\mathbf{E}\cdot\frac{\partial f_{\mathbf{k}}^{0}}{\partial\mathbf{k}}$$

is the deviation of distribution from equilibrium with electric field only, and $\widehat{\mathbf{H}}$ is the unit vector along field $\mathbf{H}$, with H the strength of $\mathbf{H}$.

we may symbolically expand the factor into a series

$$\left[1+(H\tau)\frac{e}{\hbar c}\mathbf{v}_{\mathbf{k}}\times\widehat{\mathbf{H}}\cdot\frac{\partial}{\partial\mathbf{k}}\right]^{-1}$$

$$=1+\sum_{n=1}^{\infty}\left[-(H\tau)\frac{e}{\hbar c}\mathbf{v}_{\mathbf{k}}\times\widehat{\mathbf{H}}\cdot\frac{\partial}{\partial\mathbf{k}}\right]^{n}$$

Recall that the current density $\mathbf{J}$ is generally given by

$$\mathbf{J}=\int e\mathbf{v}_{\mathbf{k}}g_{\mathbf{k}}\,\mathrm{d}\mathbf{k}$$

magnetoresistance

$$\frac{\Delta\rho}{\rho_0}=\frac{\mathbf{E}(\mathbf{J}-\mathbf{J}_0)^{-1}}{\mathbf{E}\mathbf{J}_0^{-1}}$$

a tensor equation

$$=\frac{\mathbf{E}\left[\int e\mathbf{v}_{\mathbf{k}}\boxed{(g_{\mathbf{k}}-g_{\mathbf{k}}^{0})}\mathrm{d}\mathbf{k}\right]^{-1}}{\mathbf{E}\left(\int e\mathbf{v}_{\mathbf{k}}g_{\mathbf{k}}^{0}\,\mathrm{d}\mathbf{k}\right)^{-1}}$$

$$\boxed{g_{\mathbf{k}} - g_{\mathbf{k}}^{0}} = \sum_{n=1}^{\infty} \left[-(H\tau)\frac{e}{\hbar c}\mathbf{v}_{\mathbf{k}} \times \widehat{\mathbf{H}} \cdot \frac{\partial}{\partial \mathbf{k}} \right]^{n} \times \left(-\frac{\tau e}{\hbar}\mathbf{E} \cdot \frac{\partial f_{\mathbf{k}}^{0}}{\partial \mathbf{k}} \right),$$

$$\boxed{\frac{\Delta\rho}{\rho_0} = \frac{\mathbf{E}\left\{ \int \mathbf{v}_{\mathbf{k}} \sum_{n=1}^{\infty} \left[-(H\tau)\frac{e}{\hbar c}\mathbf{v}_{\mathbf{k}} \times \widehat{\mathbf{H}} \cdot \frac{\partial}{\partial \mathbf{k}} \right]^{n} \left(\mathbf{E} \cdot \frac{\partial f_{\mathbf{k}}^{0}}{\partial \mathbf{k}} \right) \mathrm{d}\mathbf{k} \right\}^{-1}}{\mathbf{E}\left(\int \mathbf{v}_{\mathbf{k}} \mathbf{E} \cdot \frac{\partial f_{\mathbf{k}}^{0}}{\partial \mathbf{k}} \, \mathrm{d}\mathbf{k} \right)^{-1}}}$$

which is a tensor equation too.

$$\boxed{\frac{\Delta\rho}{\rho_0} = F(H\tau).}$$

the accurate form of Kohler’s rule

Kohler's rule in its approximate form

Notice the resistivity under zero magnetic field ρ_0 is related to the relaxation rate τ by $\rho_0 = m^*/ne^2\tau$, where e, m^* and n are respectively the charge, effective mass and effective density of carriers. Then Kohler's rule is re-written

$$\frac{\Delta\rho}{\rho_0} = F\left(\frac{H}{\rho_0}\frac{m^*}{ne^2}\right)$$

If the factor m^*/ne^2 does not change with T

$$\boxed{\frac{\Delta\rho}{\rho_0} = F\left(\frac{H}{\rho_0}\right)}$$

The validity requires that m^*/ne^2 be independent of T. Of these quantities, e should not change with T while m^* may vary slowly due to mass renormalization effects. Most of the problem however comes from n which could be very sensitivity to T in various conductors, even in some metals such as Bi.

We use the metal Bi for the purpose of illustration.

- Bi is a semimetal as inferred from its band structure and it is well known that the relaxation rate assumes a single T dependence from 140 K upwards [5], up to room temperature at least.
- it has both electrons and holes as charge carriers

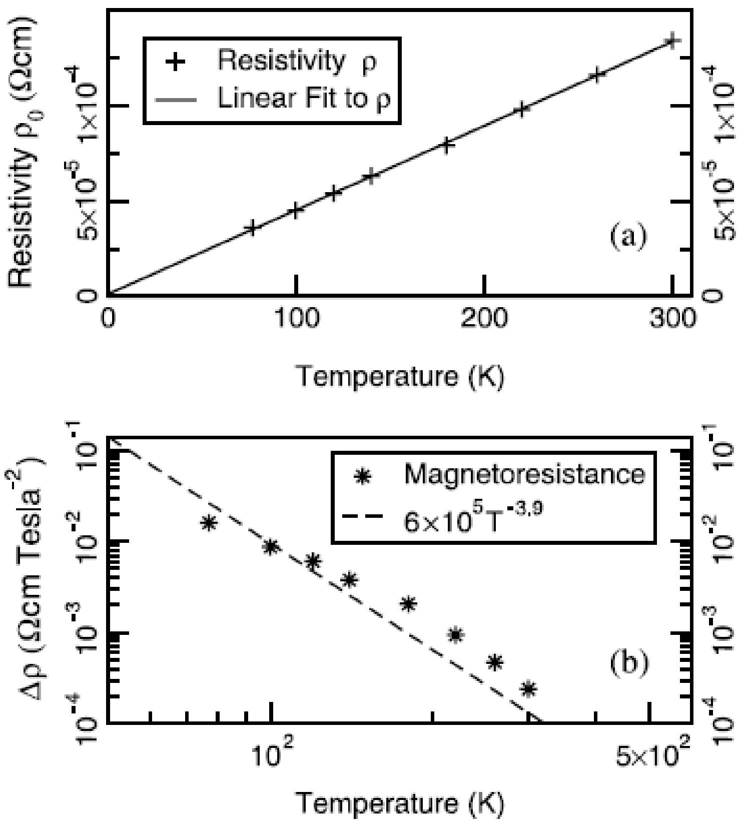


Fig. 1. (a) Resistivity ρ_0 (that of ρ_{33}) and (b) magnetoresistance $\Delta\rho$ (A_{33}) versus temperature T in bismuth. 3 here refers to the trigonal axis. Data are from Ref. [5].

n and *τ* are derived from the Hall coefficient R_H and the cotangent of Hall angle cot θ_H

$$R_{\mathrm{H}} = \frac{1}{nec},$$

$$\cot\theta_{\mathrm{H}} = \frac{m^* c}{eH\tau},$$

$$\rho_0 = \frac{m^*}{ne^2\tau}.$$

Experimentally $\Delta\rho \propto H^2 T^{-3.9}$

approximately $\Delta\rho \propto H^2 T^{-4}$

$$\frac{\Delta\rho}{\rho_0} \propto \frac{H^2 T^{-4}}{T}$$ because $\rho_0 \propto T$.

[5] J.P. Michenaud, J.P. Issi, J. Phys. C 5 (1972) 3061

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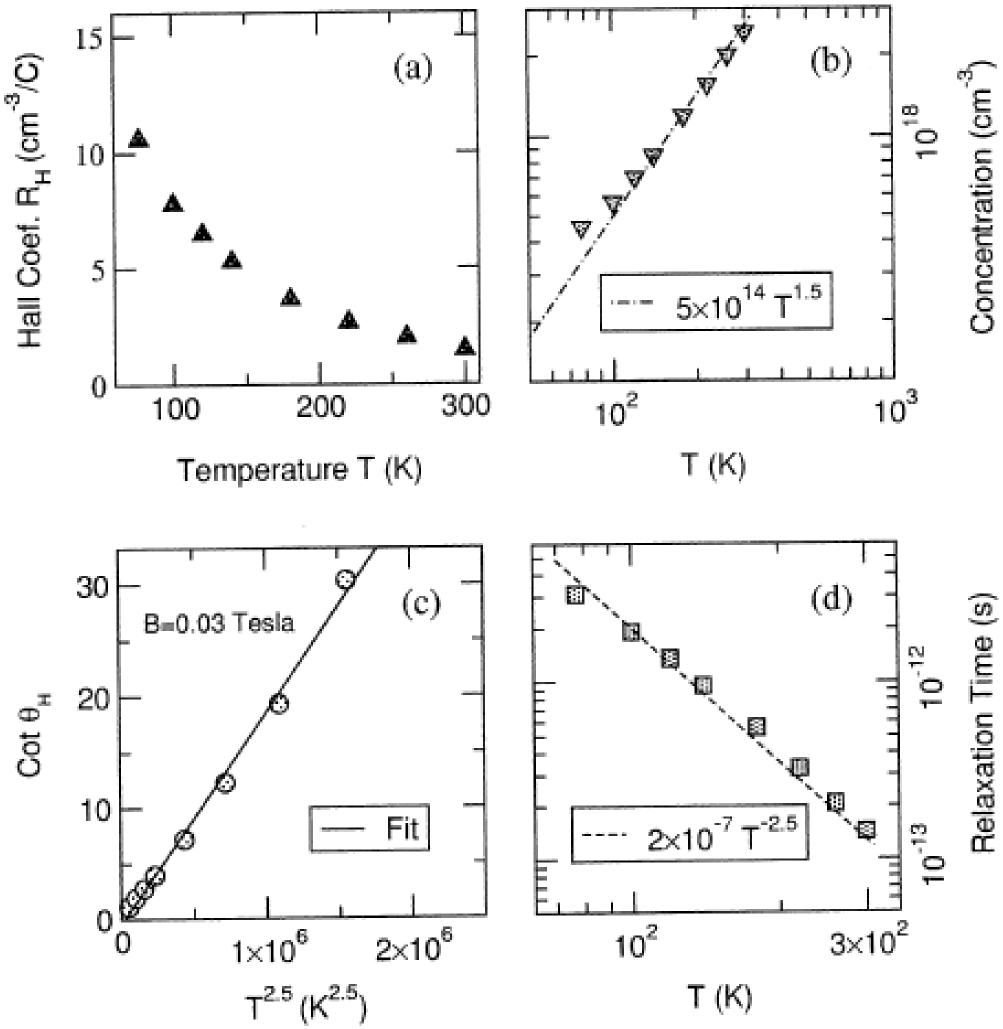


Fig. 2. Hall results versus T in bismuth. (a) The Hall coefficient R_H (R_{231}). (b) Concentrations n, of both electrons and holes. (c) $\cot\theta_H$ calculated from ρ_0 and R_H. (d) Relaxation time calculated from n, ρ_0 and a nominal electron effective mass $m^* = 1.3 \times 10^{-32}$ kg. The exact value of m^* does not affect the T scaling behavior. 1, 2, and 3 here refer to the binary, bisectrix, and trigonal axes, respectively. Data are from Ref. [5].

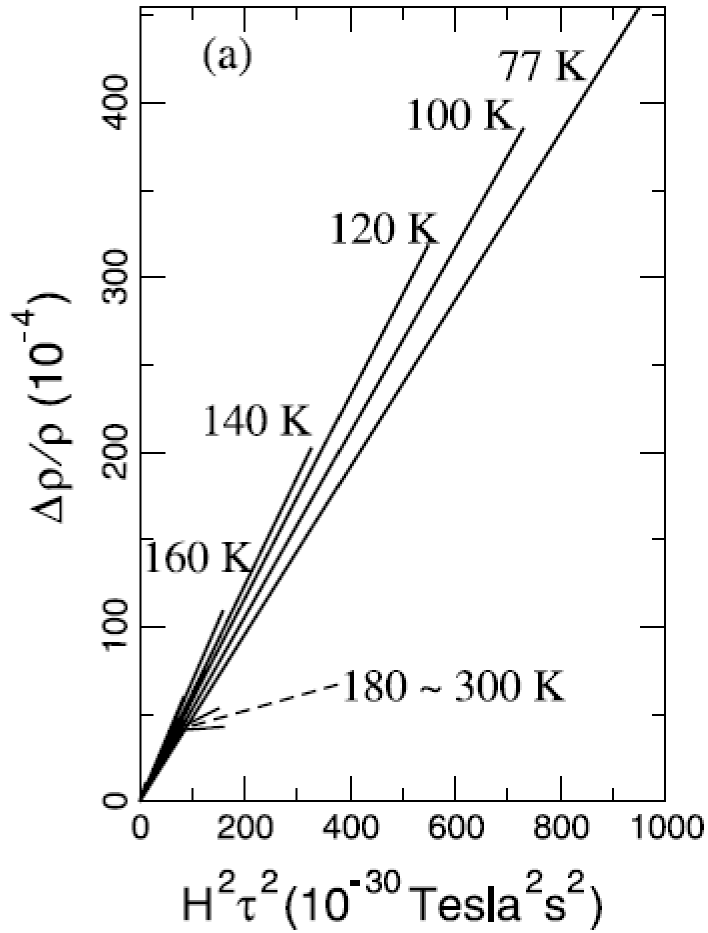
$$\frac{\Delta\rho}{\rho_0} \propto \frac{H^2 T^{-4}}{T}$$

$$\frac{\Delta\rho}{\rho_0} \propto H^2 T^{-5}.$$

$$\tau \propto T^{-2.5}$$

$$\tau^2 \propto (T^{-2.5})^2 = T^{-5}$$

$$\boxed{\frac{\Delta\rho}{\rho_0} \propto H^2 \tau^2 = (H\tau)^2}$$



T and H dependence of major transport coefficients of single crystal bismuth from 77 to 300 K

ρ_0	$\Delta\rho$	R_H	$\cot\theta_H$	n	τ
T	$H^2T^{-3.9}$	$T^{-1.5}$	$H^{-1}T^{2.5}$	$T^{1.5}$	$T^{-2.5}$

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A correct Kohler plot for bismuth using $\Delta\rho/\rho_0 = F(H\tau)$.

M. Kohler, Ann. Phys. 32 (1938) 211.

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Kohler's rule:

$$\frac{\Delta\rho}{\rho} \propto \left(\frac{H}{\rho}\right)^2$$

This increase of the resistivity of a metal is due to the Lorenz force on the electrons and thus of the same origin as the Hall effect. The Hall resistivity is linear in field, whereas the change in resitivity is quadratic to first order.

In ferromagnetic materials, the spontaneous magnetisation affects the conduction electron paths and the alignment of this magnetisation by an external magnetic field yields strongly enhanced effects compared to non-magnetic materials. These contributions to the Hall effect and magnetoresistance are called extraordinary, spontaneous or anomalous, to distinguish them from the corresponding ordinary effects.