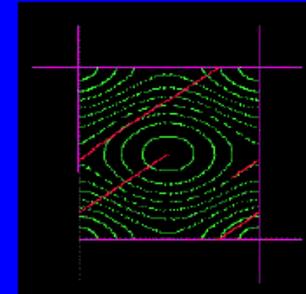
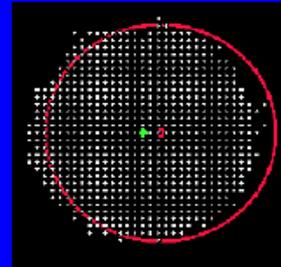
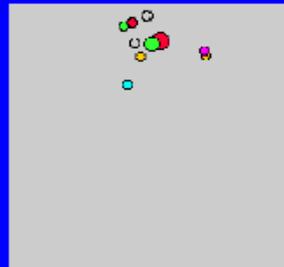
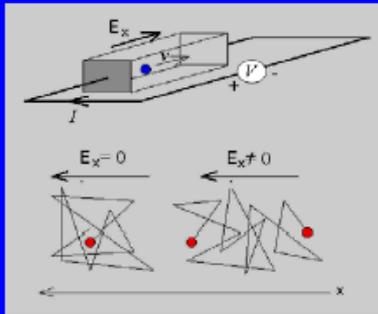


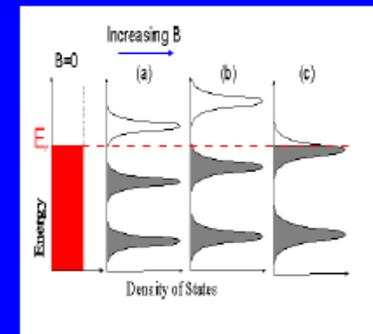
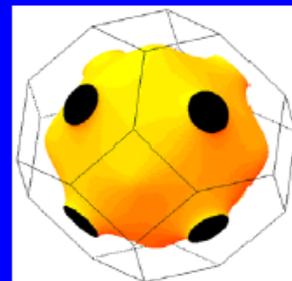
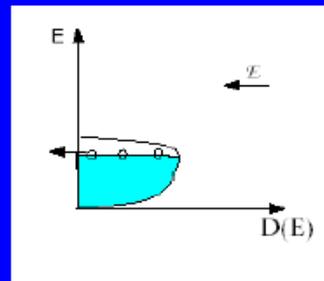
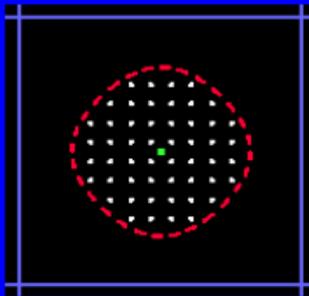
# Transport Phenomena in Solids

## Motions of electrons and transport phenomena



$$\sigma = \frac{ne^2\tau}{m}$$

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2} \sum_j \frac{\partial^2 E(\vec{k})}{\partial k_i \partial k_j}$$



# Thermoelectric effect

See Mizutani

the temperature gradient can also induce an electrical current.

linearized Boltzmann transport equation in combination with the relaxation time approximation.

$$-\mathbf{v}_{\mathbf{k}} \cdot \nabla f(\mathbf{r}, \mathbf{k}) - \frac{(-e)}{\hbar} (\mathbf{E} + \mathbf{v}_{\mathbf{k}} \times \mathbf{B}) \cdot \frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} = - \left( \frac{\partial f}{\partial t} \right)_{\text{scatter}} .$$

Boltzmann  
equation

Relaxation time  
approximation

$$- \left( \frac{\partial f}{\partial t} \right)_{\text{scatter}} = \frac{f(\mathbf{r}, \mathbf{k}) - f_0(\varepsilon_{\mathbf{k}}, T)}{\tau} = \frac{\phi(\mathbf{r}, \mathbf{k})}{\tau}$$

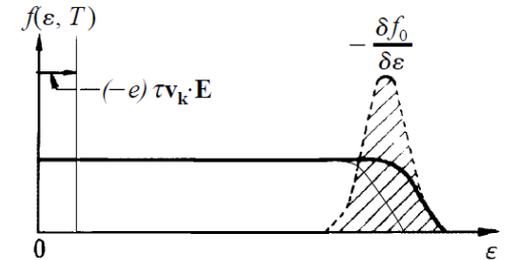
$$\phi(\mathbf{r}, \mathbf{k}) = f(\mathbf{r}, \mathbf{k}) - f_0(\varepsilon_{\mathbf{k}}, T)$$

$$f_0(\varepsilon_{\mathbf{k}}, T) = 1 / \{ \exp[(\varepsilon_{\mathbf{k}} - \zeta) / k_{\text{B}} T] + 1 \}$$

$$\frac{\partial f_0}{\partial T} = - \left( \frac{\partial f_0}{\partial \varepsilon} \right) \left[ \left( \frac{\varepsilon - \zeta}{T} \right) + \frac{\partial \zeta}{\partial T} \right],$$

$$\left( -\frac{\partial f_0}{\partial \varepsilon} \right) \mathbf{v}_{\mathbf{k}} \cdot \left[ -\left( \frac{\varepsilon(\mathbf{k}) - \zeta}{T} \right) \nabla T + (-e) \left( \mathbf{E} - \frac{\nabla \zeta}{(-e)} \right) \right]$$

$$= -\left( \frac{\partial f}{\partial t} \right)_{\text{scatter}} + \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial \phi}{\partial \mathbf{r}} + \frac{(-e)}{\hbar} (\mathbf{v}_{\mathbf{k}} \times \mathbf{B}) \cdot \frac{\partial \phi}{\partial \mathbf{k}}$$



This is the linearized Boltzmann transport equation.

$$\nabla \zeta$$

is included as an extra electric field, since it represents an effective field associated with a change in the chemical potential induced by the temperature gradient

$$\mathbf{B}=0$$

The number of electrons per unit volume

$$n = (1/4\pi^3) \iiint f_0(\mathbf{k}) d\mathbf{k}$$

The current density

$$\mathbf{J} = \frac{(-e)}{4\pi^3} \iiint \mathbf{v}_{\mathbf{k}} f(\mathbf{k}) d\mathbf{k} = \frac{(-e)}{4\pi^3} \iiint \mathbf{v}_{\mathbf{k}} [f(\mathbf{k}) - f_0(\mathbf{k})] d\mathbf{k}$$

$$= \frac{(-e)}{4\pi^3} \iiint \mathbf{v}_{\mathbf{k}} \phi(\mathbf{k}) d\mathbf{k}, \quad \int \mathbf{v}_{\mathbf{k}} f_0(\mathbf{k}) d\mathbf{k} = 0$$

$$\iiint d\mathbf{k} = \iint dS \int dk_{\perp} = \iint dS \int \frac{d\epsilon}{|\partial\epsilon/\partial k_{\perp}|} = \iint dS \int \frac{d\epsilon}{|\nabla_{k_{\perp}}\epsilon|},$$

where  $\iint dS$  indicates the integral over a constant energy surface and  $\int dk_{\perp}$  is the integral along its normal direction.

$$\begin{aligned} \mathbf{J} = & \frac{e^2\tau}{4\pi^3\hbar} \iint \mathbf{v}_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \left( -\frac{\partial f_0}{\partial \epsilon} \right) \frac{dS}{v_{k_{\perp}}} d\epsilon \cdot \mathbf{E}' \\ & + \frac{(-e)\tau}{4\pi^3\hbar} \iint \mathbf{v}_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \left( \frac{\epsilon - \zeta}{T} \right) \left( -\frac{\partial f_0}{\partial \epsilon} \right) \frac{dS}{v_{k_{\perp}}} d\epsilon \cdot (-\nabla T), \end{aligned}$$

where  $\mathbf{E}' = \mathbf{E} - [\nabla\zeta/(-e)]$

$$\mathbf{J} = \frac{e^2 \tau}{4\pi^3 \hbar} \int \int \mathbf{v}_k \mathbf{v}_k \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \frac{dS}{v_{k_\perp}} d\varepsilon \cdot \mathbf{E}'$$

$$+ \frac{(-e)\tau}{4\pi^3 \hbar} \int \int \mathbf{v}_k \mathbf{v}_k \left( \frac{\varepsilon - \zeta}{T} \right) \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \frac{dS}{v_{k_\perp}} d\varepsilon \cdot (-\nabla T),$$



$$\mathbf{J} = L_{EE} \mathbf{E} + L_{ET} \nabla T$$

$$\mathbf{U} = L_{TE} \mathbf{E} + L_{TT} \nabla T,$$

The second term indicates that the temperature gradient can also induce an electrical current.

**This is the thermoelectric effect.**

$L_{EE}$  is the electrical conductivity,

$\sigma = L_{EE}$  holds under the isothermal condition  $\nabla T = 0 \iff \mathbf{J} = \sigma \mathbf{E}$

Let us assume that a metal is in a temperature gradient but is electrically open  $\rightarrow \mathbf{J} = 0$

$$\mathbf{J} = L_{EE}\mathbf{E} + L_{ET}\nabla T$$



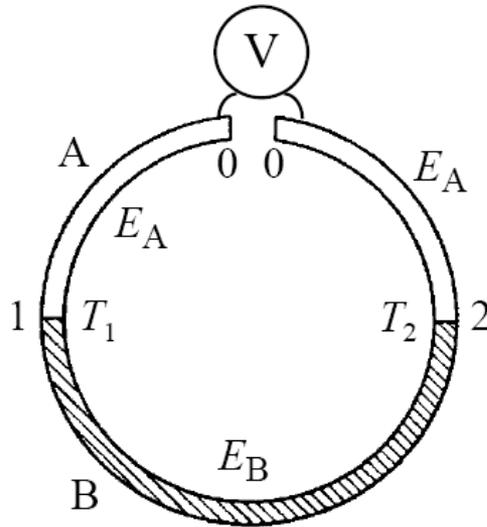
$$\mathbf{E} = Q\nabla T$$

$$Q = -\left(\frac{L_{ET}}{L_{EE}}\right)$$

The coefficient  $Q$  is called the absolute thermoelectric power or the Seebeck coefficient.

an electric field is generated due to a temperature gradient across a specimen

an electric field is generated due to a temperature gradient across a specimen



$$\mathbf{E} = Q \nabla T$$

Circuit for the measurement of the Seebeck effect.

The electromotive force

$$\begin{aligned} \phi &= \int_0^1 E_A dx + \int_1^2 E_B dx + \int_2^0 E_A dx = \int_2^1 Q_A \left( \frac{\partial T}{\partial x} \right) dx + \int_1^2 Q_B \left( \frac{\partial T}{\partial x} \right) dx \\ &= \int_{T_2}^{T_1} Q_A dT + \int_{T_1}^{T_2} Q_B dT = \int_{T_1}^{T_2} (Q_B - Q_A) dT. \end{aligned}$$

the voltage generated in the circuit is obtained by integrating the difference in the thermoelectric power of the two metals between the temperatures  $T_1$  and  $T_2$  at the two junctions.

$$\mathbf{J} = \frac{e^2 \tau}{4\pi^3 \hbar} \iint \mathbf{v}_k \mathbf{v}_k \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \frac{dS}{v_{k_\perp}} d\varepsilon \cdot \mathbf{E}'$$

$$+ \frac{(-e)\tau}{4\pi^3 \hbar} \iint \mathbf{v}_k \mathbf{v}_k \left( \frac{\varepsilon - \zeta}{T} \right) \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \frac{dS}{v_{k_\perp}} d\varepsilon \cdot (-\nabla T),$$

The expression for the thermoelectric power  $Q$  is derived as follows.

$$-L_{ET} = \frac{(-e)\tau}{4\pi^3 \hbar} \iint \mathbf{v}_k \mathbf{v}_k \left( \frac{\varepsilon - \zeta}{T} \right) \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \frac{dS}{v_{k_\perp}} d\varepsilon,$$

$$= \frac{(-e)\tau}{4\pi^3 \hbar} \cdot \frac{1}{T} \left\{ \left[ \int \mathbf{v}_k \mathbf{v}_k (\varepsilon - \zeta) \frac{dS}{v_{k_\perp}} \right]_{\varepsilon=\zeta} \right. \quad \left. \begin{array}{l} \text{from} \\ \text{Mizutani} \end{array} \right.$$

$$+ \frac{\pi^2}{6} (k_B T)^2 \left[ \int (\varepsilon - \zeta) \frac{\partial^2}{\partial \varepsilon^2} \left( \mathbf{v}_k \mathbf{v}_k \frac{dS}{v_{k_\perp}} \right) + 2 \int \frac{\partial}{\partial \varepsilon} \left( \mathbf{v}_k \mathbf{v}_k \frac{dS}{v_{k_\perp}} \right) \right]_{\varepsilon=\zeta} + \dots \left. \right\}.$$

Here the terms involving  $(\varepsilon - \zeta)$  disappear at the Fermi energy  $\zeta$ .

$$I = \int_0^\infty f(E, T) \left( \frac{dF(E)}{dE} \right) dE, \quad I = F[E_F(T)] + \left( \frac{\pi^2}{6} \right) (k_B T)^2 \left[ \frac{d^2 F(E)}{dE^2} \right]_{E=E_F(T)} + \dots$$

$$-L_{ET} = \frac{\pi^2}{3} (k_B T)^2 \cdot \frac{1}{T} \left\{ \frac{(-e)\tau}{4\pi^3 \hbar} \left[ \int \frac{\partial}{\partial \varepsilon} \left( \mathbf{v}_k \mathbf{v}_k \frac{dS}{v_{k_\perp}} \right) \right]_{\varepsilon=\zeta} \right\}.$$

$L_{EE}$  represents the electrical conductivity

$$L_{EE} = \left[ \frac{(-e)^2 \tau}{4\pi^3 \hbar} \int \mathbf{v}_k \mathbf{v}_k \frac{dS}{v_{k_\perp}} \right]_{\varepsilon=\zeta} = [\sigma(\varepsilon)]_{\varepsilon=\zeta}$$

Therefore, the thermoelectric power  $Q$  is formulated as

$$Q = \frac{\pi^2}{3(-e)} k_B^2 T \left[ \frac{\partial \ln \sigma(\varepsilon)}{\partial \varepsilon} \right]_{\varepsilon=\zeta}$$

The thermoelectric power can be calculated, once the energy dependence of the conductivity is given.

$$\mathbf{E} = Q \nabla T$$

a potential difference  $\int_{x_1}^{x_2} E dx$  is generated,  $\sigma_T \Delta T$  when a temperature difference  $\Delta T = T_2 - T_1$  exists between the two points  $x_1$  and  $x_2$  in a metal bar.

$$Q = \int_0^T (\sigma_T / T) dT$$

where  $\sigma_T$  is called the Thomson coefficient

from Mizutani

If an electronic charge ( $-e$ ) flows up through the temperature gradient  $dT$ , a heat equal to  $(-e)\sigma_T dT$  must be evolved per electron. Hence, we obtain the relation  $\sigma_T = C_{el} / n(-e)$ , where  $C_{el}$  is the electronic specific heat per unit volume and  $n$  is the number of electrons involved. Its insertion into the relation above immediately results in

$$Q = \left[ \frac{1}{n(-e)} \right] \int_0^T \frac{C_{el}}{T} dT = \frac{s}{n(-e)}$$

where  $s$  is the electronic entropy density.

Let us apply the [free-electron model](#) to the electrical conductivity formula

$$\sigma = \frac{e^2}{4\pi^3} \int \frac{\tau v_i^2 dS_F}{\hbar v_{k_\perp}} = \frac{e^2 \tau}{4\pi^3 \hbar v_F} \cdot \frac{v_F^2}{3} \int dS_F = \frac{e^2 \tau v_F S_F}{12\pi^3 \hbar} \quad \Rightarrow \quad \sigma(\epsilon) \propto \epsilon^{3/2}$$

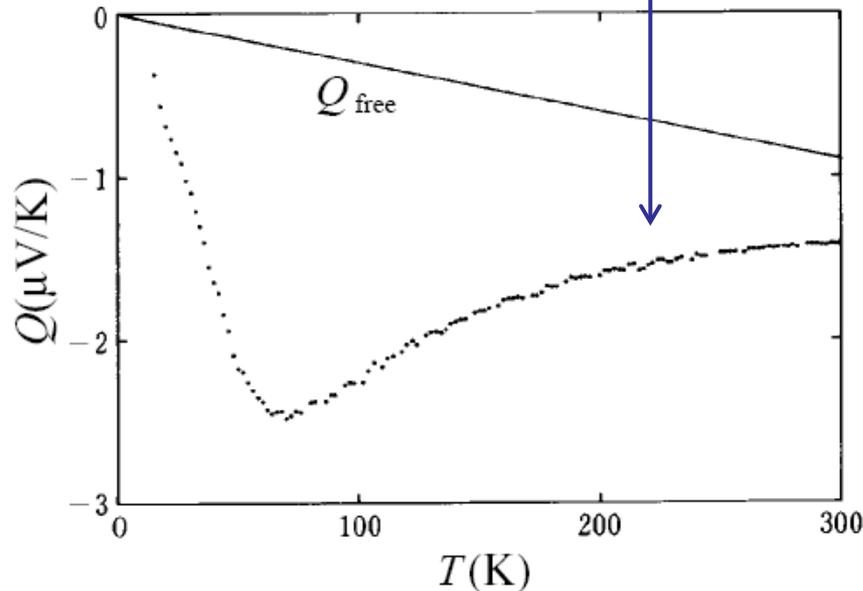
$$Q_{\text{free}} = \frac{\pi^2 k_B T}{2(-e)T_F} \approx -4.25 \times 10^2 \frac{T}{T_F} [\mu\text{V/K}]$$

where  $T_F$  is the Fermi temperature

Note that the thermoelectric power for ordinary metals is fairly small in magnitude, since  $T/T_F$  is only 0.001–0.005 at room temperature.

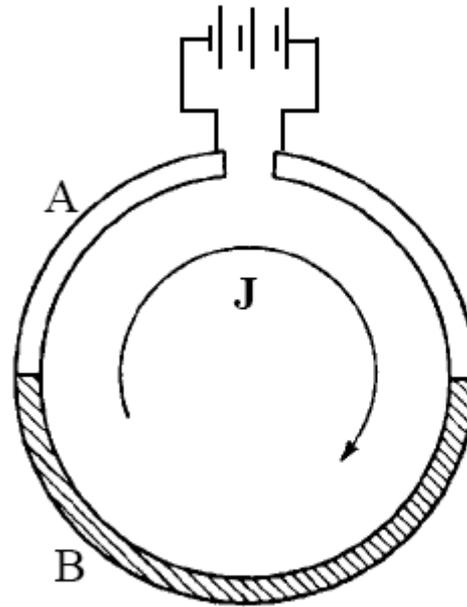
As a [typical example](#), the thermoelectric power for a well-annealed strain-free pure Al metal(99.999%), together with the free-electron behavior , Fermi temperature  $T_F=1.35 \times 10^5$  K

the experimental data deviate substantially from the free-electron model and exhibit a minimum at about 70 K. The formation of the minimum has been attributed to the phonon drag effect unique to a crystal metal, where the phonon mean free path is long.



from Mizutani

# Peltier Effect



from Mizutani

Circuit for the measurement of the Peltier effect.

Two different metals A and B are joined and connected to a battery,

An electrical current density  $\mathbf{J}$  is fed through the circuit while the circuit is maintained at a uniform temperature.

$$\begin{array}{|c|} \hline \mathbf{J} = L_{EE} \mathbf{E} + L_{ET} \nabla T \\ \hline \mathbf{U} = L_{TE} \mathbf{E} + L_{TT} \nabla T, \\ \hline \end{array}
 \quad \Rightarrow \quad
 \begin{array}{|c|} \hline \mathbf{U} = L_{TE} \mathbf{E} \\ \hline \mathbf{J} = L_{EE} \mathbf{E} \\ \hline \end{array}
 \quad \Rightarrow \quad
 \boxed{\mathbf{U} = \Pi \mathbf{J}}$$

$$\Pi = \frac{L_{TE}}{L_{EE}}$$

The coefficient  $\Pi$  is called the Peltier coefficient and is related to the thermo-electric power  $Q$  through the relation:

$$Q = -\left(\frac{L_{ET}}{L_{EE}}\right) = \left(\frac{L_{TE}/T}{L_{EE}}\right) = \frac{\Pi}{T}$$

from Mizutani

$$\mathbf{U} = \Pi \mathbf{J}$$



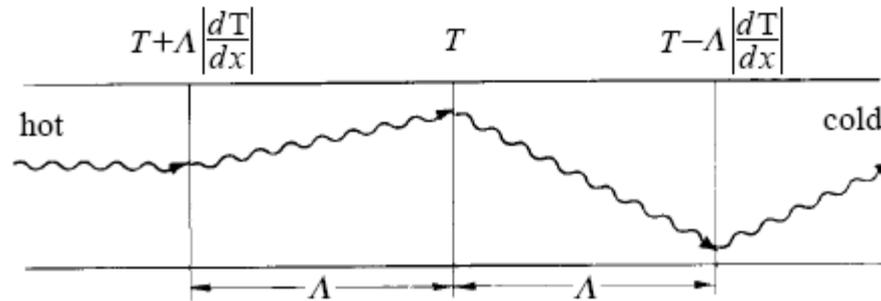
an electrical current fed to the circuit generates thermal currents  $\mathbf{U}_A = \Pi_A \mathbf{J}$  and  $\mathbf{U}_B = \Pi_B \mathbf{J}$  in the metals A and B, respectively.

Thus, a heat flux  $(\Pi_A - \Pi_B) \mathbf{J}$  will be emitted at one junction and absorbed at the other junction. As a consequence, the one junction becomes **hotter**, the other junction **colder**. This is the Peltier effect.

# Phonon drag effect

a voltage is generated between the two ends of a sample across which a temperature gradient  $\nabla T$  exists.

- there exists no current flow due to conduction electrons because of an open circuit.
- phonons at the high-temperature end are driven to the colder end under a finite temperature gradient.
- If the mean free path of the **phonon** is very long, then the collision of one phonon with other phonons is so scarce that its energy cannot be released to the lattice system.
- Instead, phonons can exchange their energy with **electrons**, since the relaxation time for the **phonon–electron** interaction is much shorter than that for the **phonon–phonon** interaction.
- the extra local energy carried by a phonon is fed back to the electron system, resulting in a new extra electric field because of  $\mathbf{J}=0$ .
- The generation of the electric field in the electron system due to the flow of the non-equilibrium phonon is called the **phonon drag effect**
- electrons are carried along by the flow of phonons caused by the temperature gradient.**



Phonon drag effect. [Reproduced from H. M. Rosenberg, *Low Temperature Solid State Physics* (Clarendon Press, Oxford, 1963)]

thermal current density of the phonon at the “hot” end:  $U(T + \Lambda |dT/dx|)$

thermal current density of the phonon at the “cold” end:  $U(T - \Lambda |dT/dx|)$

The difference in thermal energy in the region over  $2\Lambda$ :

$$U\left(T + \Lambda \left|\frac{dT}{dx}\right|\right) - U\left(T - \Lambda \left|\frac{dT}{dx}\right|\right) = 2\Lambda C_{\text{lattice}} \left|\frac{dT}{dx}\right|$$

$$C_{\text{lattice}} = dU/dT$$

This extra energy has to be absorbed in this region, where the only sink available is that provided by  $2\Lambda n$  electrons. Thus, the extra energy must be converted into an electric field  $\Delta E$  and its magnitude is derived from the relation:

$$2\Lambda n(-e)\Delta E = 2\Lambda C_{\text{lattice}} \left| \frac{dT}{dx} \right| = 2\Lambda C_{\text{lattice}} \nabla T$$



from Mizutani

$$\Delta E = \frac{C_{\text{lattice}}}{(-e)n} \nabla T.$$

The thermoelectric power due to phonon drag,  $\Delta E = Q_{\text{ph. drag}} \nabla T$ , is given by

$$Q_{\text{ph. drag}} = \frac{C_{\text{lattice}}}{(-e)n}$$

$$Q_{\text{ph. drag}} = \frac{C_{\text{lattice}}}{(-e)n}$$

$$T > \Theta_D \rightarrow$$

$$C_{\text{lattice}} = 3R$$

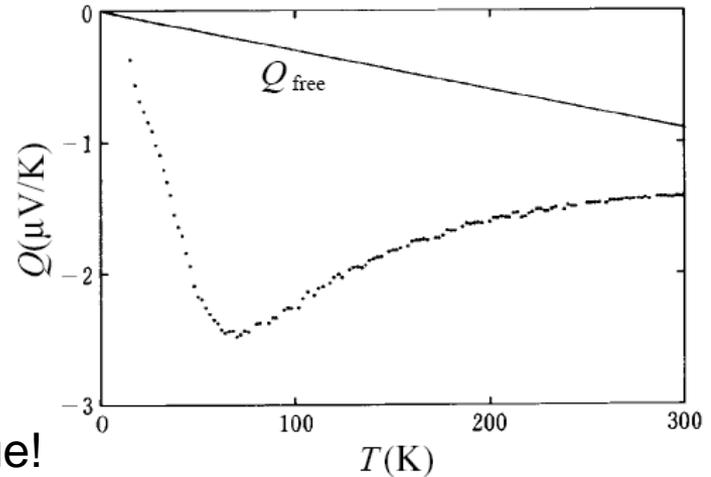
Dulong–Petit law

$$Q_{\text{ph. drag}} = -86 \mu\text{V/K}$$

much larger than the measured value!

Not valid at

$$T > \Theta_D$$



the mean free path of the phonon becomes so short that the phonon drag effect is known to become unimportant at such high temperatures.

$$Q_{\text{ph. drag}} = \frac{C_{\text{lattice}}}{(-e)n}$$

$$T < \Theta_D$$

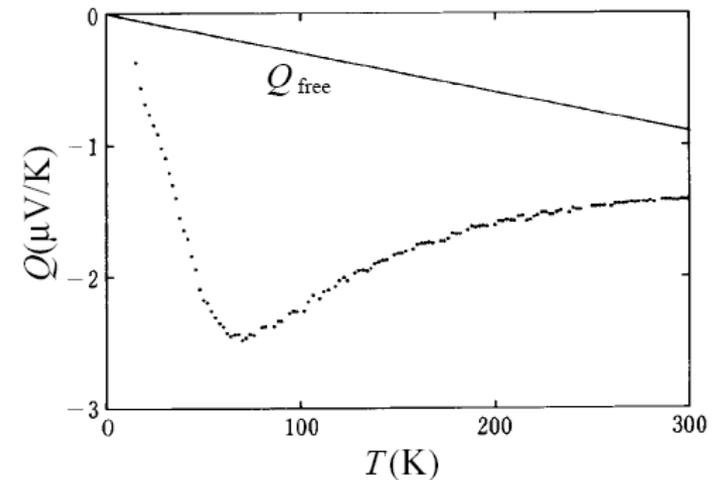
The lattice specific heat decreases as  $T^3$  below about 20 K



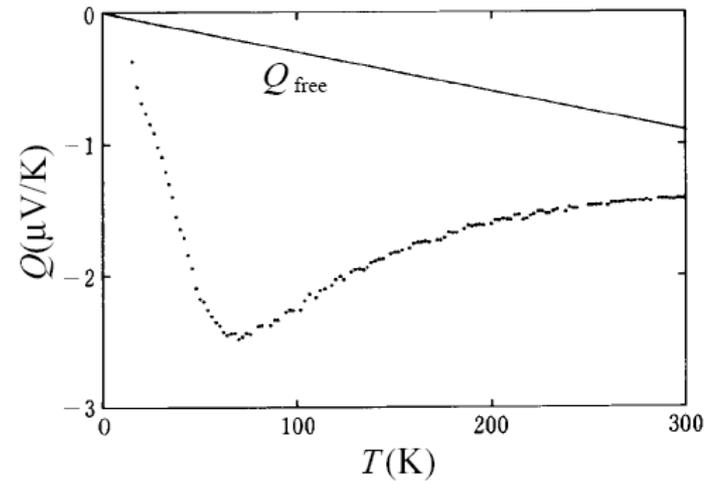
the phonon drag effect becomes ineffective again at low temperatures.

it is most significant in the intermediate temperature range around  $T/\Theta_D \approx 0.2$

is responsible for the formation of a deep valley



The **valley** becomes shallower in alloys because of the shortening of the mean free path of phonons due to the disruption of the periodic lattice.



The phonon drag effect is essentially absent in amorphous alloys because of the lack of lattice periodicity.

Hence, the temperature dependence of the thermoelectric power in amorphous alloys is attributed to other effects like the inelastic electron–phonon interaction and the energy dependence of the relaxation time.

# Thermoelectric power in metals and semiconductors

The interpretation of the measured thermoelectric power is not straightforward even in simple metals

e.g. the sign of the thermoelectric power  $Q$  in the alkali metals cannot be correctly predicted from the free electron model.

positive for Li but is negative for Na and K, though all these metals possess a single-electron Fermi surface.

$$Q = \frac{\pi^2}{3(-e)} k_B^2 T \left[ \frac{\partial \ln \sigma(\varepsilon)}{\partial \varepsilon} \right]_{\varepsilon=\zeta}$$

from Mizutani

the energy dependence of the relaxation time and inelastic electron–phonon interaction

substance	$Q$ ( $\mu\text{V/K}$ ) at 273 K <sup>a</sup>
Ag	1.38
Al	−1.6
Au	1.74
Ca	10.3
Cd	2.56
Cs	−0.9
Cu	1.5
Fe	16.2
K	−12.8
Li	10.6
Mg	−1.47
Na	−5.8
Pb	−1.25
Pd	−9.7
Rb	−9.47
Bi <sub>2</sub> Te <sub>3</sub>	+250 (p); −300 (n)
FeSi <sub>2</sub>	+250 (p); −250 (n)

*Note:*

<sup>a</sup> The measuring temperatures for Bi<sub>2</sub>Te<sub>3</sub> and FeSi<sub>2</sub> are 300–500 K and above 900 K, respectively.

- positive thermoelectric power has been observed in monovalent noble metals

Contact of the Fermi surface with the {111} zone planes has been suggested to play an important role in its behavior.

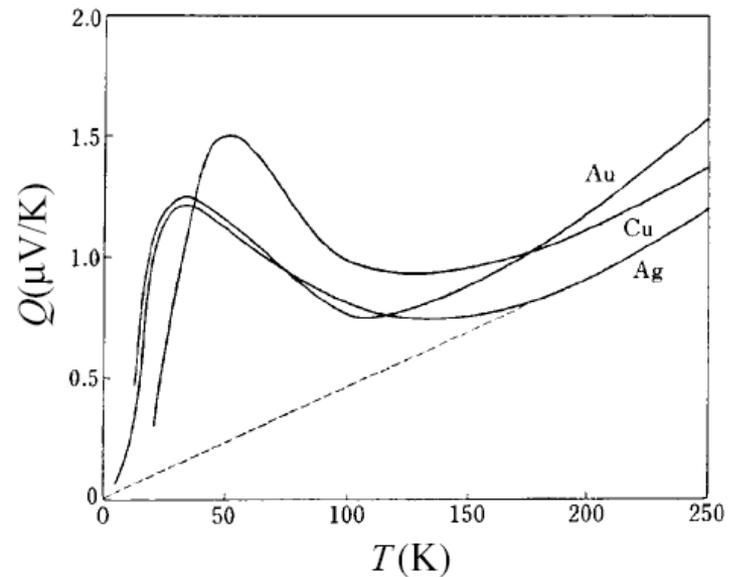


Figure 11.9. Temperature dependence of the thermoelectric power of the noble metals Cu, Ag and Au. [D. K. C. MacDonald, *Principles of Thermoelectricity* (John Wiley & Sons, Inc., New York 1962) p. 71]

to synthesize thermoelectric device materials to convert efficiently **heat to electricity or vice versa.**

Ordinary metals possess the Fermi temperature of  $10^4$ – $10^5$ K and, thus, the resulting thermoelectric power is, at most, 10–20  $\mu$ V/K.

the heat–current conversion efficiency for a thermoelectric material.

Figure of merit

$$ZT = TQ^2 / \kappa\rho$$

**Intrinsic** semiconductors are not important for practical thermo-electric devices since the contributions of electrons and holes are of opposite signs and tend to cancel.

A large value of  $Q$  should be achievable, not in metals, but in **heavily doped semiconductors**.

$$ZT = TQ^2 / \kappa\rho$$

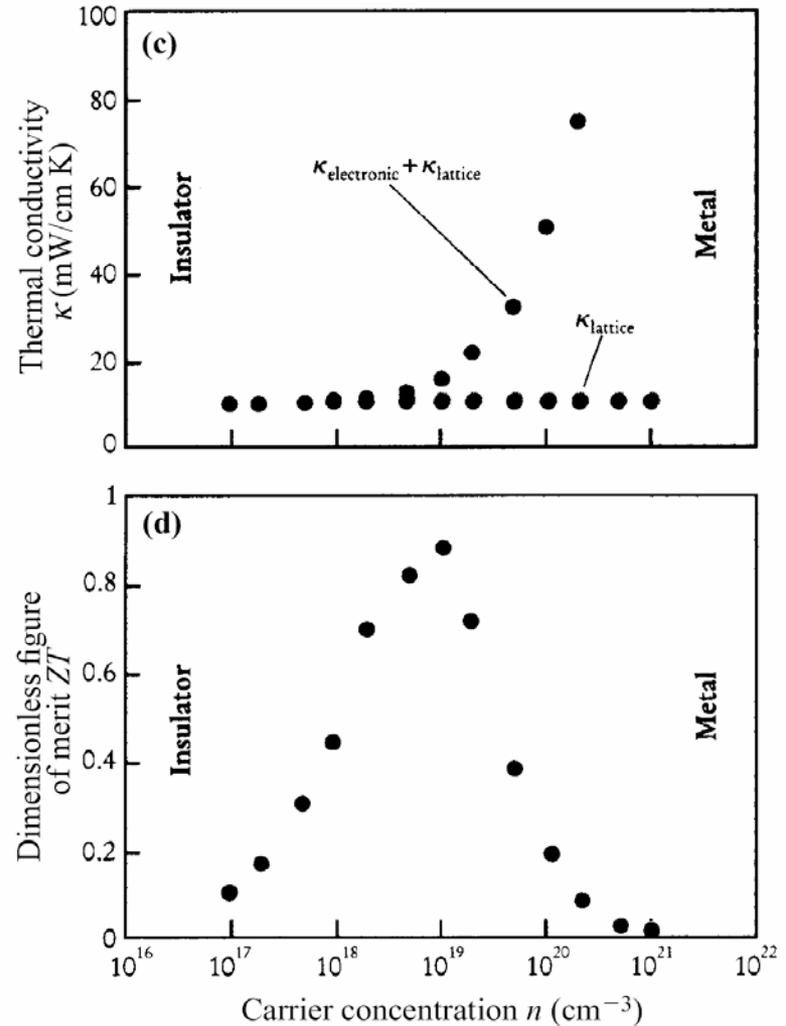
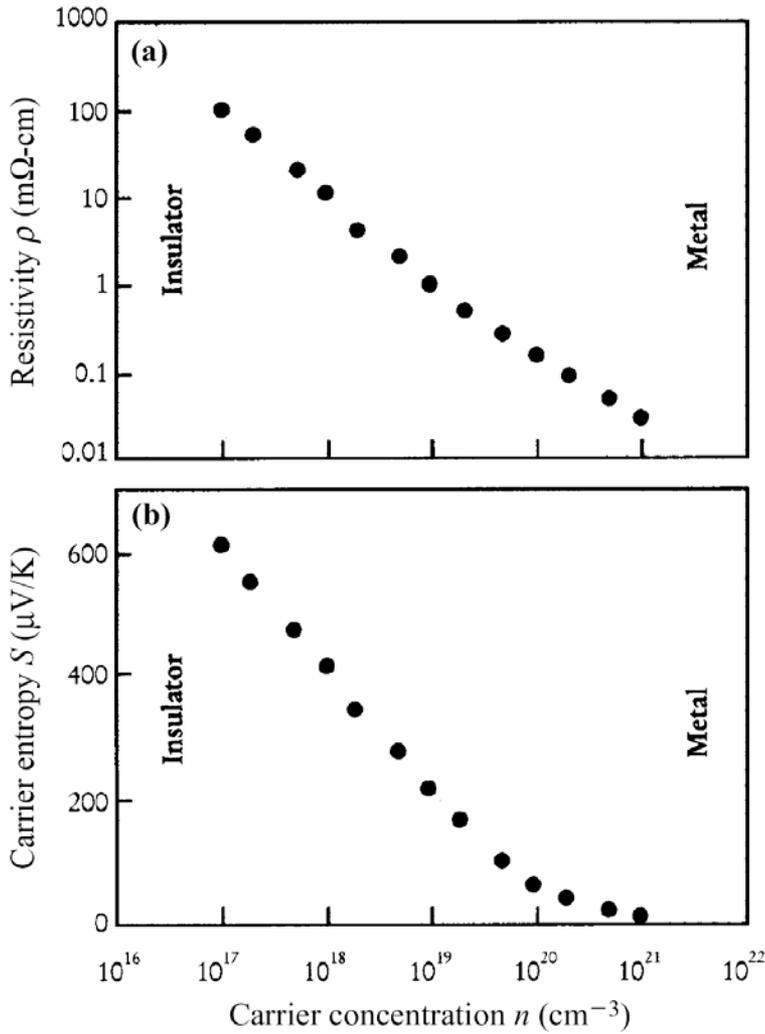


Figure 11.10. Carrier concentration dependence of (a) resistivity, (b) carrier entropy (or thermoelectric power), (c) thermal conductivity, and (d) figure of merit for an idealized semiconductor at room temperature. [G. Mahan, B. Sales and J. Sharp, *Physics Today*, March (1997) pp. 42–47]

- The **thermoelectric power**  $Q$  is expected to reach a value as high as **500–600  $\mu\text{V}/\text{K}$**  for a carrier concentration of  $10^{17}$ – $10^{18}$   $\text{cm}^3$ .
- $\text{Bi}_2\text{Te}_3$  and  $\text{FeSi}_2$  exhibit a thermoelectric power of a few hundreds  $\mu\text{V}/\text{K}$  and are considered as **the most efficient thermoelectric device** materials available at present.

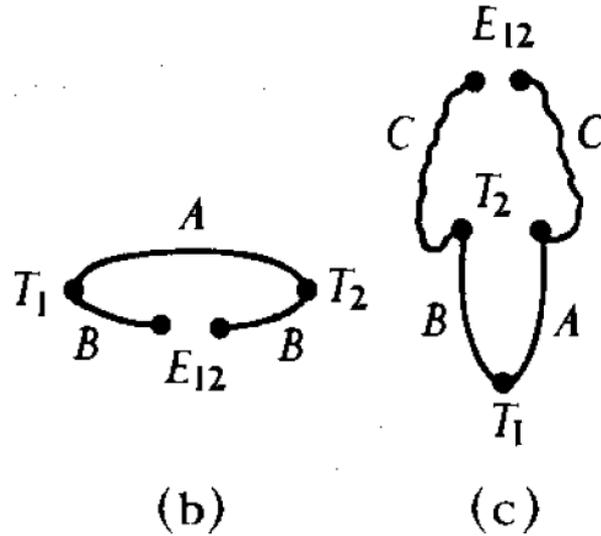
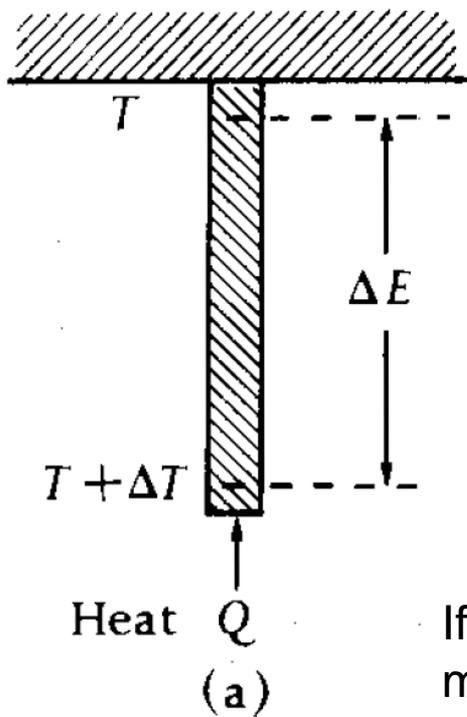
Further increase in  $Q$  beyond several hundreds  $\mu\text{V}/\text{K}$ , while suppressing the **electrical resistivity** and **thermal conductivity** to be as low as possible, is of urgent need from the viewpoint of practical applications.

# THERMOELECTRIC THERMOMETERS

$$E = Q \nabla T$$

$$E = S \nabla T$$

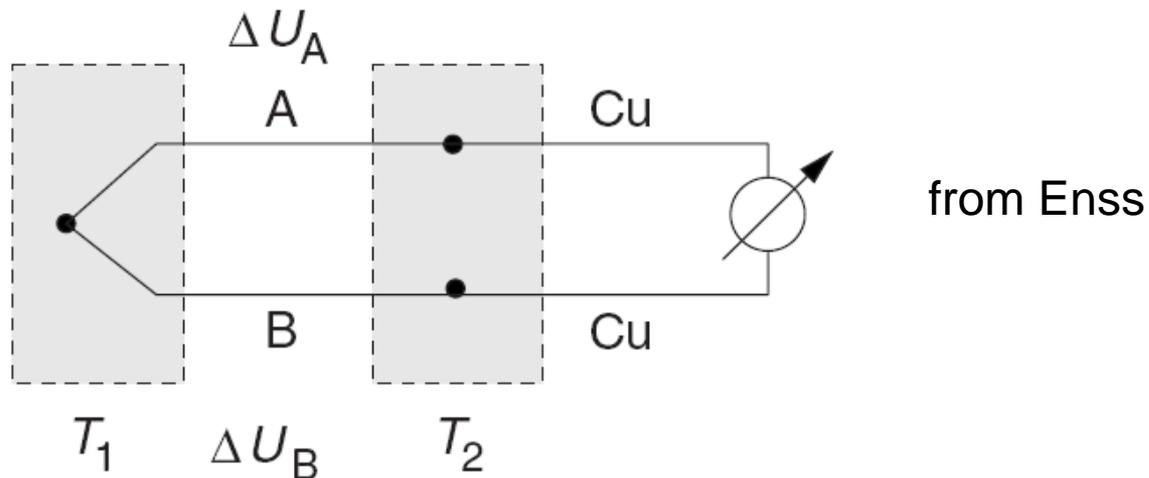
$$\Delta U = S \Delta T$$



If a thermal gradient is generated in a metallic conductor, a voltage between the warm and cold end occurs, termed the *Seebeck effect*.

A precise measurement of the **thermoelectric voltage** using a single piece of conducting material is very difficult because the contacts to a voltmeter through the electrical leads are at different temperatures and therefore can influence the measurement. To avoid these complications, the difference between the thermoelectric voltage of two different conductors is usually measured.

$$V_{AB} = \int_{T_1}^{T_2} (\mathcal{S}_A - \mathcal{S}_B) dT$$



Schematic illustration of the classical setup to determine the temperature with a thermoelectric element

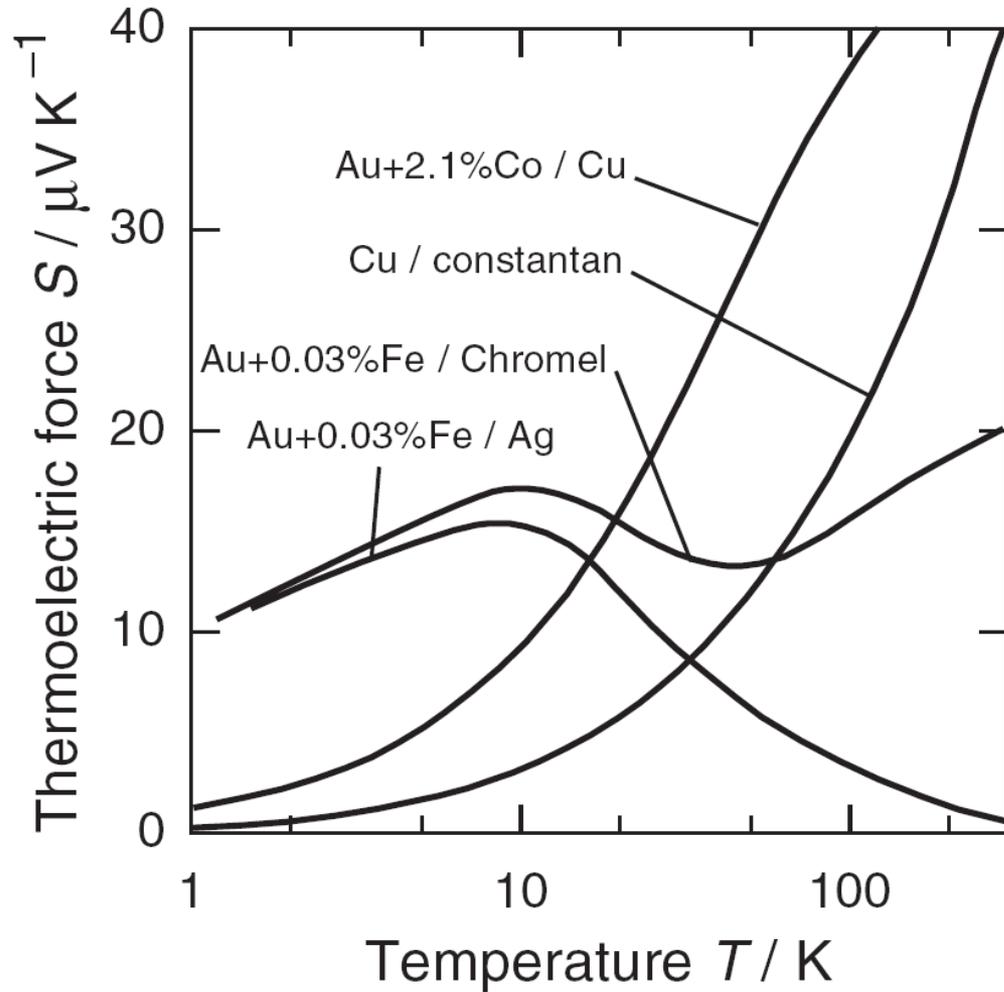
A disadvantage is, however, that the thermoelectric power vanishes for  $T \rightarrow 0$ .

Emf,  $E$ , of copper-constantan thermocouple with one junction maintained at the ice point

$E_{\mu V}$	$T^{\circ} \text{K}$	$E_{\mu V}$	$T^{\circ} \text{K}$
5200	90.206	6000	29.637
5300	84.384	6020	27.002
5400	78.290	6040	24.134
5500	71.872	6050	22.590
5600	65.039	6060	20.952
5700	57.674	6070	19.190
5800	49.636	6080	17.242
5850	45.293	6085	16.171
5900	40.637	6090	15.017
5950	35.503	6095	13.770

NIST-  
data  
base

the thermoelectric power of this system is generally too small for thermometry below about 10 K.



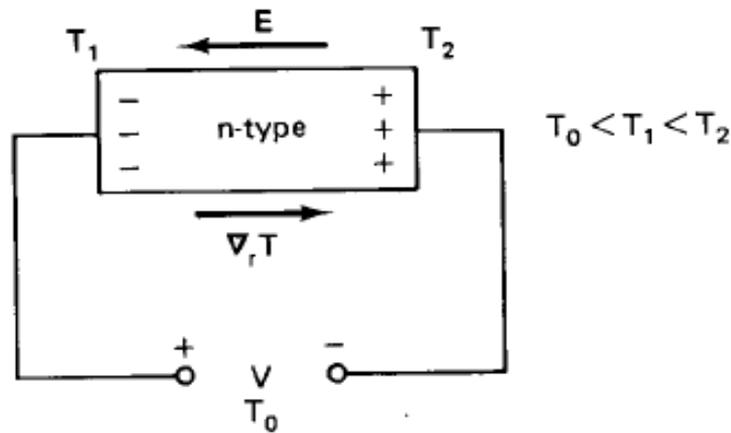
reproduced from  
Enss

Some special materials, certain metals with magnetic impurities, show thermoelectric powers that are sufficiently large at low temperatures. They can be used for thermometry down to about 1 K.

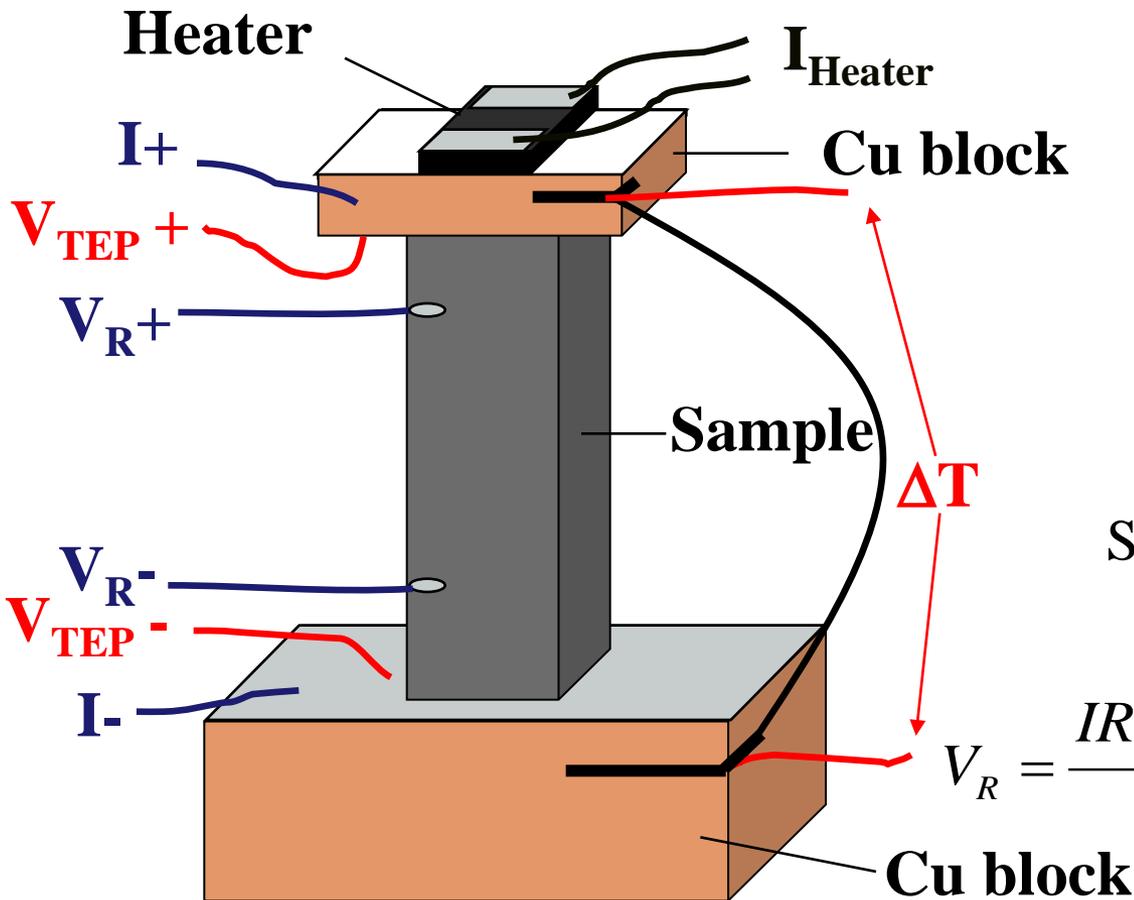
# Thermoelectric Applications

- Power Generation
  - Waste Heat Recovery
  - Active Cooling/Warming

- At the hot junction the Fermi level is higher than at the cold junction. Electrons will move from the hot junction to the cold junction in an attempt to equalize the Fermi level, thereby creating an electric field which can be measured in terms of the open circuit voltage  $V$



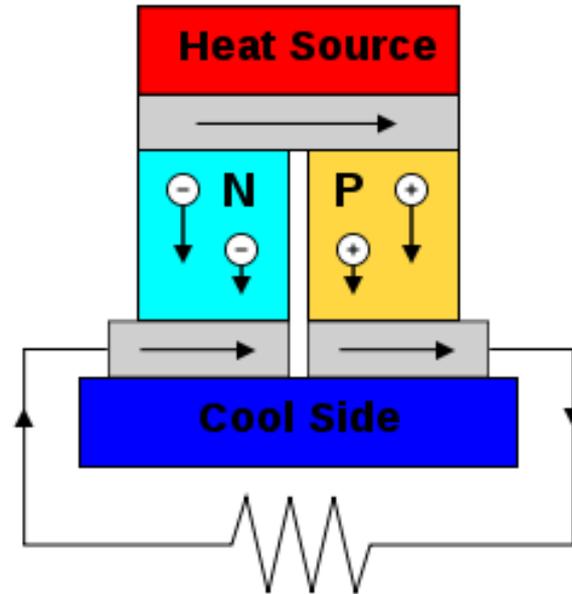
# Resistivity and Thermopower



Heater Power,  
 $P = I^2 R$ , creates  $\Delta T$   
 for Thermopower  
 Measurement

4-probe Resistivity  
 Measurement:  
 Current Reversed to  
 Subtract Thermoelectric  
 Contribution

$$V_R = \frac{IR + V_{\text{TEP}} - (-I)R - V_{\text{TEP}}}{2}$$

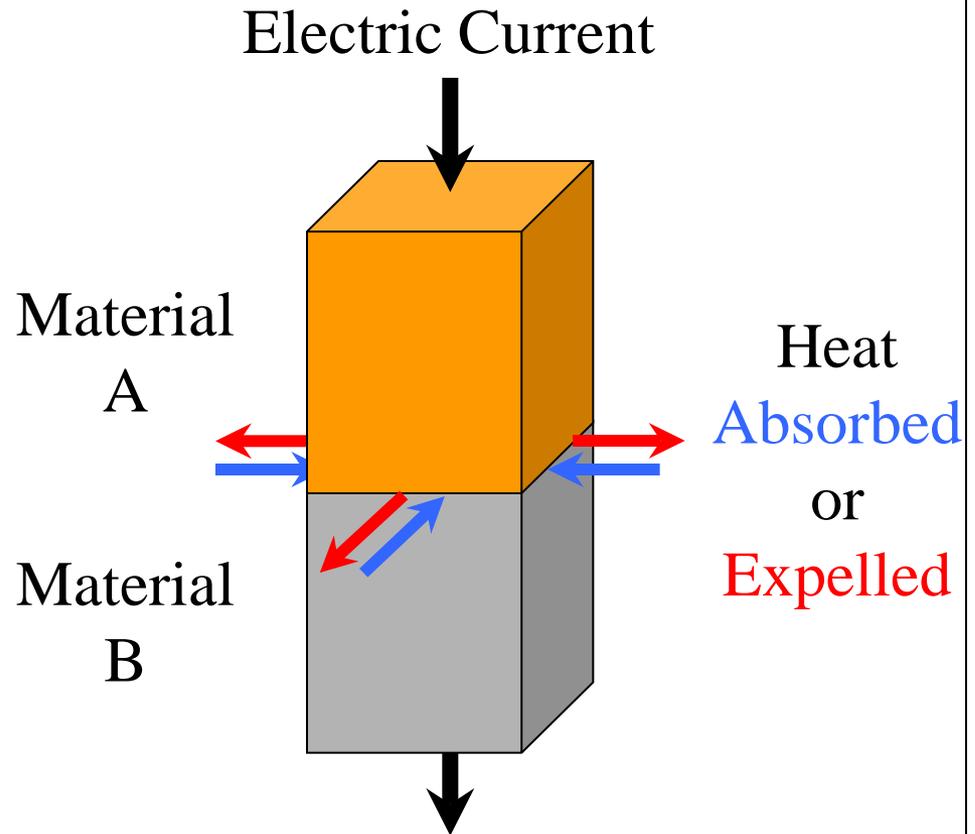


If a heat source is provided, the thermoelectric device may function as a **power generator**. The heat source will drive electrons in the n-type element toward the cooler region, thus creating a current through the circuit. Holes in the p-type element will then flow in the direction of the current. The current can then be used to power a load, thus converting the thermal energy into electrical energy.

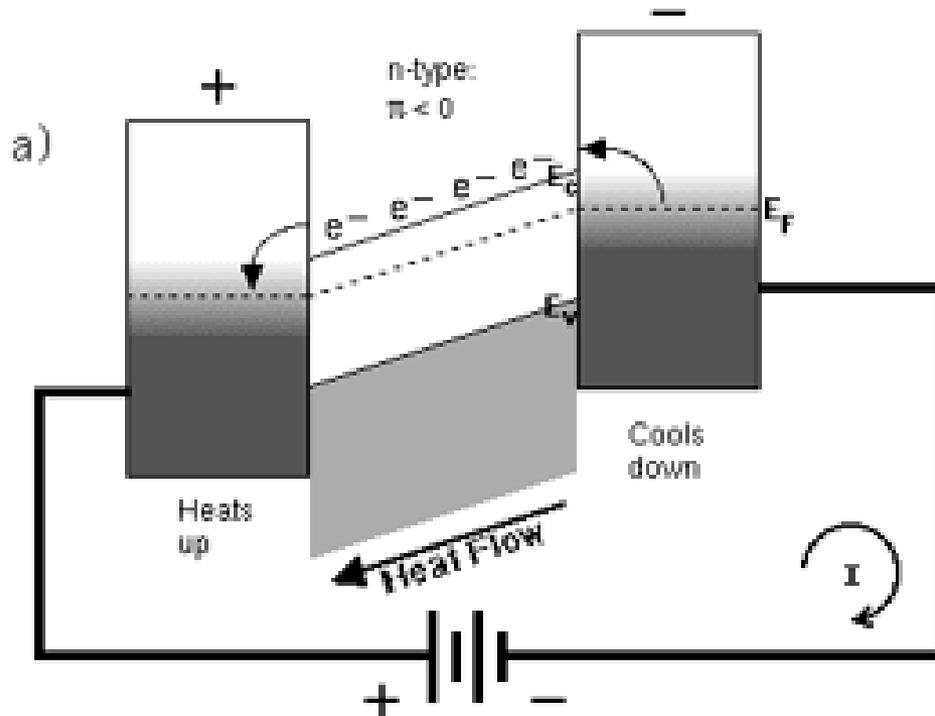
# TE Effects

## Peltier Effect

Difference in  $\epsilon_F$  between Materials A and B



# Peltier Effect



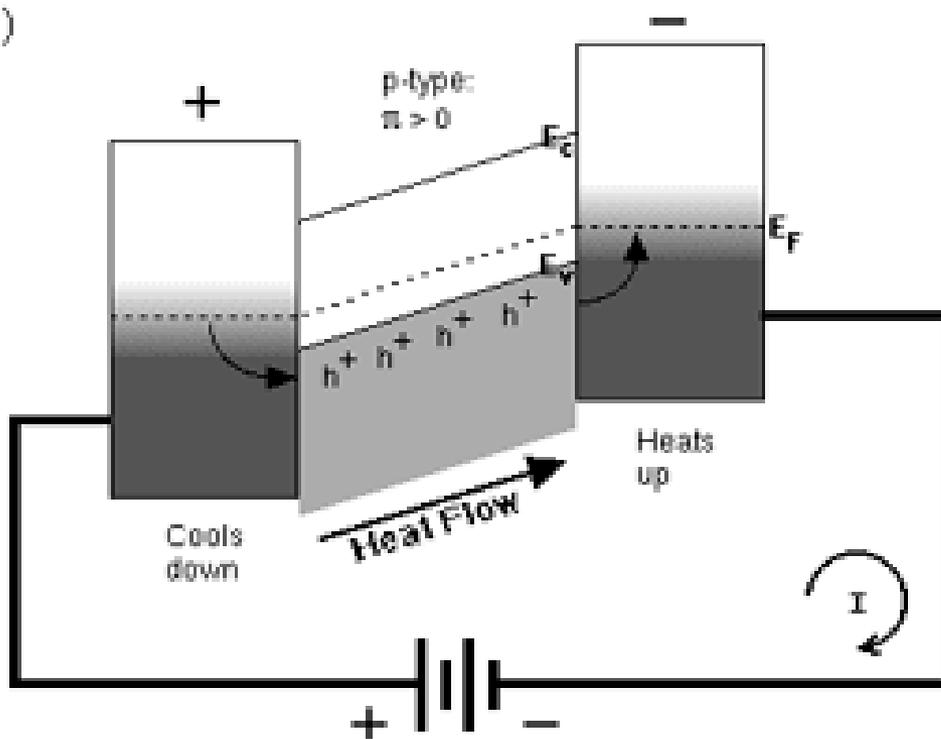
$\Pi < 0$  ; Negative Peltier coefficient

High energy electrons move from right to left.

Thermal current and electric current flow in opposite directions.

(electronic)

# Peltier Cooling



$\Pi > 0$  ; Positive Peltier coefficient

High energy holes move from left to right.

Thermal current and electric current flow in same direction.

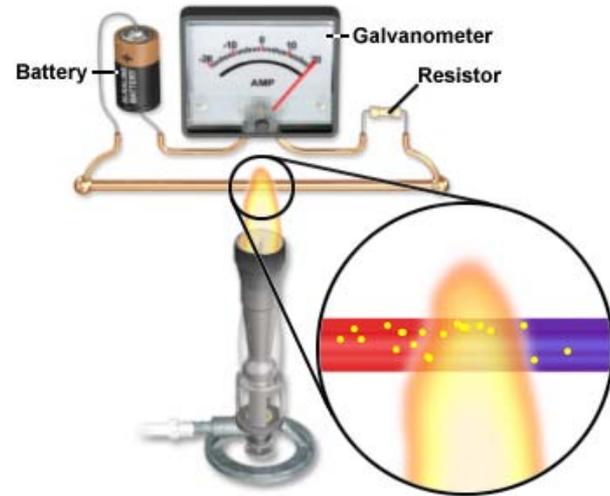
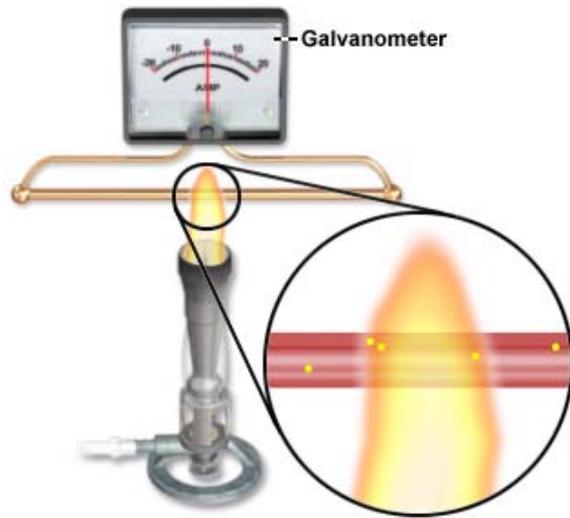
$q = \Pi * j$ , where  $q$  is thermal current density and  $j$  is electrical current density.

$\Pi = S * T$  (Volts)       $S \sim 2.5 k_B / e$  for typical TE materials

$T$  is the Absolute Temperature

## Thomson effect

the **evolution** or **absorption** of heat when electric current passes through a circuit composed of a single material that has a temperature difference along its length. This transfer of heat is superimposed on the common production of heat associated with the electrical resistance to currents in conductors. If a copper wire carrying a steady electric current is subjected to external heating at a short section while the rest remains cooler, heat is absorbed from the copper as the conventional current approaches the hot point, and heat is transferred to the copper just beyond the hot point. This effect was discovered (1854) by the British physicist William Thomson (Lord Kelvin).



[http://www.daviddarling.info/encyclopedia/T/Thomson\\_effect.html](http://www.daviddarling.info/encyclopedia/T/Thomson_effect.html)

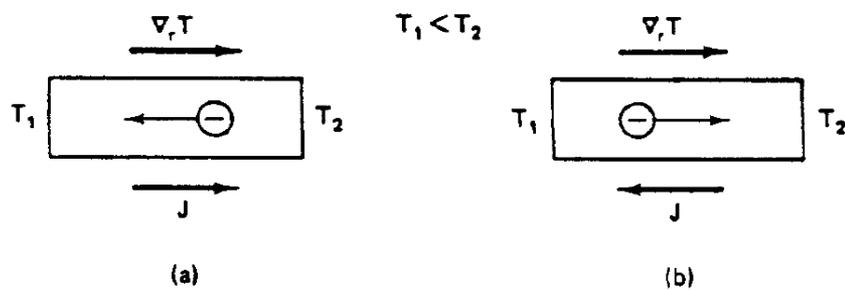


Figure 5.7: The Thomson term in an  $n$ -type semiconductor produces (a) heating when  $\vec{j}$  and  $\vec{\nabla}T$  are in the same direction and (b) cooling when  $\vec{j}$  and  $\vec{\nabla}T$  are in opposite directions.

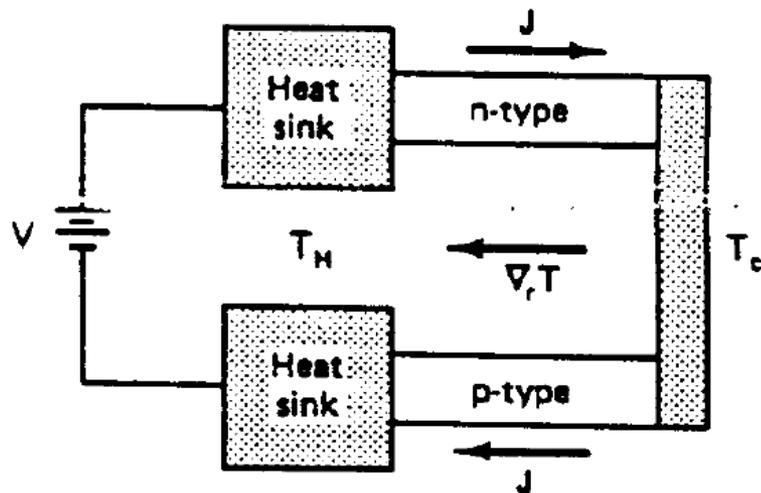
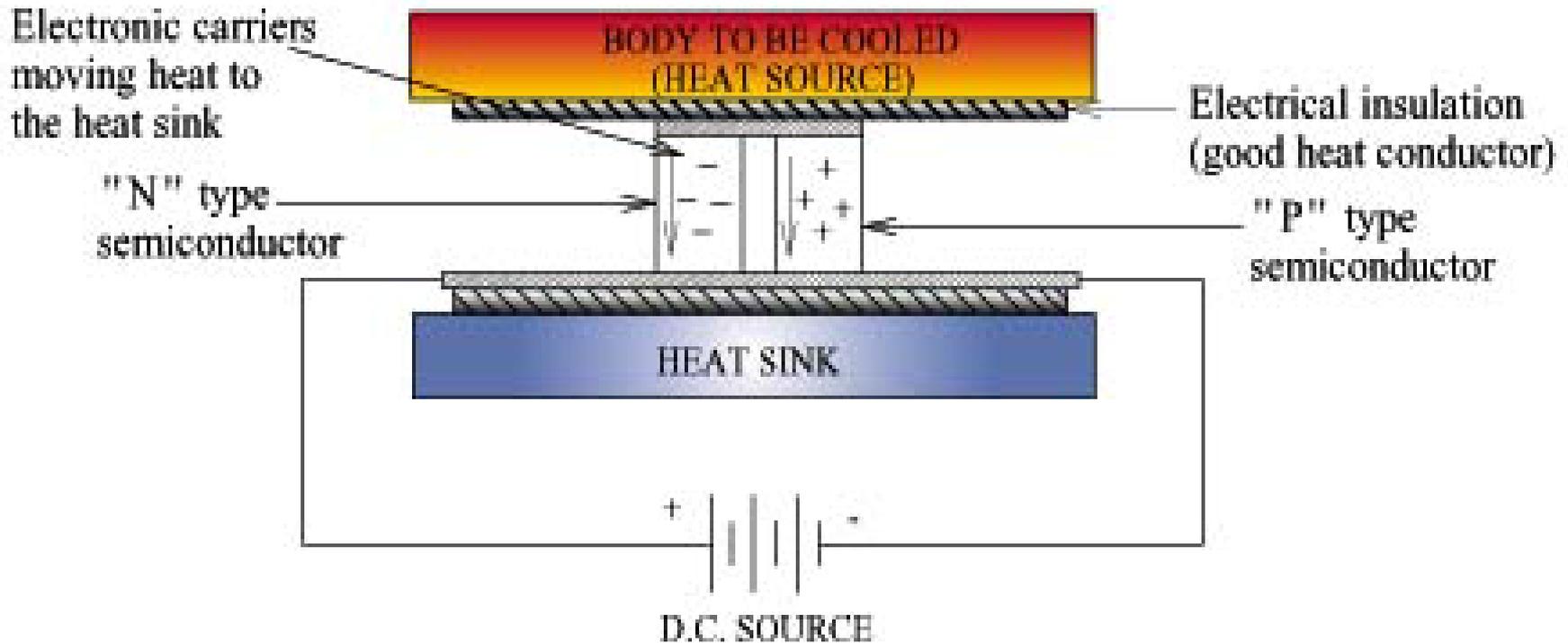
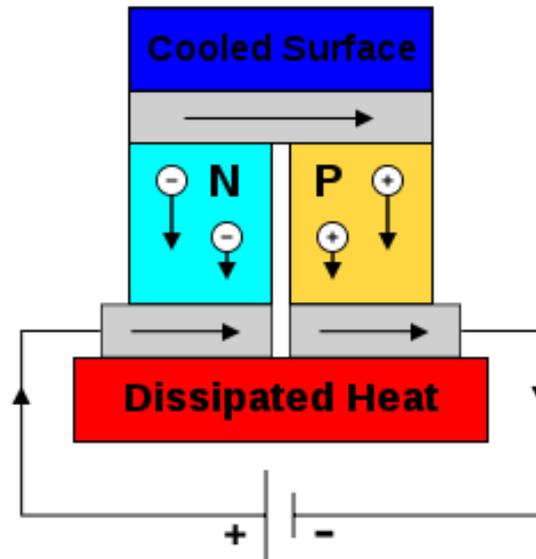


Figure 5.8: Schematic diagram of a thermoelectric cooler. The heat sinks and cold junctions are metals that form ohmic contacts to the active thermoelectric  $n$ -type and  $p$ -type semiconductors.

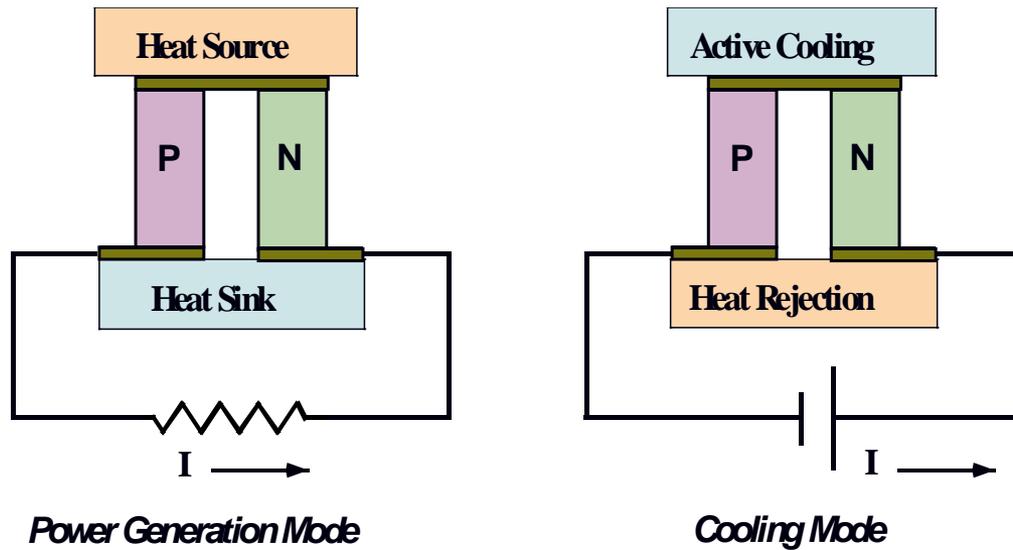
# Thermoelectric Refrigeration





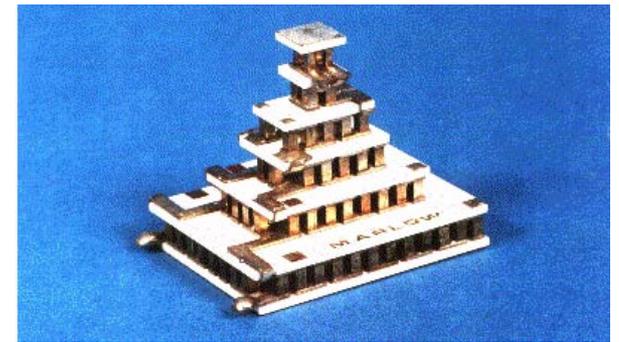
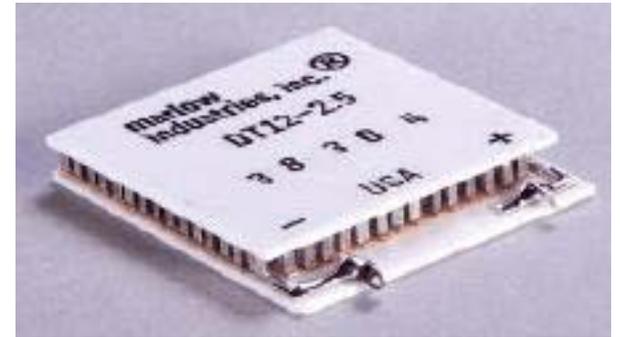
Charge flows through the n-type element, crosses a metallic interconnect, and passes into the p-type element. If a power source is provided, the thermoelectric device may act as **a cooler**. This is the Peltier effect. Electrons in the n-type element will move opposite the direction of current, and holes in the p-type element will move in the direction of current, both removing heat from one side of the device.

# TE Couple and Module



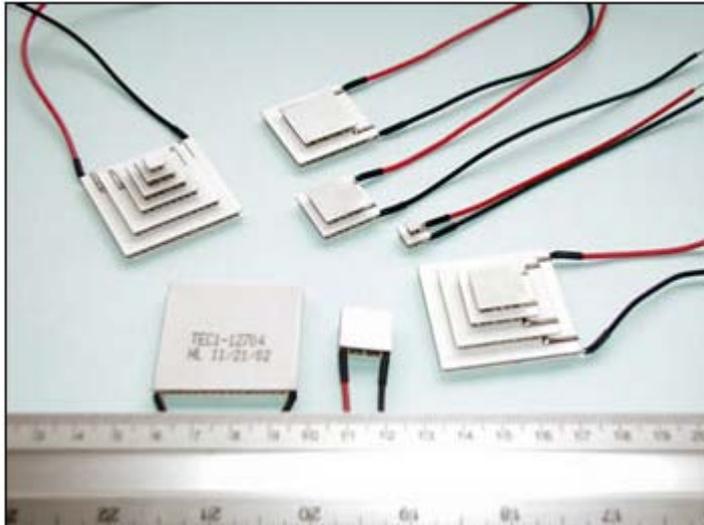
Operating Modes of a  
Thermoelectric Couple

T. M. Tritt, *Science* 31, 1276 (1996)



Modules

[www.marlow.com](http://www.marlow.com)



**thermoelectric**  
**supplier.com**