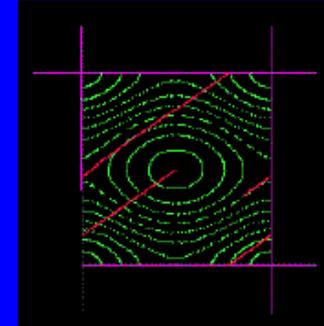
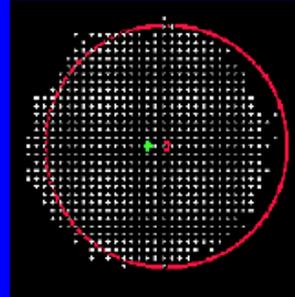
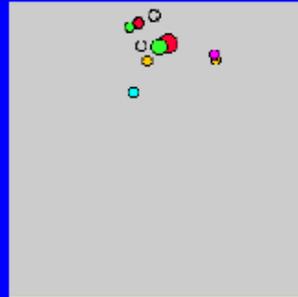
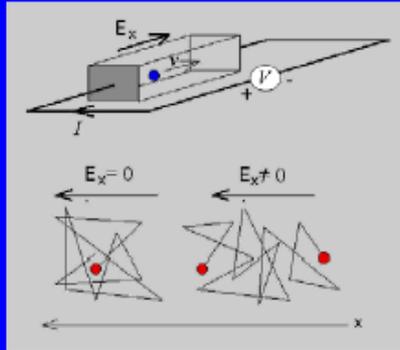


Conduction electron scattering  
and the resistance of the  
magnetic elements

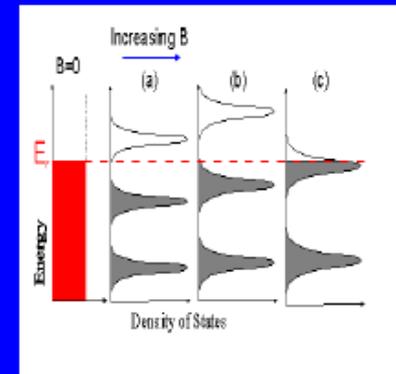
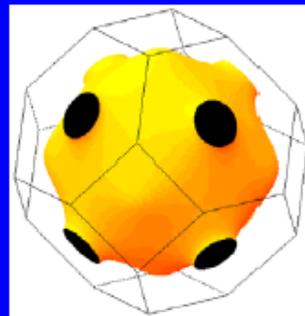
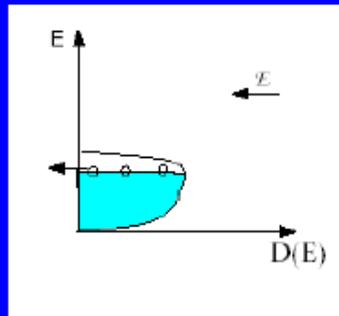
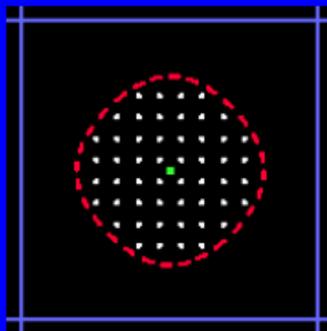
# RECAP

## Motions of electrons and transport phenomena



$$\sigma = \frac{ne^2\tau}{m}$$

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2} \sum_j \frac{\partial^2 E(\vec{k})}{\partial k_i \partial k_j}$$

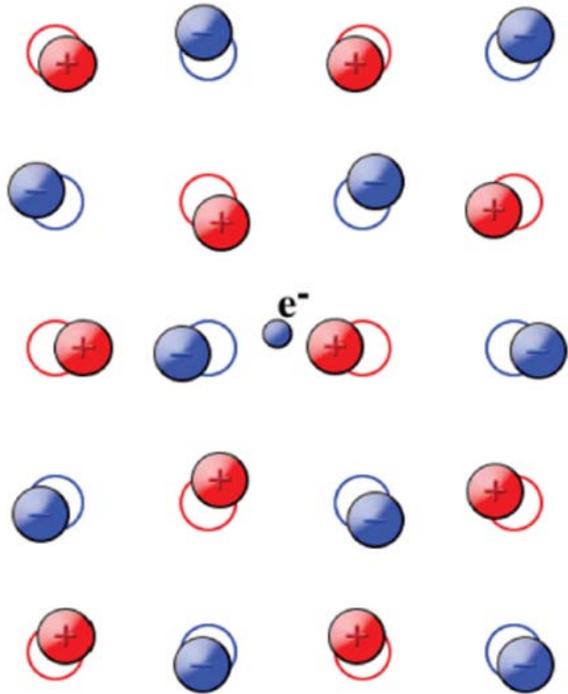


# Electronic Transport

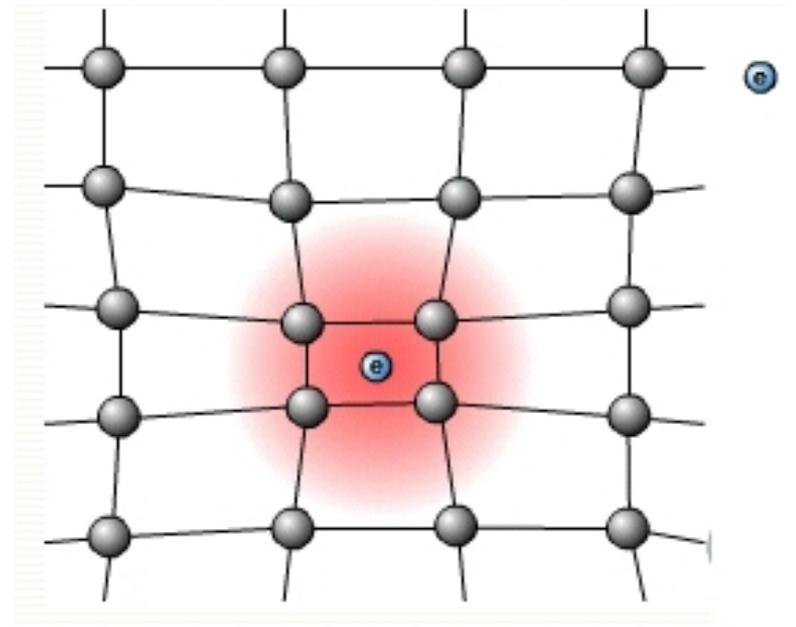
- transport in **insulators** (of heat mostly) is dominated by **phonons**.
- The **thermal conductivity** of some insulators can be quite large (cf. diamond).
  
- **Metals**, with transport dominated by **electrons**
- generally conduct both **heat** and **charge** quite well.
- In addition the ability to conduct thermal, charge, and entropy currents leads to interesting phenomena.

# Electrical conductivity

- Electrical resistivity due to electron-phonon interaction.
- Bloch-Gruneisen law
- Residual resistivity of metals
- Impurity effect in a metal



## electron-phonon interaction



from Solid State Course by Mark Jarrel (Louisiana Univ.),

# Boltzmann equation

Boltzmann equation describes the evolution of the *distribution function* in space, momentum, and time.

**Definition 1** The *distribution function*,  $f(\mathbf{r}, \mathbf{p}, t)$ , is such that a quantity  $f(\mathbf{r}, \mathbf{p}, t) d^3r d^3p$  gives the number of particles located in volume  $d^3r$  near point  $\mathbf{r}$  whose momenta occupy volume  $d^3p$  near a point in the momentum space  $\mathbf{p}$  at time  $t$ .

$$\int d^3r \int \frac{d^3p}{(2\pi)^3} f(\mathbf{r}, \mathbf{p}, t) = N(t)$$

$$f_0(\vec{k}) = f(\vec{r}, \vec{k}, t) \Big|_{\vec{E}=0} = \frac{1}{e^{(E(\vec{k}) - E_F)/k_B T} + 1}$$

In the absence of scattering,

$$\frac{d}{dt} f(\mathbf{r}, \mathbf{p}, t) = 0$$



$$\begin{aligned} \frac{\partial f}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \vec{\nabla}_{\mathbf{r}} f + \frac{d\mathbf{p}}{dt} \cdot \nabla_{\mathbf{p}} f &= 0 \\ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \vec{\nabla}_{\mathbf{r}} f + \vec{F} \cdot \nabla_{\mathbf{p}} f &= 0, \end{aligned}$$

In the presence of scattering, the RHS of the equation is different from zero because particles change their momentum states due to scattering.

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \vec{\nabla}_{\mathbf{r}} f + \vec{F} \cdot \nabla_{\mathbf{p}} f = I_{\text{coll}} [f]$$

*An equilibrium distribution function,  $f_0(\mathbf{p})$ , nullifies the collision integral*

$$I_{\text{coll}} [f_0] = 0.$$

# Relaxation time approximation:

This is based on the assumption that a nonequilibrium distribution  $f(\mathbf{k}, \mathbf{r}, t)$  gradually returns to its equilibrium value within a characteristic time, the relaxation time  $\tau(\mathbf{k})$ , by the scattering of electrons with the wave vector  $\mathbf{k}$  into states  $\mathbf{k}'$ , and vice versa.

$$\left(\frac{\partial f}{\partial t}\right)_s = -\frac{f(\vec{k}) - f_0(\vec{k})}{\tau(\vec{k})}$$

The rate of return to equilibrium is proportional to

$$f(\vec{k}) - f_0(\vec{k})$$

The proportionality coefficient

$$1/\tau$$

is a function of  $\vec{k}$  only

$$f = f_0 + f_1$$

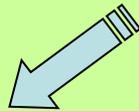
$$f_1 = e \cdot \tau \cdot E \cdot v \cdot \frac{df_0}{d\varepsilon}$$

For steady state:  $\left(\frac{\partial f}{\partial t}\right) = 0$

For a homogeneous Sommerfeld metal:  $\nabla_r f = 0$

$$-\frac{e}{\hbar} \vec{E} \cdot \vec{\nabla}_k f = \left(\frac{\partial f}{\partial t}\right)_s$$

$$-\frac{e}{\hbar} \vec{E} \cdot \vec{\nabla}_k f = -\frac{f(\vec{k}) - f_0(\vec{k})}{\tau(\vec{k})}$$



$$f = f_0 + \frac{\tau(\vec{k}) e E}{\hbar} \cdot \nabla_k f_0$$

$$\frac{\partial f_0}{\partial k} = \frac{\partial f_0}{\partial \varepsilon} \cdot \frac{\partial \varepsilon}{\partial k}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2m}$$

$$\frac{\partial \varepsilon}{\partial k} = \frac{\hbar^2 k}{m}$$

$$v = \frac{p}{m} = \frac{\hbar k}{m}$$

# Electrical conductivity

- Electrical resistivity due to electron-phonon interaction.
- Bloch-Gruneisen law
- Residual resistivity of metals
- Impurity effect in a metal

# Electrical conductivity of metals

## Matthiessen's rule

$$\frac{1}{\tau} = \frac{1}{\tau_{def}} + \frac{1}{\tau_{e-e}} + \frac{1}{\tau_{ph}} + \frac{1}{\tau_{mag}} + \frac{1}{\tau_{CEF}} + \dots$$

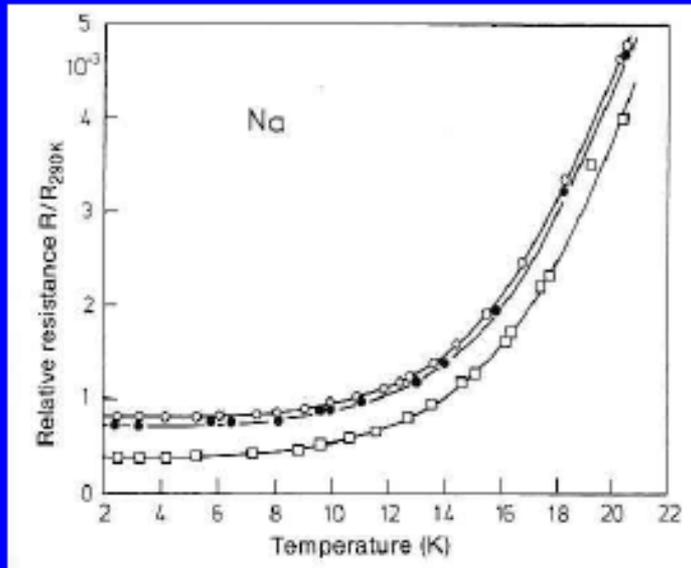
$$\rho = \rho_0 + \rho_{e-e} + \rho_{ph} + \rho_{mag} + \rho_{CEF} + \dots$$

$$\rho_0 = \text{constant}$$

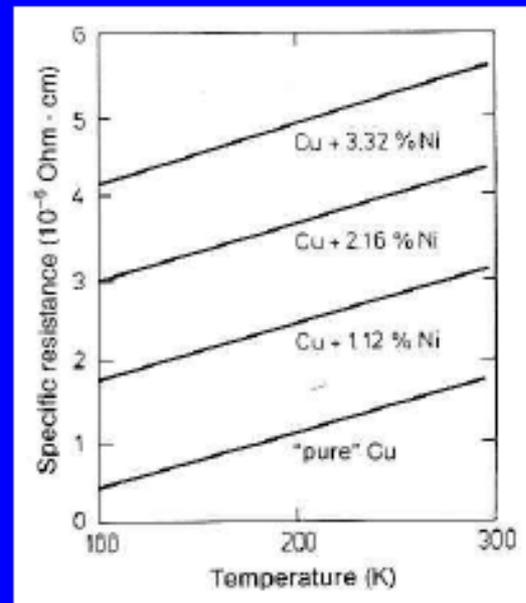
$$\rho_{e-e} = AT^2$$

$$\rho_{ph} = a(T/\theta)^5 \int_0^{\theta/T} \frac{x^5 dx}{(e^x - 1)(1 - e^{-x})}$$

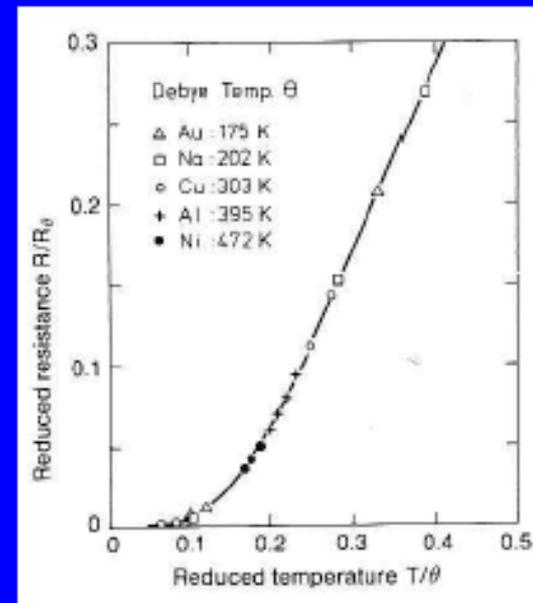
Bloch-Gruneisen



resistance of sodium  
3 diff. defect concentrations

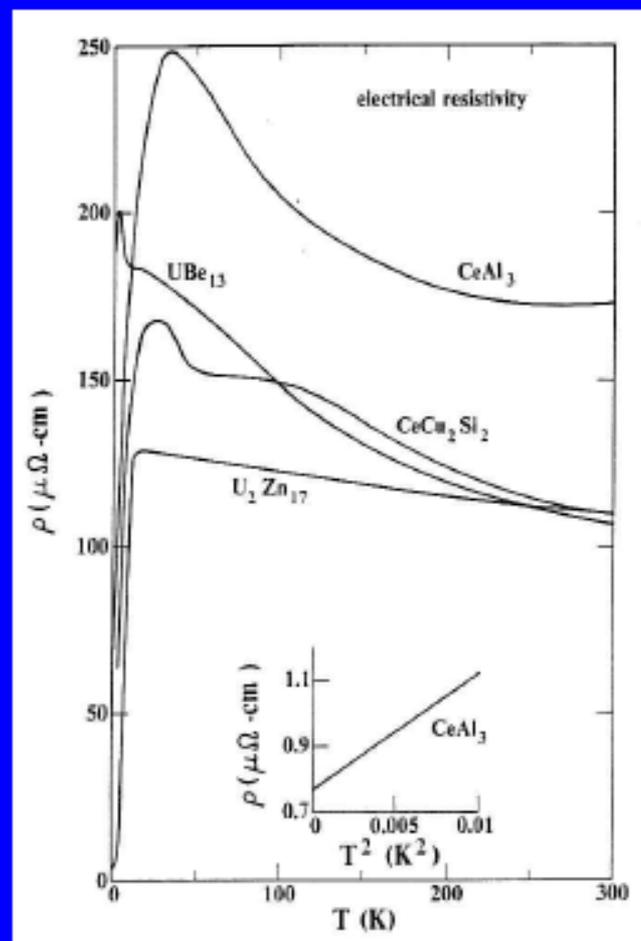


resistivity of copper-nickel alloys

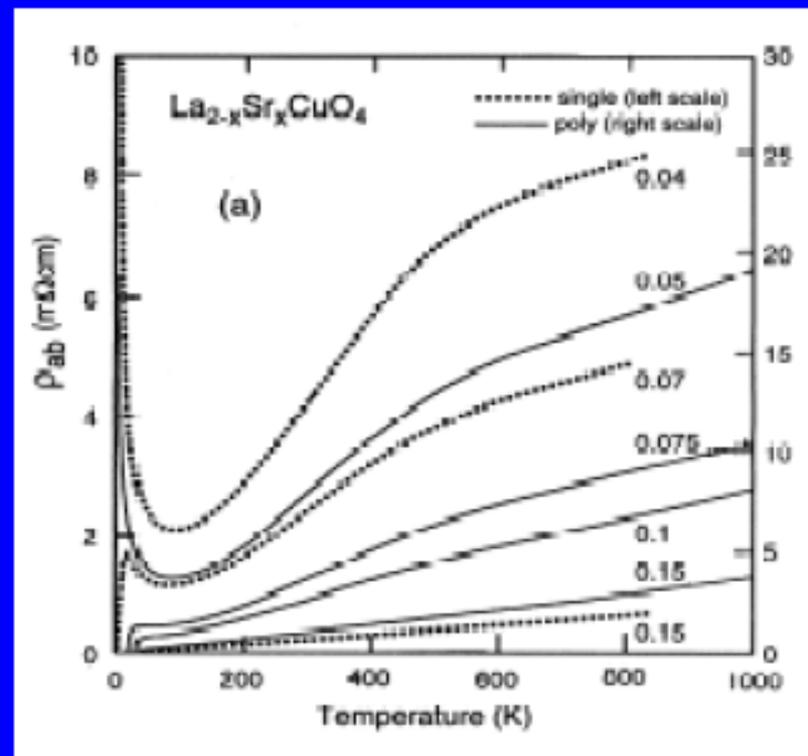


phonon (Debye) resistance

# Electrical conductivity of metals: examples



resistivity of heavy-fermion compounds



resistivity of superconducting cuprates:  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

# Magnetic scattering

Many alloy systems of interest may contain one or more magnetic phases

- Disorder within itinerant or localized spin systems will cause scattering of conduction electrons.
- The spin state of the conduction electrons  $\sigma$  can adopt only one of the two values,  $\pm 1/2$  (spin 'up' and 'down') and interacts with other spins  $S$  through the effects of symmetry and the Pauli exclusion principle.
- The energy associated with this exchange interaction:

$$E_{ex} = -J(r) \vec{\sigma} \cdot \vec{S}$$

Mizutani  
Enss

The exchange parameter  $J(r)$  falls off rapidly with distance  $r$

Below some critical temperature the itinerant or localized spins may condense into an ordered array (**ferromagnetic or antiferromagnetic order**).

Any **disorder** in this array will cause **electron scattering**

This scattering may be **elastic** → no change in energy (no spin-flip)

**inelastic** → **spin state of the electron changes**.

In both situations the scattering rate will be different for the spin-up and spin-down conduction electrons.

Table 13.1. *Classification of crystalline metals and alloys in terms of magnetism*

group	magnetic system			non-magnetic system	
	I ferromagnetism	II weak ferromagnetism	III spin-glass or magnetically dilute alloys	IV paramagnetism	V weak paramagnetism or diamagnetism
representative metals and alloys	Fe, Co, Ni	ZrZn <sub>2</sub> , Sc <sub>3</sub> In	Au-(Fe), Cu-(Fe)	Ti, Zr, Nb	Na, Mg, Al, Cu, Ag, Au, Zn, Pb
characteristic features of magnetism	( $T_C > 300$ K) Fe 1043 K Co 1400 K Ni 631 K	( $T_C \ll 300$ K) ZrZn <sub>2</sub> 21K Sc <sub>3</sub> In 5.5 K	the spin freezing temperature in spin-glass or Curie-Weiss type magnetic susceptibility in dilute alloys (Kondo effect)	weak temperature dependence of magnetic susceptibility due to Pauli paramagnetism	temperature independent magnetic susceptibility
resistivity at 300 K ( $\mu\Omega$ -cm)	Fe 9.7 Co 6.2 Ni 6.8			Nb 14.5 Mo 5.7 Pd 10.8	Al 2.69 Cu 1.67 Na 4.6

Metals in groups (I) to (IV) are characterized by the possession of a Fermi level either in the d band or the d states.

- those in groups (I) to (III) are magnetic
- those in group (IV) are non-magnetic.
- Metals in group (V) possess a Fermi level in the sp band and are non-magnetic.

Group (I) → ferromagnetic metals and alloys with a Curie temperature well above room temperature.

- The metals Fe, Co, Ni and their alloys belong to this group.
- Antiferromagnetic metals and alloys with a Néel temperature  $T_N$  above 300K are also included in group (I).

- Metals and alloys in group (II) exhibit spontaneous magnetization only at low temperatures.
- Metals and alloys in group (III) carry a localized moment but exhibit no spontaneous magnetization down to the lowest temperature available.
- **Spinglass** is defined as a substance obeying the Curie–Weiss law down to the spin freezing temperature  $T_f$ , below which the randomly oriented magnetic moments are “frozen” in motion without resulting in any spontaneous magnetization.

**Paramagnetic metals** and **alloys** obeying the Curie law down to the lowest temperature available are also included in group (III).

Among these are dilute alloys, in which very small amounts of impurity atoms carrying finite magnetic moments are dissolved.

As a typical example, we cite a **Cu metal** containing only **a few ppm of Fe** atoms.

A **resistivity minimum** phenomenon is often observed at low temperatures in these magnetically dilute alloys and is known as the **Kondo effect**. Here the **s–d interaction** plays a critical role and has been discussed as one of the most exciting topics in the electron theory of metals in 1970s and 80s.

- Metals and alloys in group (IV) carry no localized magnetic moments though the Fermi level is situated in the middle of the d band.

- The magnetic susceptibility shows only a weak temperature dependence of the Pauli paramagnetism .Typical transition metals like Ti, V, Zr, Nb,

- Mo, Pd, Pt and their alloys are included in this group.

Metals and alloys possessing a Fermi level in the sp band are classified within group (V). All nonmagnetic metals like Na, Cu, Ag, Au, Mg, Zn, Al, Pb and their alloys belong to this group.

from Mizutani

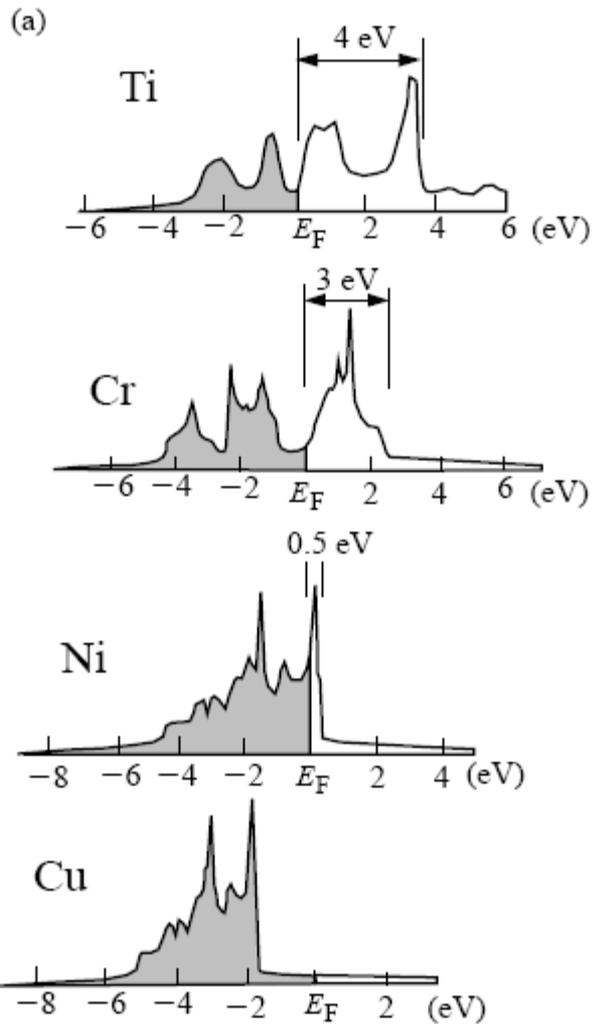
Table 10.1. *Electron transport properties of metals at 273 K*

element	electrical conductivity, $\sigma$ ( $\times 10^6 / \Omega\text{-m}$ )	resistivity, $\rho$ ( $\mu\Omega\text{-cm}$ )	TCR <sup>a</sup> , $\alpha$ ( $\times 10^{-3}/\text{K}$ )
Li	11.8	8.5	4.37
Na	23.4	4.27	5.5
Cu	64.5	1.55	4.33
Ag	66	1.5	4.1
Au	49	2.04	3.98
Mg	25.4	3.94	4.2
Ca	28	3.6	4
Zn	18.3	5.45	4.20
Al	40	2.50	4.67
Pb	5.17	19.3	4.22
Bi	0.93	107	
Ti	2.38	42	5.5
V	0.54	18.2	
Fe	11.5	8.71	6.57
Zr	2.47	40.5	4.0
W	20.4	4.89	4.83

Note:

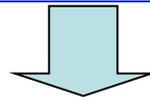
<sup>a</sup> Temperature coefficient of resistivity (TCR):  $\alpha_{273\text{ K}} = (1/\rho)(d\rho/dT)_{T=273\text{ K}}$

the resistivity of transition metals like Ti, Cr and Fe is always higher than that of simple sp-electron metals like Al and Cu.



- the valence band of the 3d-transition metals like Ni consists of the superposition of a narrow 3d band over a wide 4s band
- the density of states at the Fermi level is shared by both 3d electrons and 4s or 4p electrons.

- a transition metal always possesses a larger resistivity than a simple metal in group (V).
- **sp electrons** at the Fermi level exclusively convey the electrical current
- the density of states at the Fermi level in the transition metal is very high because of its location in the middle of the **d band**.
- the scattering probability  $1/\tau$  of the conduction electron is proportional to the final density of states at the Fermi level.



This means that the higher the **density of states** at the Fermi level, the more frequently sp conduction electrons are scattered into it. **A higher final density of states at the Fermi level would result in a shorter relaxation time  $\tau$  and, in turn, a higher resistivity in transition metals.**

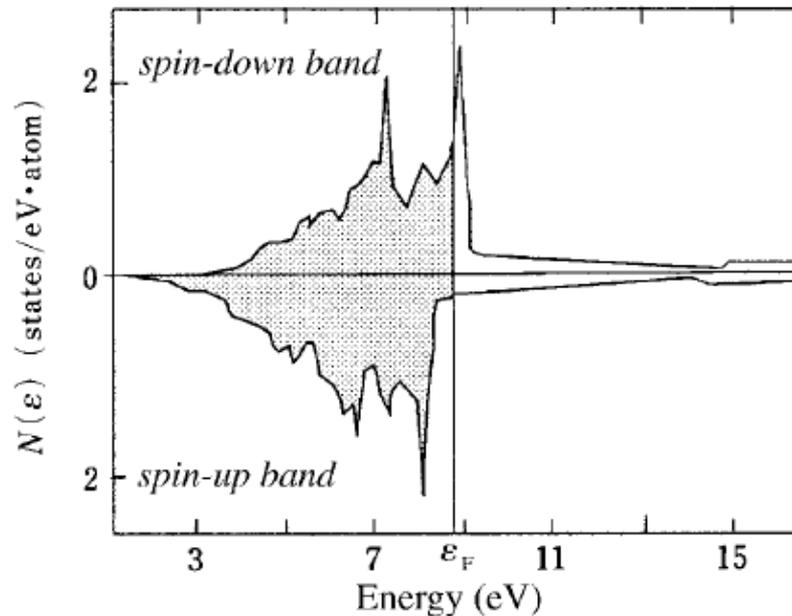


Figure 13.6. Valence band in the ferromagnetic state of pure Ni. The spin-up band is shifted to lower binding energies relative to the spin-down band due to the exchange energy. [D.A. Papaconstantopoulos, *Handbook of the Band Structure of Elemental Solids* (Plenum Press, 1986)]

the spin-up band in pure Ni is displaced relative to the spin-down band owing to the exchange interaction in the ferromagnetic state, resulting in a complete filling of the spin up band.

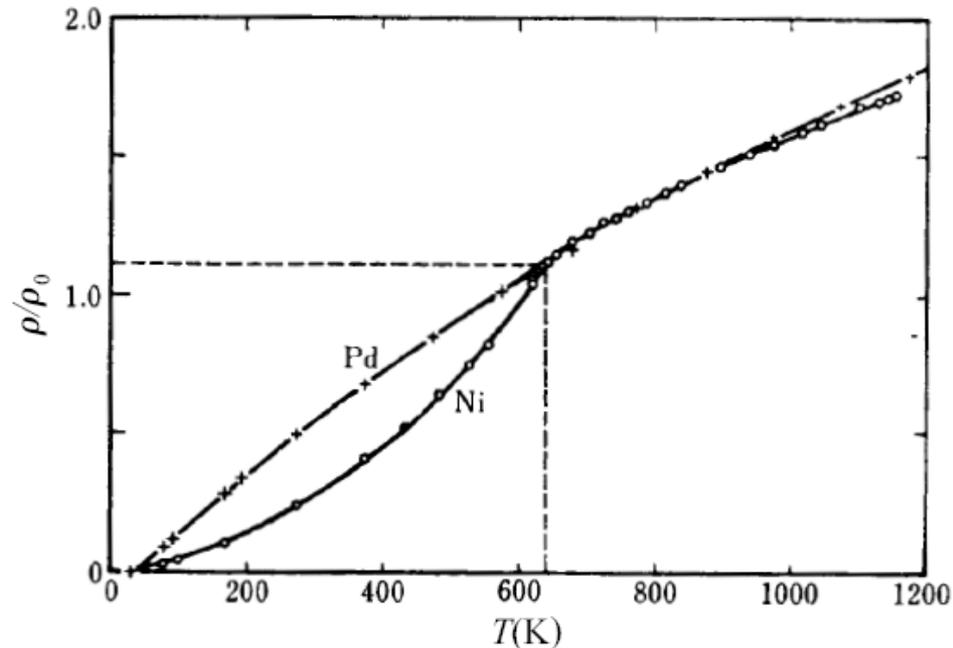


Figure 13.9. Temperature dependence of the normalized electrical resistivity in Ni and Pd. The data are shown such that both sets of data coincide with each other at the Curie point of pure Ni. [J. M. Ziman, *Electrons and Phonons* (Clarendon Press, Oxford 1962) p. 380]

Mott pointed out that, at low temperatures, **spin-orientation** of the **conduction electron** must be **unchanged** upon **scattering** so that spin-up conduction electrons in Ni cannot make transitions to the spin-up d band because it is full. This implies that these electrons would have a longer mean free path than those with the opposite spin.

Table 13.2. *Magnetic properties of several ferromagnetic metals and intermetallic compounds*

substance	spontaneous magnetization extrapolated to 0 K		magnetic moment extrapolated to 0 K ( $\mu_B$ ) <sup>a</sup>	Curie temperature (K)
	(gauss/cm <sup>3</sup> )	(Wb/m <sup>2</sup> )		
Fe	1744	0.175	2.216	1043
Co	1435	0.145	1.72	1400
Ni	512	0.051	0.616	631
Gd	1980	0.198	7.0	293
Dy	3030	0.292	10.2	85
MnBi	675	0.068	3.52	630
Cu <sub>2</sub> MnAl	580	0.058	4.0	603

*Note:*

$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}$  (SI) and  $\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M}$  (CGS)

<sup>a</sup> See footnote 6 on p. 46 in Chapter 3.

See Mizutani

# Electron–Magnon Scattering

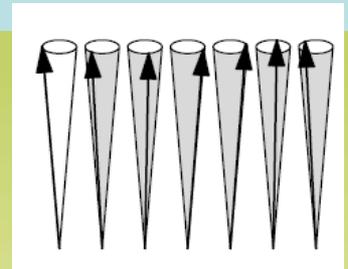
**Spin waves** are the collective excitations of the aligned spins in ferromagnets

The quasiparticles assigned to these excitations are called **magnons**.

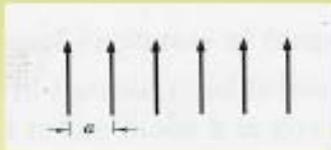
A simple Hamiltonian for spin waves is:

$$H = -J \sum_{i,j} \mathbf{s}_i \mathbf{s}_j,$$

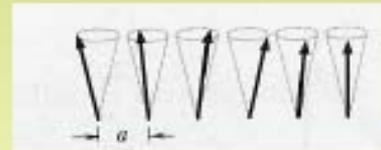
with  $J$  the magnetic exchange integral, which can be measured with neutrons.



Take a simple ferromagnet,

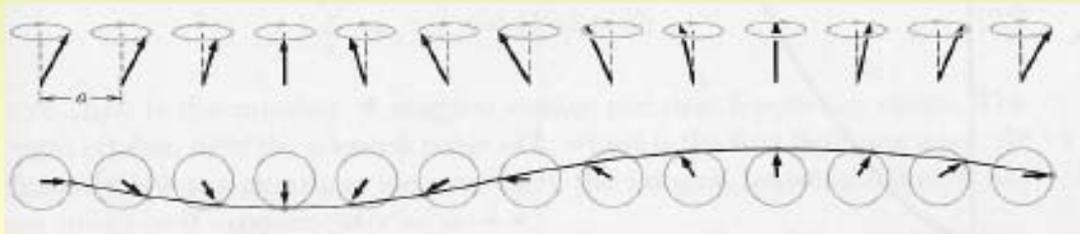


... then ...

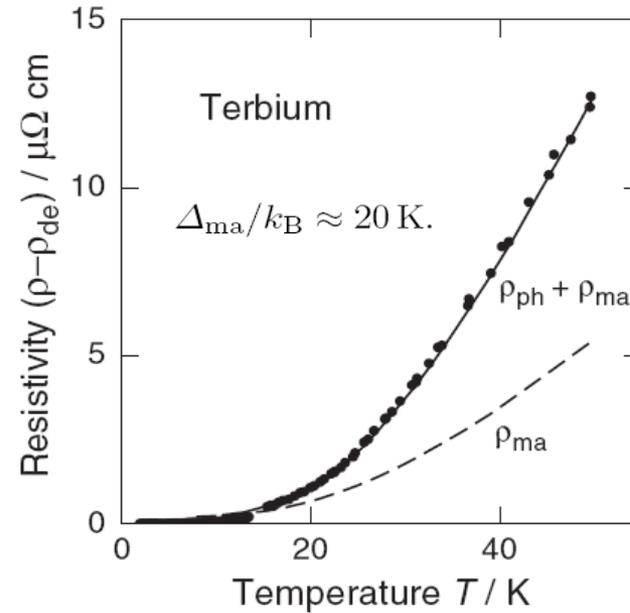
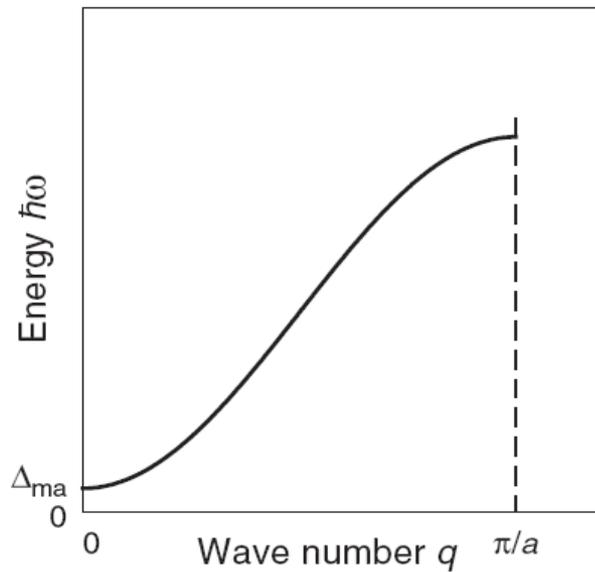


the spin waves might look like this.

Spin waves have a frequency and a wavelength...



... and they are directly measurable with neutrons



**Fig. 7.9.** (a) Illustration of the dispersion relation of spin waves. (b) Electrical resistivity of terbium as a function of temperature. The temperature-independent residual resistivity  $\rho_{de}$  has been subtracted. The *full line* depicts the sum of the resistance caused by phonon and magnon scattering, the *dashed line* shows the calculated magnon contribution [312]

The energy gap  $\Delta_{ma}$  at  $q = 0$  is caused by the anisotropy of the exchange interaction

from Enss

$$\tau^{-1} \propto n_{ma} \propto e^{-\Delta_{ma}/k_B T},$$

and for the electrical resistivity

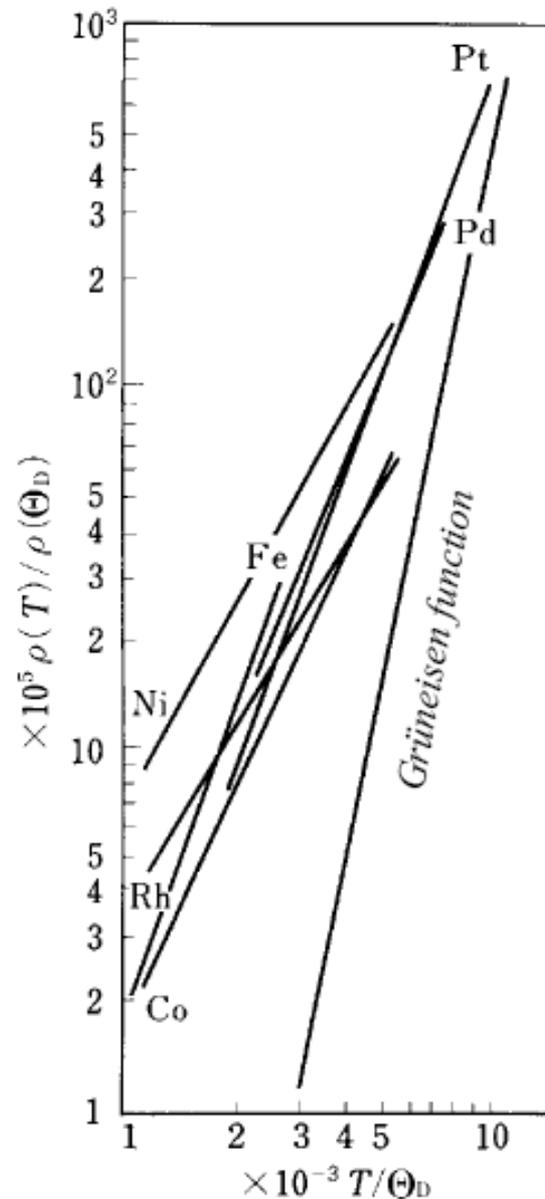
$$\rho_{ma} \propto e^{-\Delta_{ma}/k_B T}.$$

Bloch–Grüneisen law,

$n = 5$  in  $\rho_0 + AT^n$  at low temperatures

In contrast to non-magnetic metals like pure Na with  $n=5$ , the exponent  $n$  for ferromagnetic metals is distributed in the range 2–3. A smaller exponent is believed to originate from the electron–magnon interaction in ferromagnetic metals.

See Mizutani



log–log scale.

Figure 13.10. Normalized electrical resistivity versus normalized temperature for several transition metals at low temperatures. [K. Mendelssohn, *Can. J. Phys.* **34** (1956)]

# Magnetic impurities in metals

By magnetic impurities we mean impurities that contribute a Curie-Weiss term to the susceptibility.

## Fe embedded in pure Cu

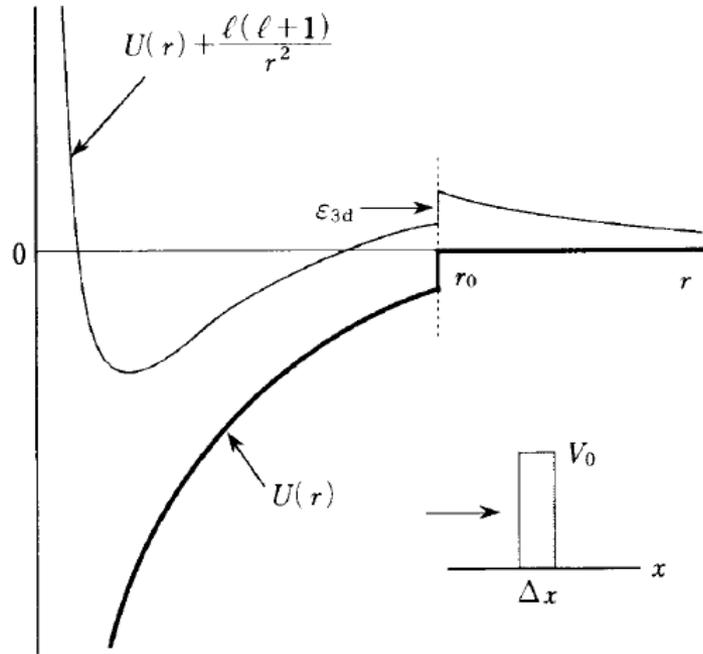
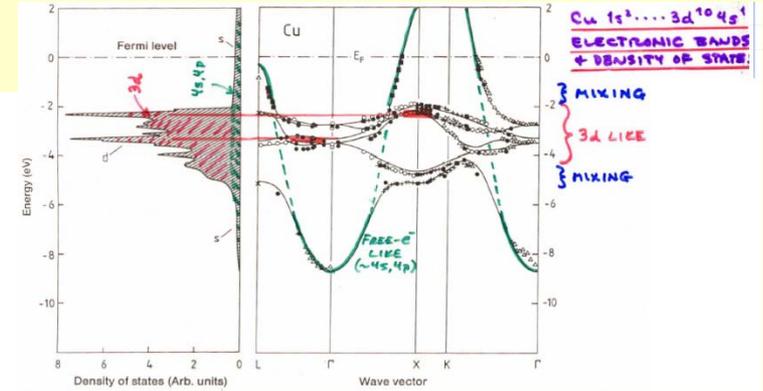


Figure 13.11. Impurity potential due to an Fe atom embedded in pure Cu. Its potential  $U(r)$  is approximated as the Coulomb potential in the range  $r \leq r_0$  and zero in  $r > r_0$ . A potential barrier is formed for electrons with  $\ell \geq 1$  as a result of the centrifugal potential. Its role may be approximated by the positive square-well potential shown inset.



See Mizutani

Iron

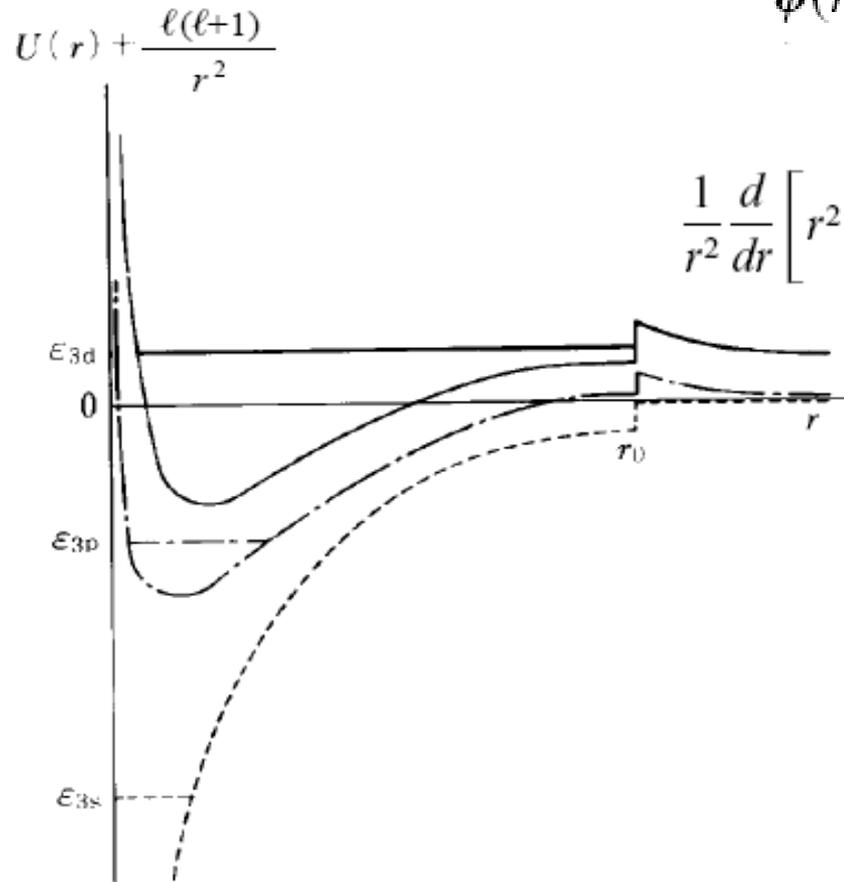
$[\text{Ar}].3d^6.4s^2$

Non-magnetic metals containing magnetic impurities

$$\psi(r, \theta, \varphi) = R(r) Y_{\ell m}(\theta, \varphi),$$

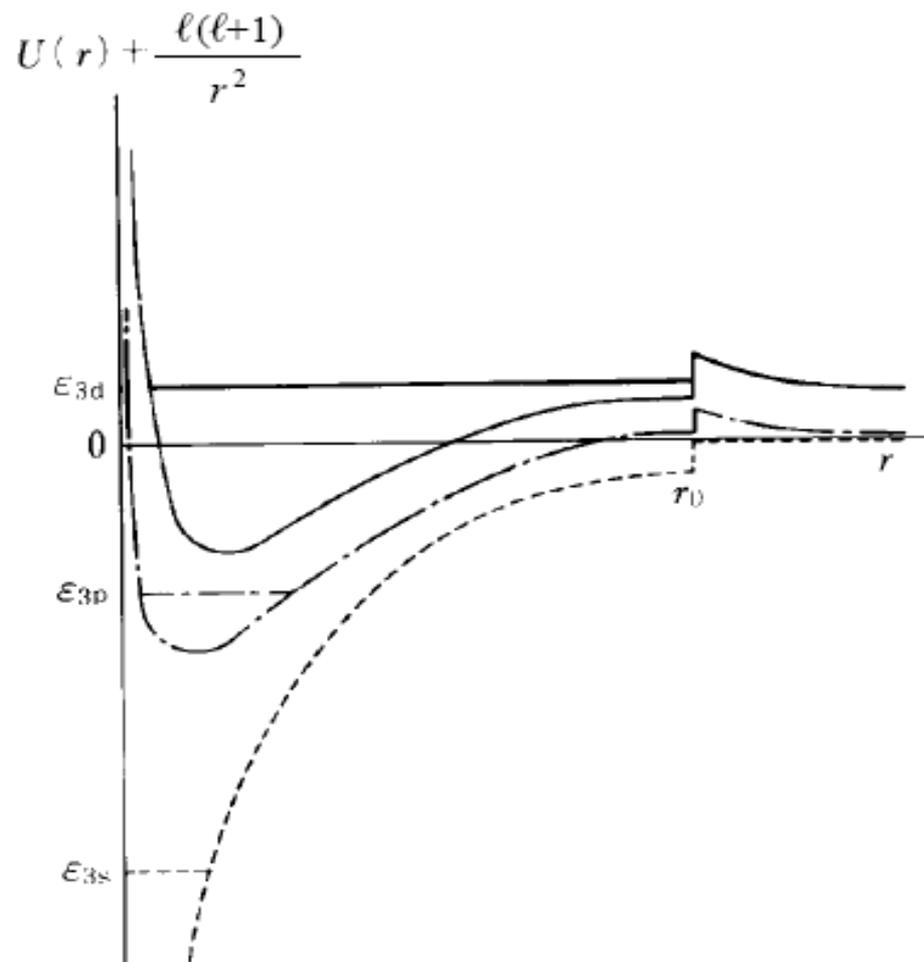
Schrödinger equation:

$$\frac{1}{r^2} \frac{d}{dr} \left[ r^2 \frac{dR(r)}{dr} \right] + \left\{ k^2 - \left[ U(r) + \frac{\ell(\ell+1)}{r^2} \right] \right\} R(r) = 0,$$



the repulsive  
centrifugal potential

Impurity potential and 3s, 3p and 3d energy levels of an Fe atom embedded in pure Cu.



Impurity potential and 3s, 3p and 3d energy levels of an Fe atom embedded in pure Cu.

The 3s and 3p electrons of the Fe atom will form core levels with negative energies but the energy level of the 3d electrons will be formed at a positive energy.

If the potential barrier were absent, the conduction electron with a positive energy would have been only weakly scattered by the impurity potential.

Instead, the existing potential barrier would enhance the tendency of the incident electron to localize in the range  $r \leq r_0$ .

the incident electron with a positive energy interacts strongly with the Fe 3d states because of the presence of the potential barrier.

However, some portion of the incident electron can escape from the impurity potential and mix with the wave function of the electrons forming the valence band of pure Cu. Thus the conduction electron coupled with the 3d electron is extended over a certain range in both real space and energies and results in **a narrow energy band near the Fermi level.**

the formation of the virtual bound state is believed to be responsible for an increase in the residual resistivity when small amounts of 3d-transition metal atoms are uniformly distributed within the matrix of a non-magnetic metal like pure Cu.

**FRIEDEL 1956**

$$\psi(r, \theta, \varphi) = R(r) Y_{\ell m}(\theta, \varphi),$$

Far from the impurity atom

$$\psi(r, \theta, \varphi) \xrightarrow{r \rightarrow \infty} A \left[ e^{ikz} + \frac{1}{r} f(\theta, \varphi) e^{ikr} \right].$$

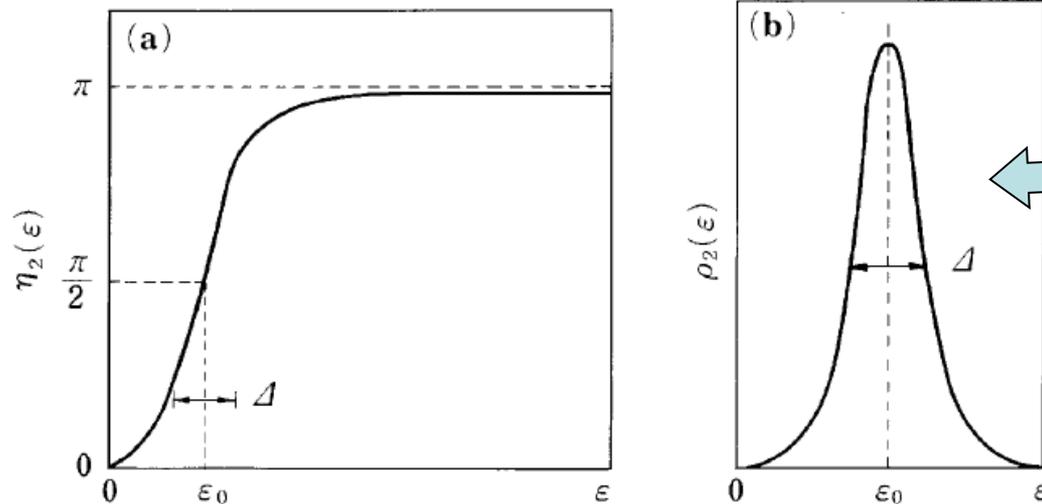
The impurity atom introduces an excess nuclear charge  $e\Delta Z$ , which must be screened by the same amount of the electronic charge, as required from the charge neutrality condition.

Here  $\Delta Z$  represents the difference in the valency between the impurity and matrix,

$$\Delta Z = 2 \int_0^{\varepsilon_F} \Delta n(\varepsilon) d\varepsilon = \frac{2}{\pi} \sum_{\ell} (2_{\ell} + 1) \eta_{\ell}(\varepsilon_F),$$

the excess nuclear charge around the impurity atom is screened by conduction electrons on an atomic scale.

from Mizutani



Virtual bound state

Figure 13.14. Energy dependence of (a) phase shift and (b) partial density of states for  $\ell=2$ . [J. Kondo, *Kinzoku Densi Ron* (Shokabo, Tokyo 1983) (in Japanese)]

$$\phi(r) \approx \frac{1}{r^3} \cos 2k_F r$$

When the **incident electron** has an energy  $\epsilon_0$  near  $\epsilon_{3d}$

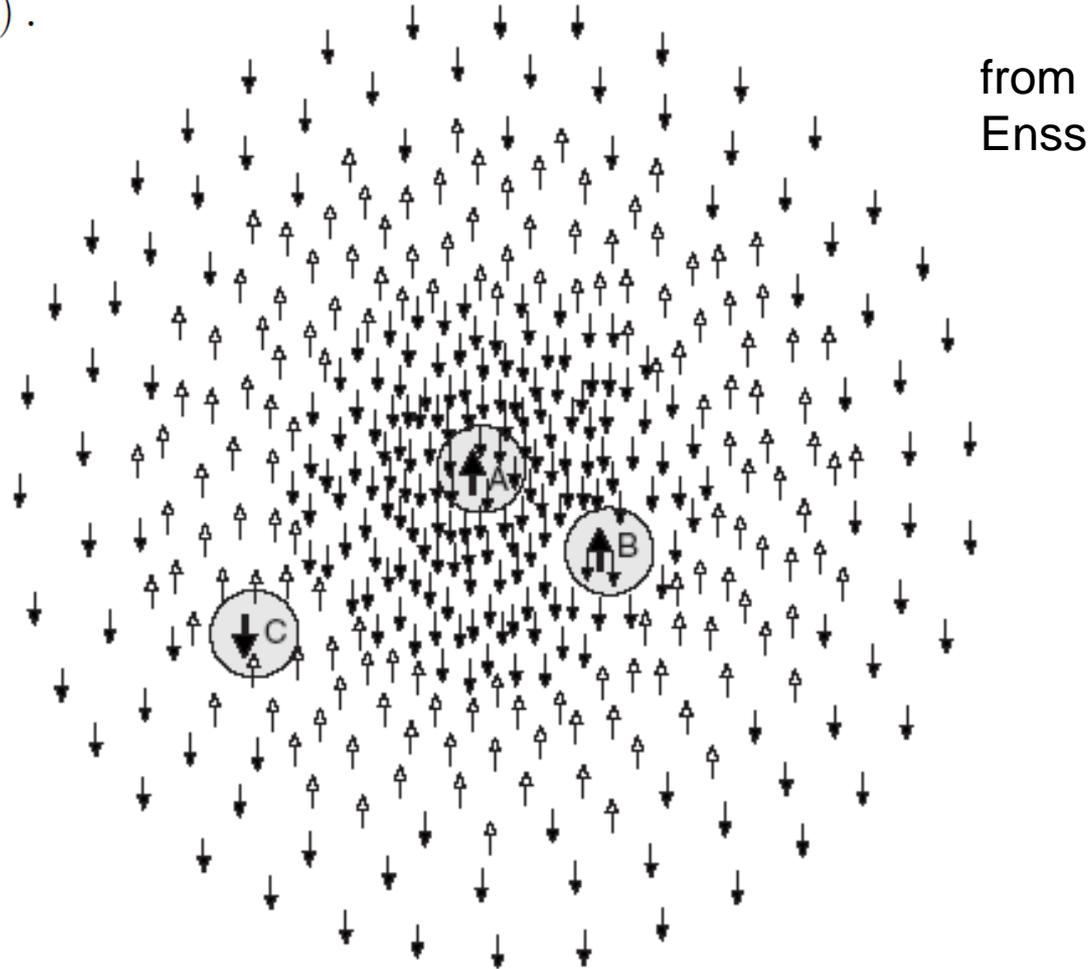
This is called **resonant scattering** and enhances the tendency for these electrons **to localize** inside the potential barrier as a result of the centrifugal potential.

a redistribution of conduction electrons occurs and results in a change in the density of states in the energy range  $\epsilon$  to  $\epsilon+d\epsilon$ .

$$\mathcal{H}_{sd} = -J \mathbf{S} \cdot \mathbf{s} \delta(\mathbf{r} - \mathbf{R}).$$

spatial  
variation of the  
magnetization of  
the form:

$$\frac{1}{r^3} \cos 2k_F r$$



**Fig. 7.17.** Friedel oscillation of the spin polarization. The localized magnetic moment A causes an oscillation of the spin polarization of the neighboring conduction electrons, resulting in an indirect exchange interaction between neighboring impurities. Depending on the distance, a ferro- (B) or antiferromagnetic (C) alignment of the magnetic moments of the impurities is favored. For clarity, only the spin polarization due to the magnetic moment A is shown

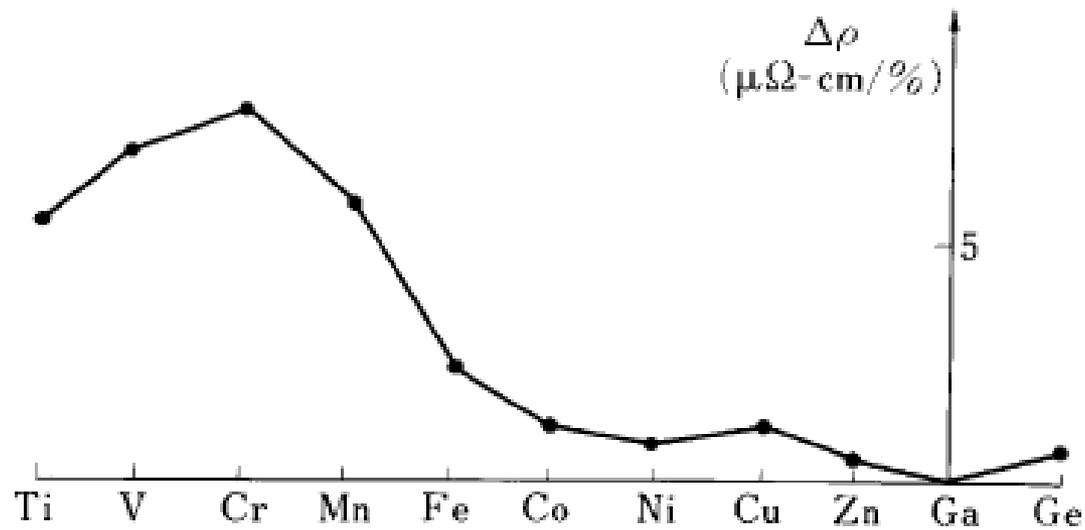


Figure 13.15. Residual resistivity due to addition of 1 at.% transition metal impurity to pure Al. [From ref. 10.]

from  
Mizutani

# Anderson model



## Magnetic impurities

Magnetic ions have interesting properties when they are found as impurities in nonmagnetic crystals.

- They usually retain their magnetic moment, so small magnets are distributed randomly throughout the crystal. If the host crystal is a **metal**, the magnetic impurities make an interesting contribution to the electrical resistivity.

- The conduction electrons scatter from the magnetic impurity.

Depending on the position of the ion levels with respect to the Fermi energy of the host, **electrons from the ions** might join the conduction electrons or, vice versa, **electrons from the conduction band** might drop into lower-lying **ionic levels**.

from Mizutani



Let us consider again a system where a single 3d-transition metal atom is embedded in a non-magnetic metal like Al and assume that this impurity atom possesses only one spin-up 3d electron. Suppose that a spin-down 3d electron is added to the same orbit as the already existing spin-up 3d electron. The addition of this electron increases the energy of the system by  $U$  as a result of the Coulomb repulsive interaction. In the atomic 3d orbitals, the Coulomb interaction is very large and is of the order of 30eV. Its magnitude is reduced to 1–7eV for 3d electrons in a metallic environment due partly to the delocalization of the orbital and due partly to screening by the other electrons. This is still fairly large and plays a key role in the following discussions.

the energy of a second spin-up electron increases by the **Coulomb energy  $U$**  if *the first spin-down electron has already occupied the lowest 3d orbital with the energy  $\varepsilon_0$ .*

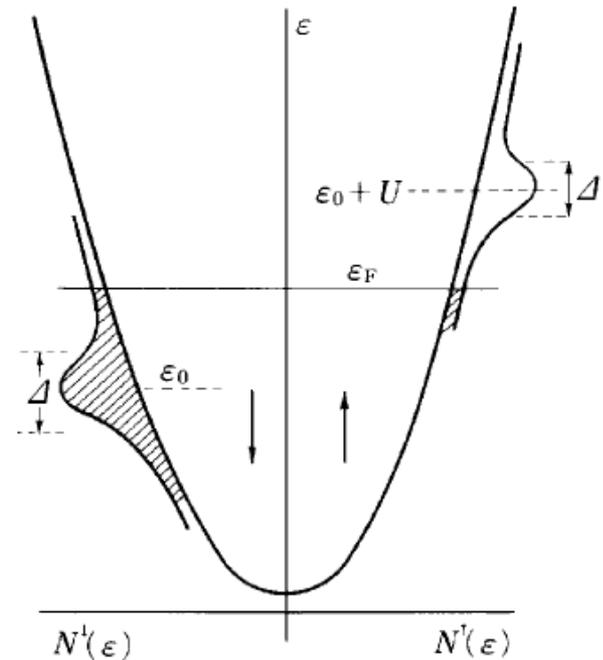
Anderson predicted the local moment to appear,

$\varepsilon_0 + U \gg \varepsilon_F$  and  $\varepsilon_0 \ll \varepsilon_F$ , together with  $|\varepsilon_0 + U - \varepsilon_F| \gg \Delta$  and  $|\varepsilon_F - \varepsilon_0| \gg \Delta$ .

$$\pi\Delta/U < 1.$$

- Since the energy of the second spin-up electron is  $\varepsilon_0 + U$ , this state will not be occupied.
- Therefore, the 3d orbital is filled only by the spin-down 3d electron, resulting in the appearance of the magnetic moment.

the magnetic moment will not appear, if both energy levels  $\varepsilon_0$  and  $\varepsilon_0 + U$  appear above or below the Fermi level.



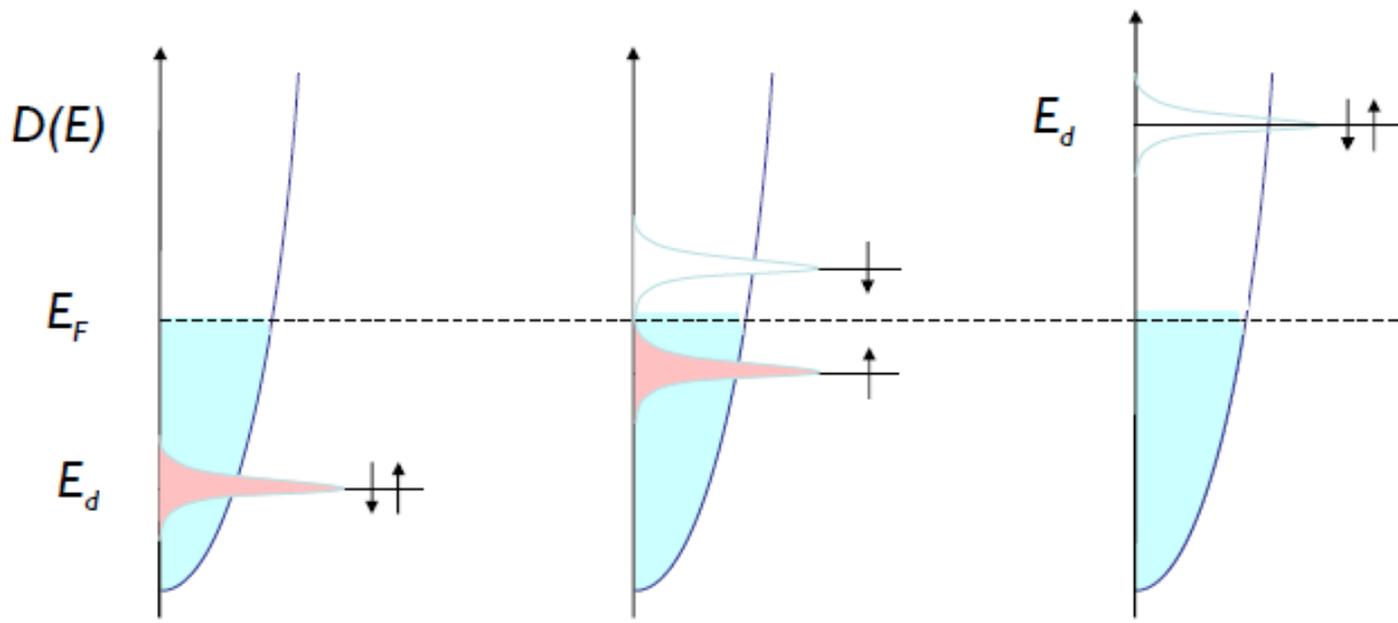
Anderson model for the formation of localized moment.

see Mizutani

non-magnetic

local magnetic moment

non-magnetic



The phenomenon associated with the exchange of the spin orientation between the magnetic moment of the impurity atom and that of the conduction electron is called **spin fluctuation**.

a localized moment will not be observed at low temperatures:

Here the frequency of spin fluctuations is so high over the thermal fluctuation time-scale  $\tau$  that only an average of up- and down-spins is observed.

Let us summarize the magnetic properties of a magnetically dilute alloy as a function of temperature and Coulomb energy  $U$ . Obviously, a localized moment will not appear and the temperature-independent Pauli paramagnetic susceptibility will dominate, when  $U$  is zero. Its value is given by  $N(\epsilon_F)\mu_B^2$ . In contrast, a localized moment appears, when  $U$  becomes large and satisfies the condition  $\pi\Delta/U < 1$ . Thus, the magnetic susceptibility obeys the Curie law owing to the moment localized at the impurity atom. However, the moment will apparently disappear at low temperatures where the condition  $T < T_{sf} \ll T_\Delta$  is satisfied. This behavior is schematically illustrated in Fig. 13.17.

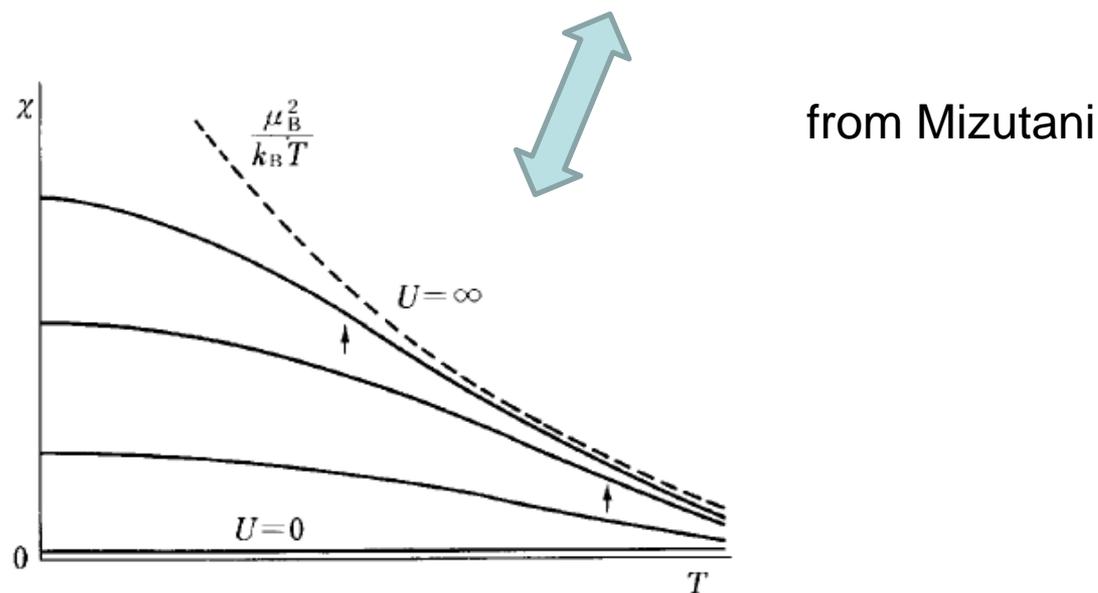


Figure 13.17. Temperature dependence of magnetic susceptibility with Coulomb energy  $U$  as a parameter. The arrows indicate the temperatures above which spinfluctuations dominate. [J. Kondo, *Kinzoku Densi Ron* (Shokabo, Tokyo 1983) (in Japanese)]

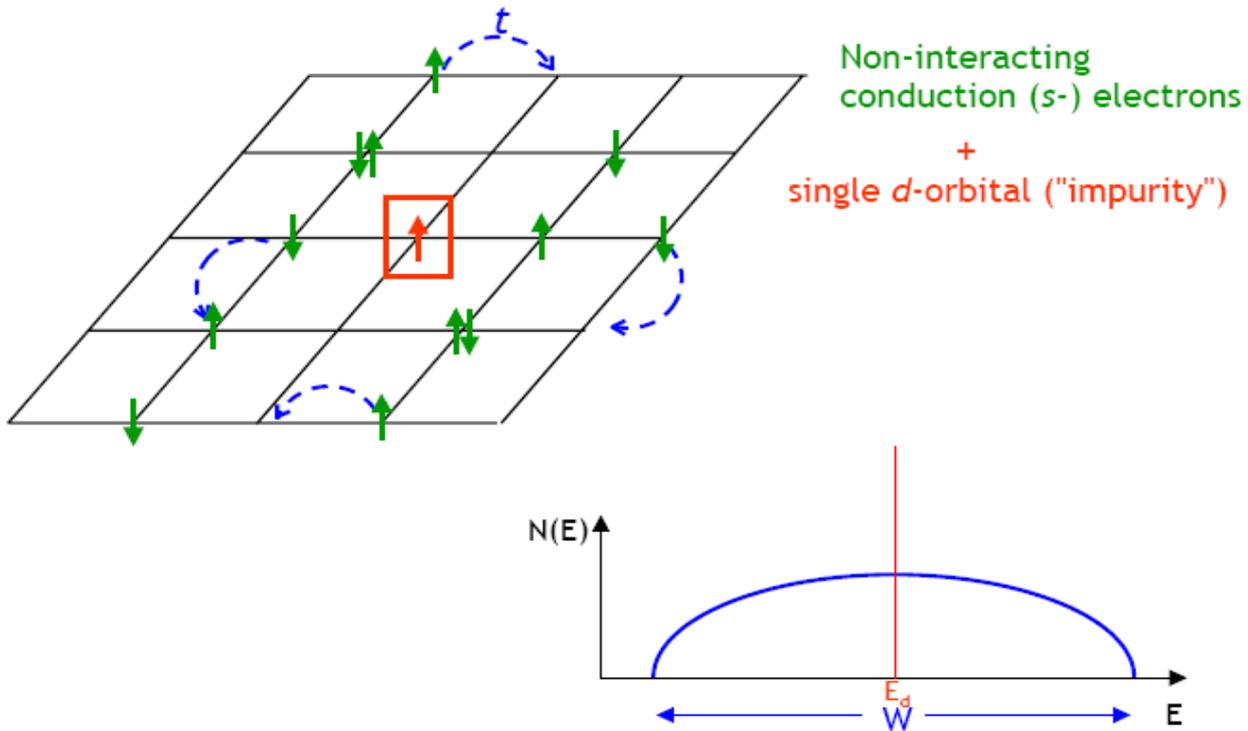
**local orbital  
electron with  
magnetic moment**

**conduction electrons**

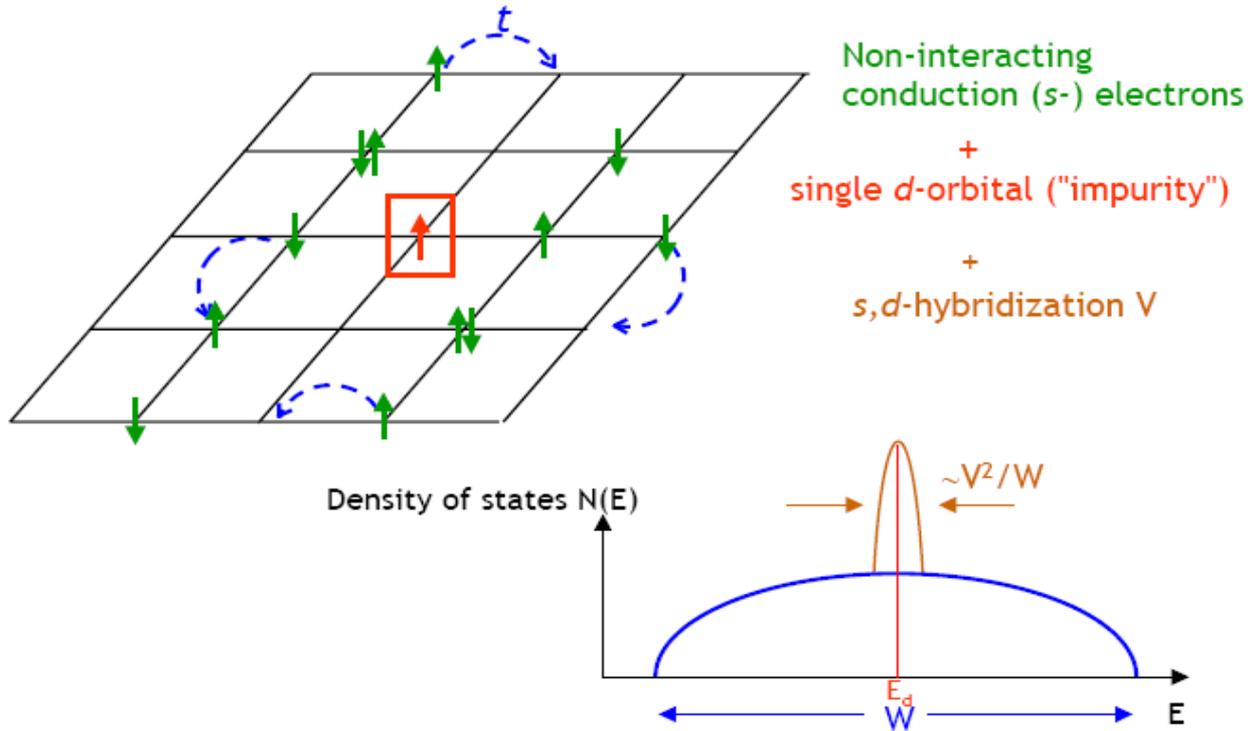


**And.  
Imp.**

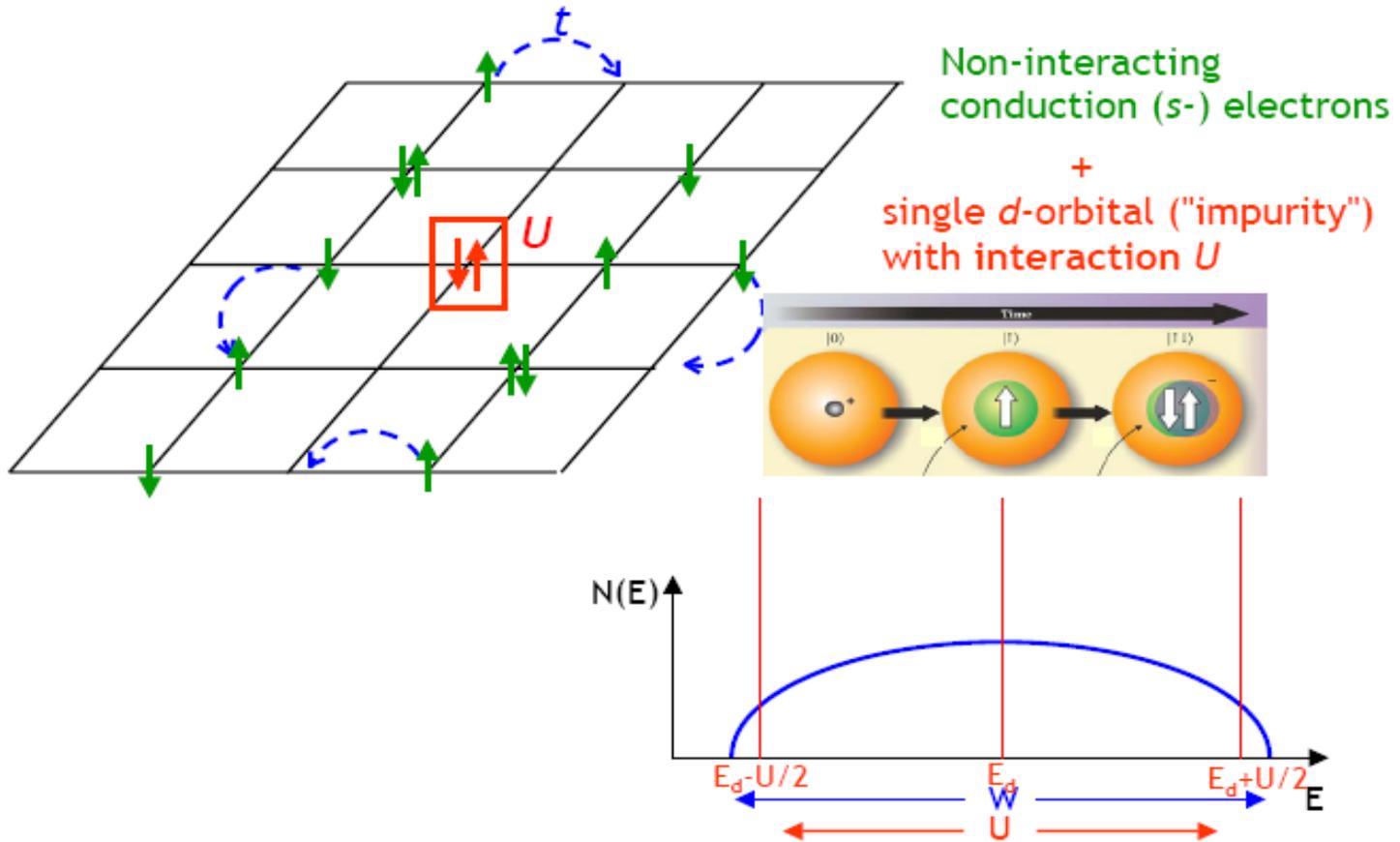
## Excursion: Single-impurity Anderson model



## Excursion: Single-impurity Anderson model

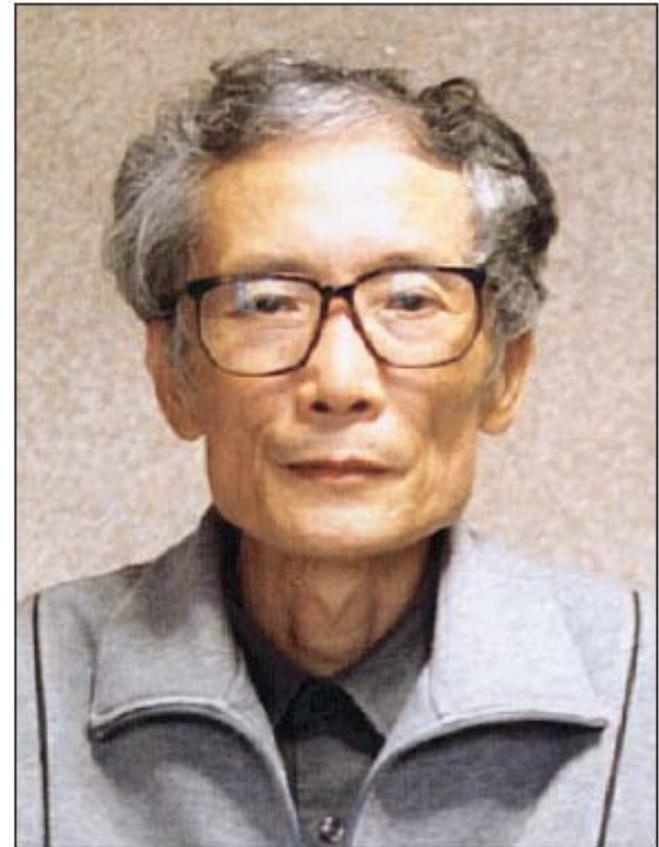
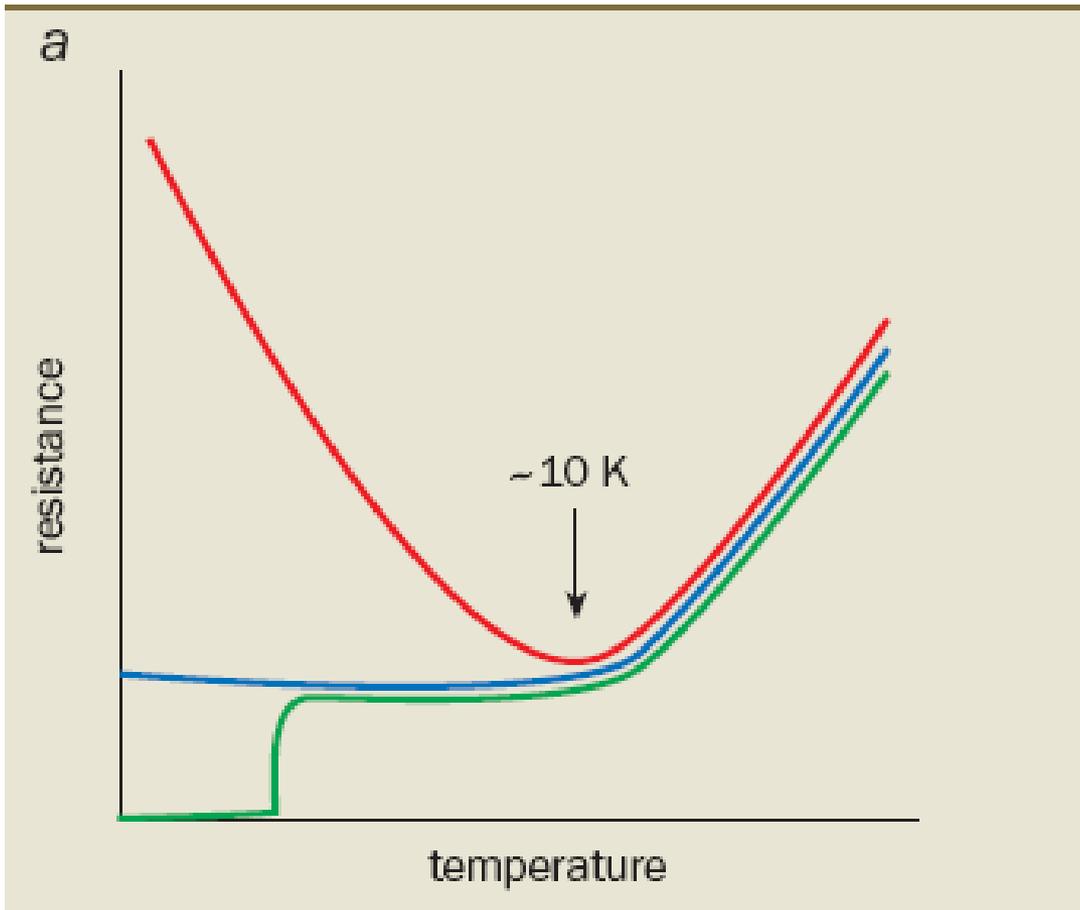


## Excursion: Single-impurity Anderson model





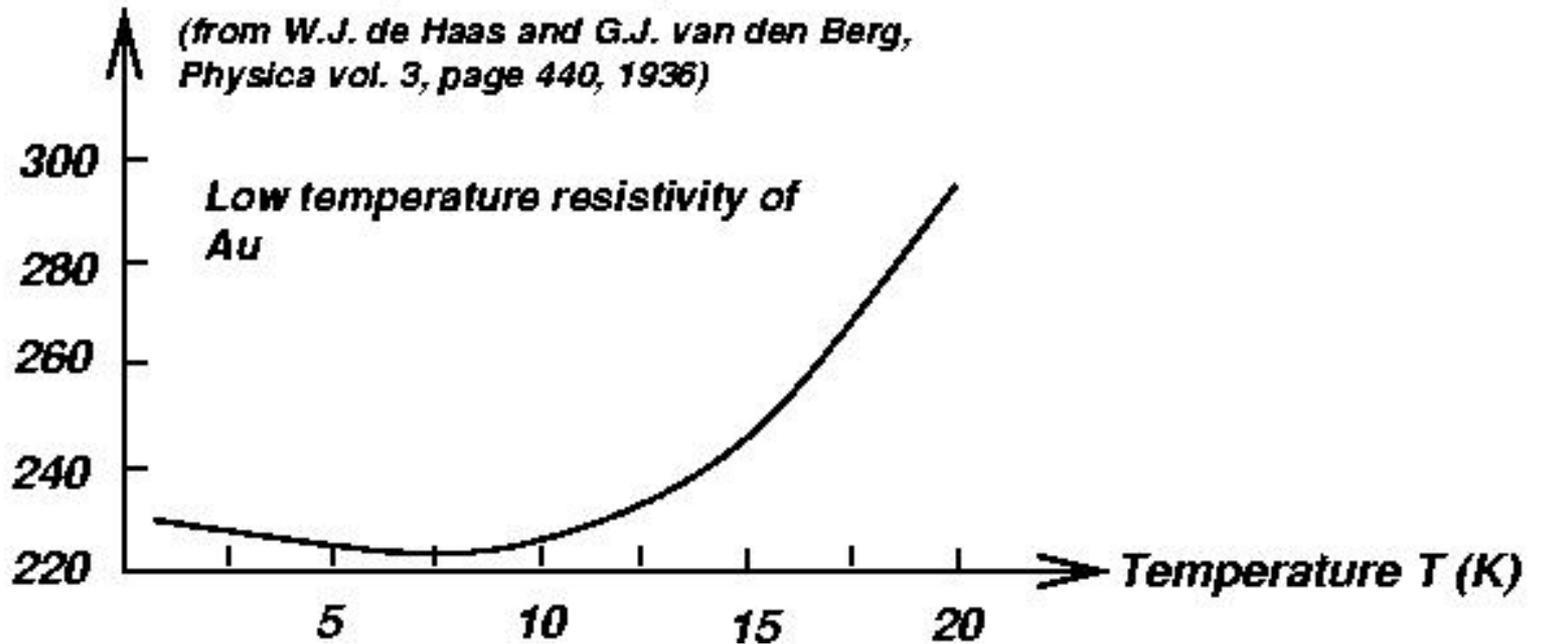
# Kondo effect



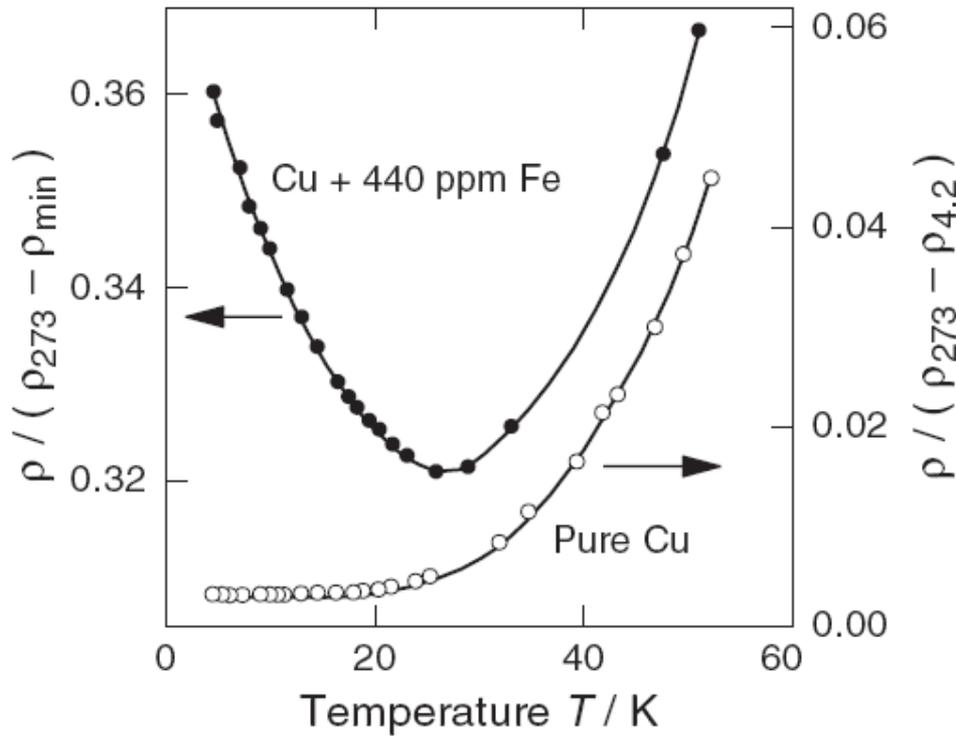
The theory that describes the scattering of electrons from a localized magnetic impurity was initiated by the work of Jun Kondo in 1964

**Resistance/Resistance(T=0 Celsius) x 10000**

*(from W.J. de Haas and G.J. van den Berg,  
Physica vol. 3, page 440, 1936)*



Probably gold with iron impurities

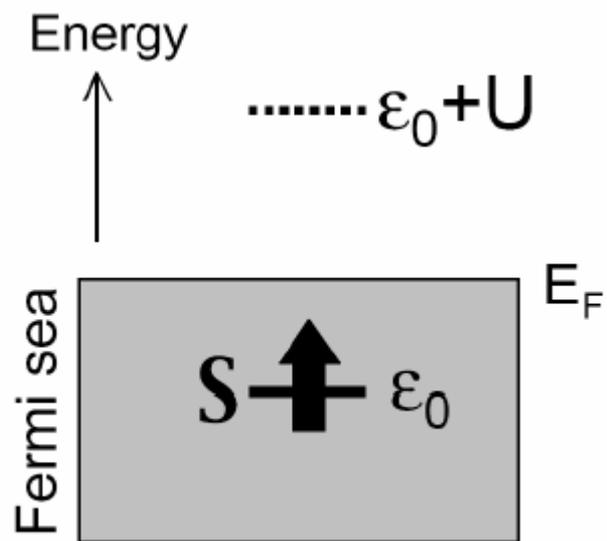


**Fig. 7.15.** Reduced electrical resistivity of pure copper and copper doped with 440 ppm Fe.  $\rho_{4.2}$ ,  $\rho_{273}$ , and  $\rho_{\min}$  represent the resistivity at helium temperature, room temperature, and at the minimum of the resistance curve, respectively. The *full lines* connect the data points [315]

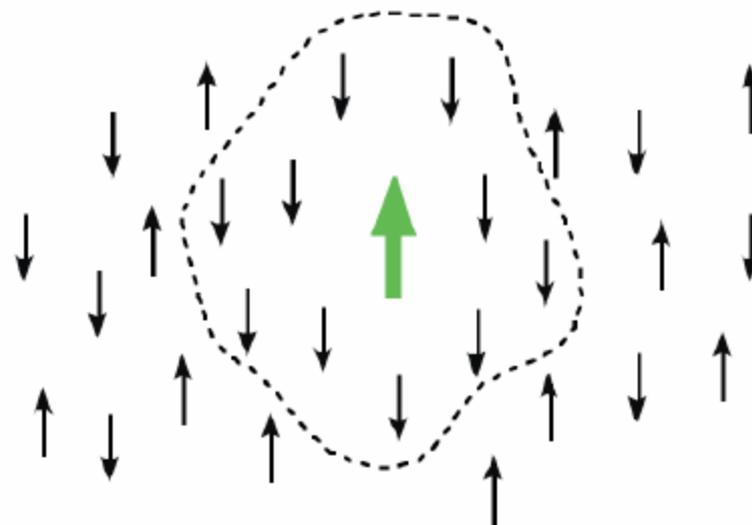
from Enss

In 1964, it was shown by *Kondo* that this phenomenon reflects the **spin dependent scattering** of the conduction electrons by the magnetic moments of **impurity atoms**

- simple metals containing a small amount of a transition metal
- With decreasing temperature the **exchange interaction** between the **conduction electrons** and the **localized  $d$ -electrons of the impurity** atoms becomes more and more significant, resulting in a rising electrical resistivity.
- Together with the  $T^5$ -dependent resistivity due to the electron-phonon interaction this specific scattering mechanism leads to the above-mentioned minimum.

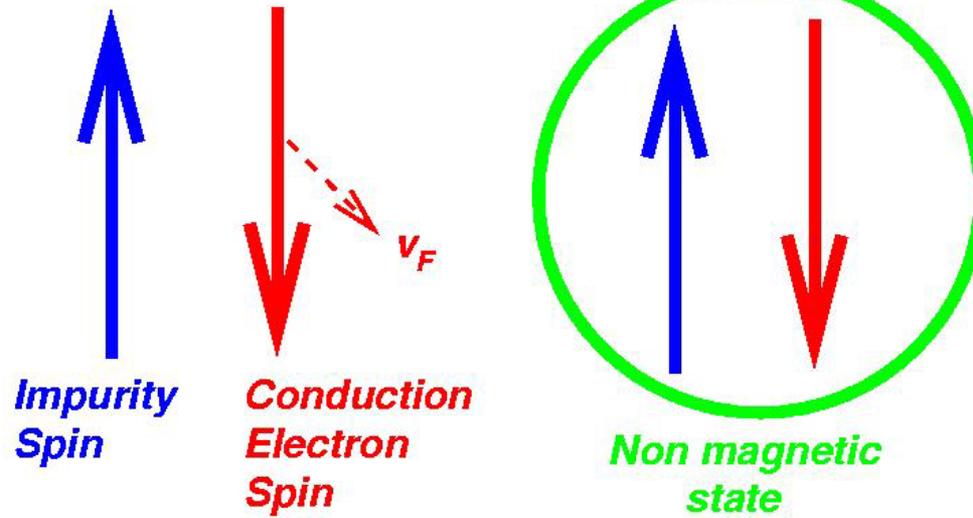


new energy scale: Kondo temperature  $T_K$   
 formation of spin-singlet screening cloud



cloud: more effective scatterer  
 increase in resistance

*High T – weak coupling      Low T – strong coupling*



$$H_{\text{sd}} = -2J(\mathbf{r} - \mathbf{R}_n)(\mathbf{s} \cdot \mathbf{S}_n),$$

Perturbation theory in the 3<sup>rd</sup> ordin.

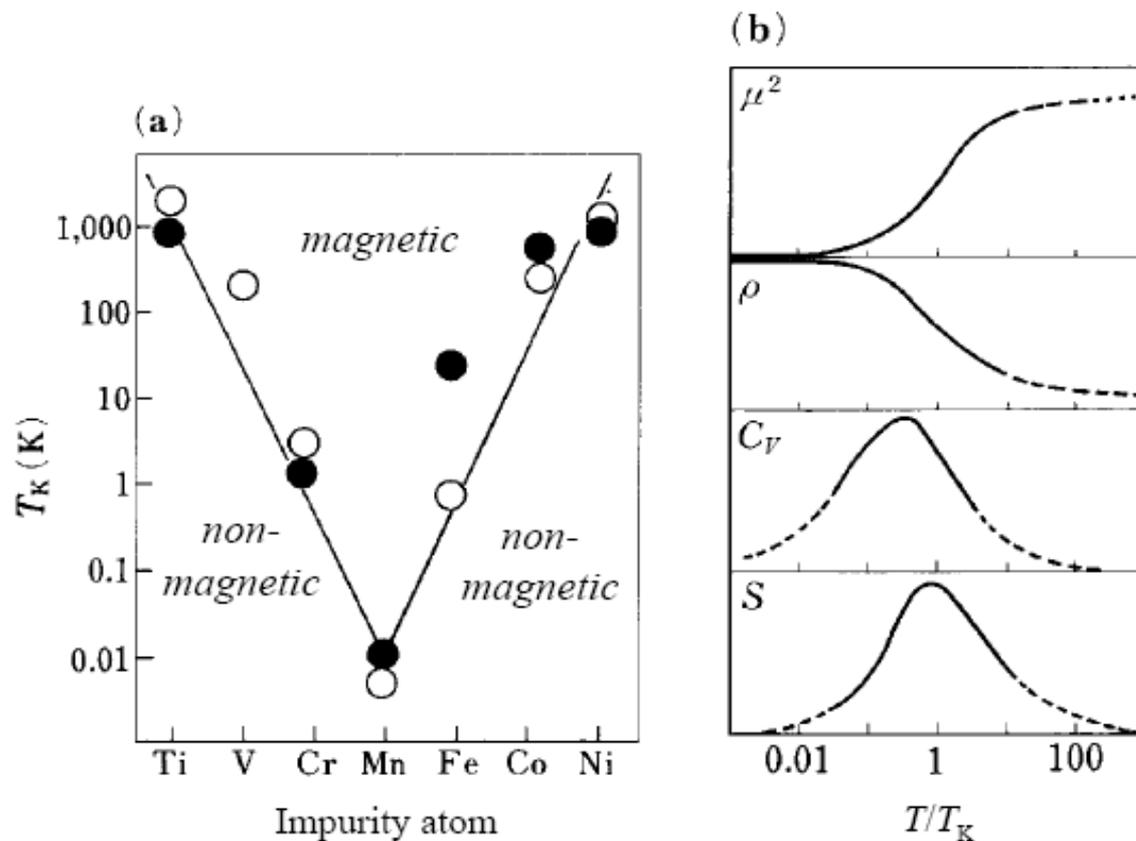
$$\rho_{\text{spin}} = c\rho_0 \pi^2 [N(\epsilon_F)]^2 J^2 S(S+1) \left[ 1 + 4JN(\epsilon_F) \log \frac{k_B T}{D} \right],$$

$$\rho = \rho_0 + AT^5 + B \log T,$$

$$T_{\text{min}} = (|B|/5A)^{1/5}$$

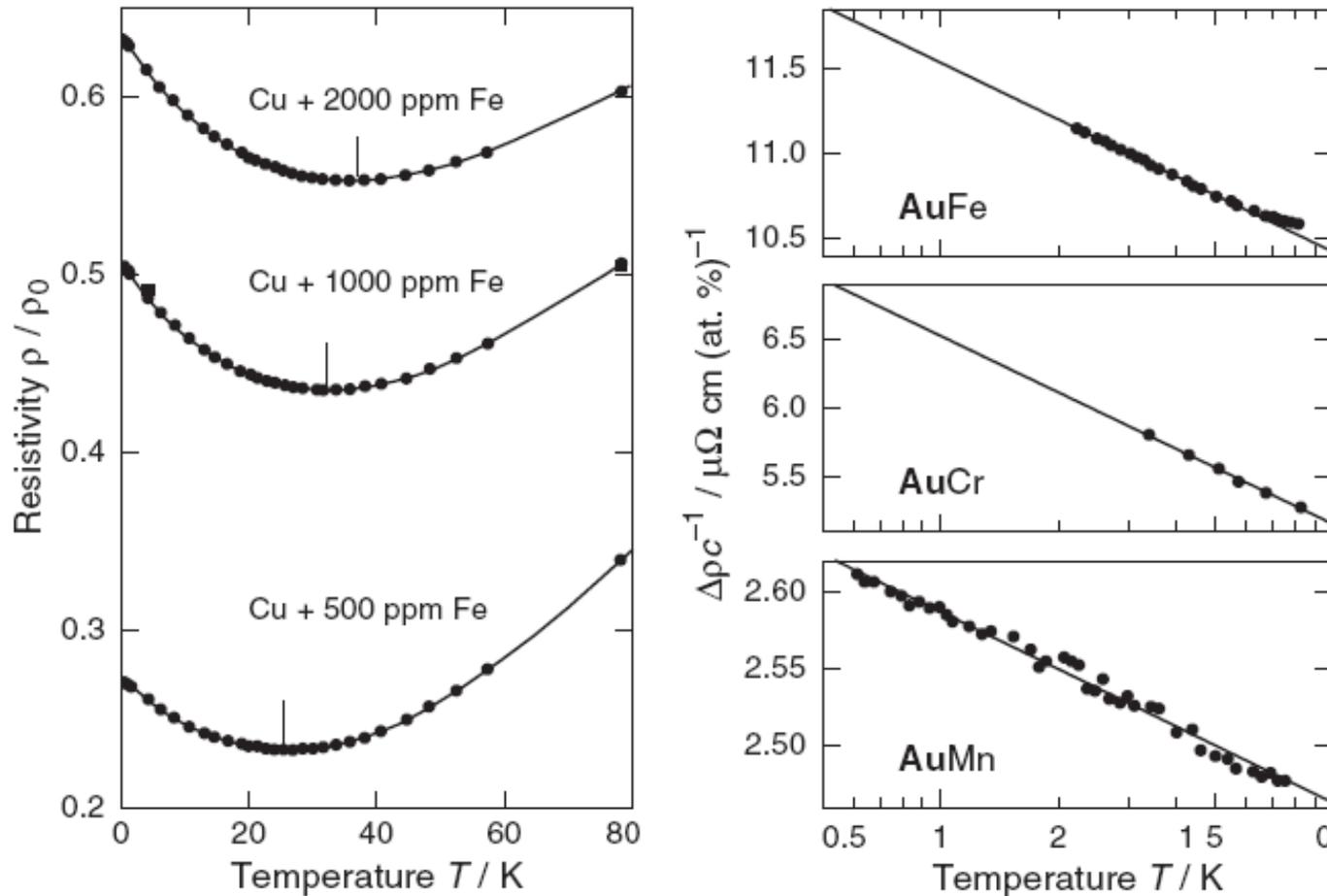
$$T_K = \frac{D}{k_B} \exp[-1/|J|N(\epsilon_F)].$$

Briefly, an **antiferromagnetic coupling** between the **localized spin** and the **spin of the conduction electron** is essential in the Kondo effect. It induces spin polarization of the conduction electron in a direction opposite to that of the localized moment and cancels completely the localized moment, leading to the formation of a singlet state  $S=0$  at absolute zero.



from  
Mizutani

Figure 13.19. (a) Kondo temperature for Cu (●) and Au (○) alloyed with various 3d-transition metal impurities. (b) Temperature dependence of the magnetic moment  $\mu^2$ , electrical resistivity  $\rho$ , specific heat  $C_V$  and thermoelectric power  $S$ . Temperature is normalized with respect to the Kondo temperature. Solid curves are based on experimental behavior with the properties of the pure host material subtracted. The dashed portions have not been examined experimentally and represent predictions based on theory. [M. D. Daybell, *Magnetism* edited by H.Suhl, (Academic Press, 1973) vol.5, pp. 121–147]



**Fig. 7.19.** (a) Temperature variation of the electrical resistivity of copper containing different concentrations of iron. The *full lines* represent the theoretical results [322]. (b) Electrical resistivity of gold samples with different magnetic impurities. The logarithmic variation of the resistivity below the resistance minimum is clearly visible [323]

from Enss