UNDE ELASTICE

particule care interacționează

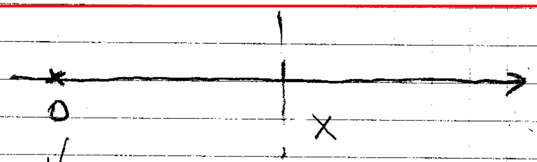
- medii continue: gaze, lichide, solide
- propagarea perturbațiilor

Unde plane

considerăm | fără atenuare

- particulele situate într-un plan perp. pe direcția de propagare oscilează identic

elongația



$u(0,t) = f(t)$; viteza c

$u(x,t) = u(0, t - x/c) = f(t - x/c) = F(x - ct)$

F(x-c

longitudinală / transversală

unda plană progresivă în sensul Ox

$c \to -c$ $\leftarrow$

$u(x,t) = f(t + x/c)$

$= F(x + ct)$

Unda plană progresivă monocromatică

osc. armonice de o frecv. ω

$u(0,t) = A\cos\omega t = f(t)$

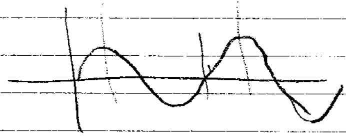
$$\boxed{u(x,t) = f(t - x/c) = A\cos\omega(t - x/c)}$$

periodică în spațiu și timp

$$\cos\omega\left(t-\frac{x+\lambda}{c}\right)=\cos\omega\left(t-\frac{x}{c}\right)\to\frac{\omega\lambda}{c}=2\pi$$

$$\lambda=\frac{2\pi c}{\omega}=cT=\frac{c}{\nu}$$

lungimea de undă



$$u(x,t)=A\cos\omega(t-x/c)=A\cos2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$$

$$=A\cos(\omega t-kx)$$

număr de undă $k=\frac{2\pi}{\lambda}=\frac{2\pi}{cT}=\frac{\omega}{c}$

vectorul de undă $\vec{k}=k\vec{n}=\frac{2\pi}{\lambda}\vec{n}=\frac{\omega}{c}\vec{n}$

$$u(\vec{r},t)=A\cos(\omega t-\vec{k}\cdot\vec{r})=\mathrm{Re}\left\{Ae^{i(\omega t-\vec{k}\vec{r})}\right\}$$

faza $\varphi=\omega t-\vec{k}\vec{r}$

suprafețe de undă – $\boxed{\text{supraf. de fază const.}}$

– perp. pe direcția de propagare

$\omega t-\vec{k}\vec{r}=\text{const.}$ ec. unui __plan__

normalele pe supraf de undă → $\boxed{\text{raze}}$

pt. unde monocromatice:

viteza undei = viteza de fază

$$\varphi = \omega t - kx = \omega(t+dt) - k(x+dx)$$

$$d\varphi = \omega dt - k dx = 0 \qquad \boxed{v_f = \frac{dx}{dt} = \frac{\omega}{k} = \frac{\lambda}{T} = c}$$

dacă faza din x la t
a ajuns $x+dx$ la $t+dt$

unde elastice → unde sonore

infrasunete $< 16\,Hz$

audibile $16\,Hz - 20\,kHz$

ultrasunete $> 20\,kHz \rightarrow 10\,GHz$

Ecuația undelor

$$u(x,t) = f(\underbrace{t-\tfrac{x}{c}}_{z})$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial t}$$

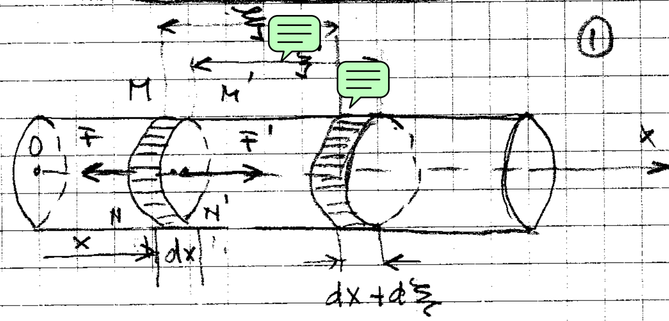
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x} = -\frac{\partial f}{\partial z}\cdot\frac{1}{c} = -\frac{1}{c}\frac{\partial f}{\partial t}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = -\frac{1}{c}\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial t}\right) = -\frac{1}{c}\frac{\partial^2 f}{\partial x \partial t}$$

$$= -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial x}\right) = \frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} \quad (3)$$

d'Alembertian

Unde elastice într-un corp solid elastic



1

deformarea coloanei dintre cele 2 secțiuni

$$d\xi = \xi' - \xi = (\partial \xi / \partial x) \cdot dx$$

$$\frac{\partial \xi}{\partial x} = \varepsilon$$ deformația specifică normală

ca urmare a deformării – forțe elastice F, F'

l. Hooke

$$F = EA \frac{\partial \xi}{\partial x} \qquad \sigma = E\varepsilon$$

când coloana nu este în echilibru

$$F' - F = (\partial F / \partial x)\, dx$$ aproximare

legea a doua a dinamicii

$$(\rho A \cancel{dx}) \cdot \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial F}{\partial x} \cdot \cancel{dx}$$

$$\frac{\partial F}{\partial x} = \rho A \frac{\partial^2 \xi}{\partial t^2} \longrightarrow \boxed{\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 \xi}{\partial x^2}}$$

$$\boxed{c = \sqrt{\frac{E}{\rho}}}$$

3D care nu se def. transversal (volum constant) ②

pentru un mediu elastic ~~infinit~~

$$c' = \left\{E(1-\mu)/[(1-2\mu)(1+\mu)\rho]\right\}^{1/2}$$

μ – coef. de contracție transversală a lui Poisson

E modulul de elasticitate longitudinal

– în cazul perturbațiilor transversale:

$$c_t = \sqrt{\frac{G}{\rho}}$$

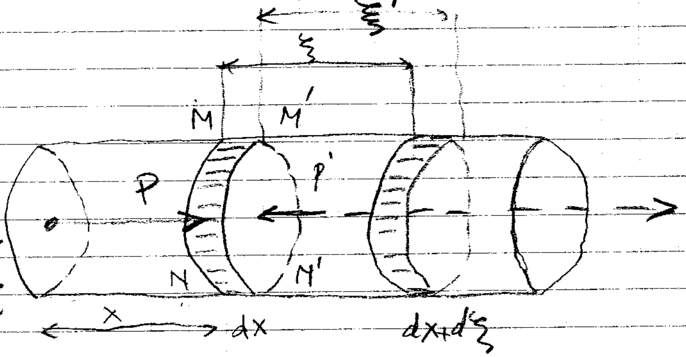
G – modul de forfecare

$G \sim 0,4E$

c_l, c', c_t

pt. metale $\mu \cong 0,3$ ex. $\rightarrow c_l/c_t = 1,87$

$\boxed{\text{Unde de presiune într-un gaz}}$



gazele compresibile

fluctuații de presiune

–||– densitate

perturbație volumele $p \mp p'$

$\cancel{A\,dx \rightarrow A(dx+d\xi)}$; $d\xi = \xi' - \xi$

conservare masei

$$(\cancel{\rho'}+\rho_0)A\,dx = \rho A(dx+d\xi)$$

$$\rho = \rho_0\left(1+\frac{\partial \xi}{\partial x}\right)^{-1} \simeq \rho_0\left(1-\frac{\partial \xi}{\partial x}\right)$$

②

~~Alamă~~	E $10^{10}\ \frac{N}{m^2}$	ρ g/cm^3	μ	c_l m/s 20°C	c_l' m/s	c_t m/s
Alamă	9	8.4-8,7	0.35	3400	4500	2100
Aluminiu	7	2,7	0,34	5240	6400	3130
Oțel	20-22	7,8	0,28	5100	6000	3200
Sticlă	6	2,4-2,8	0,25	5000	5200	3300

$$\frac{\Delta b}{b_0} = -\mu \varepsilon$$

$$\rho - \rho_0 = -\rho_0 \frac{\partial \xi}{\partial x} \quad \Big| \quad \frac{\partial \rho}{\partial x} = -\rho_0 \frac{\partial^2 \xi}{\partial x^2}$$ (3)

$$p = p(\rho)$$

$$p = p_0 + (\rho - \rho_0)\left(\frac{dp}{d\rho}\right)_0 + \frac{1}{2}(\rho - \rho_0)^2 \left(\frac{d^2 p}{d\rho^2}\right)_0 + \dots$$

$$p = p_0 + (\rho - \rho_0)\left(\frac{dp}{d\rho}\right)_0$$

$$\boxed{K = \rho_0 \left(\frac{dp}{d\rho}\right)_0}$$ modul de elasticitate volumic

$$[K]_{SI} = N m^{-2}$$

$$(\rho - \rho_0)/\rho_0 = -\frac{\partial \xi}{\partial x}$$

$$\Delta p = \frac{K}{\rho_0} \Delta \rho$$

$$p = p_0 + K \frac{\rho - \rho_0}{\rho_0}$$ | diferențiem cu t

$$\Delta p = -K \frac{\partial \xi}{\partial x}$$

$$\frac{\partial p}{\partial t} = \frac{K}{\rho_0} \frac{\partial \rho}{\partial t} \quad \Big| \Rightarrow \boxed{p = p_0 - K \frac{\partial \xi}{\partial x}}$$

corespunde legii lui Hooke pt. fluide

$$\Delta p = c^2 \Delta \rho$$

$$p' = -K \frac{\partial \xi}{\partial x}$$ → presiune instantanee (unda de presiune) presiune sonoră instantanee

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$$p = p_0 + p'$$

$$\frac{\partial p}{\partial x} = -K \frac{\partial^2 \xi}{\partial x^2}$$

(3)

$$dF = A(p' - p) = -A\,dp$$

legea a II a lui Newton

$$-A\,dp = (\rho_0 A\,dx)\frac{\partial^2 \xi}{\partial t^2}$$

$$\left(\frac{\partial p}{\partial x} = -k\frac{\partial^2 \xi}{\partial x^2}\right)$$

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial^2 \xi}{\partial t^2} = -\rho_0 \frac{\partial v_x}{\partial t}$$

!

$$\boxed{\Delta p = c^2 \Delta p}$$

$$\boxed{\frac{\partial^2 \xi}{\partial t^2} = \frac{k}{\rho_0}\cdot\frac{\partial^2 \xi}{\partial x^2}}$$

$$c = \sqrt{\frac{k}{\rho_0}}$$

verso →

$$\frac{\partial^2 p}{\partial t^2} = \frac{k}{\rho_0}\cdot\frac{\partial^2 p}{\partial x^2}$$

$$\frac{\partial^2 p}{\partial t^2} = c^2 \Delta p$$

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{k}{\rho_0}\cdot\frac{\partial^2 \rho}{\partial x^2}$$

ex. lichide →

Viteza undelor în gaze

Newton → proc. izoterm

$pV = \text{const} \rightarrow \ln p + \ln V = \text{const.}$

$$\frac{dp}{p} + \frac{dV}{V} = 0, \quad \frac{dp}{dV} = -\frac{p}{V}$$

	°C	c m/s
apă	0	1407
mercur	20	1451
alcool	20	1177

Viteza de propagare a undelor în lichide

$$\left(\frac{dp}{d\rho}\right) = \frac{K}{\rho_0}$$

$$\Delta p = p - p_0 = \left(\frac{\partial p}{\partial \rho}\right)\cdot\Delta\rho$$

$$\boxed{\Delta p = c^2\Delta\rho} = -\frac{K}{\rho_0}\cdot\rho_0\frac{\partial\xi}{\partial x} = +\frac{K}{\rho_0}\cdot\rho_0\overset{v}{\frac{\partial\xi}{\partial t}}\cdot\overset{c}{\frac{\partial t}{\partial x}}$$

$$\boxed{\Delta\rho = -\rho_0\frac{\partial\xi}{\partial x}}$$

$$\boxed{\Delta p = \rho_0 c v}$$

$$K = \rho_0 \left(\frac{\partial p}{\partial \rho}\right)_0 = -V \frac{dp}{dV} \qquad V \quad \rho \quad \frac{d\nu}{d\frac{m}{V}}$$

$$pV = \frac{m}{\mu} R T \quad \Rightarrow \quad p = \frac{\rho}{\mu} R T$$

$$\frac{dp}{d\rho} = \frac{1}{\mu} R T$$

$$\rho \frac{dp}{d\rho} = \frac{m}{V\mu} R T = \frac{\nu R T}{V} = \frac{pV}{V} = p$$

$$K_{izot} = p \quad \rightarrow \quad \underline{c = \sqrt{p/\rho}}$$

infirmat de experiment

Laplace $\rightarrow$ procesul adiabatic

$$pV^{\gamma} = \text{const.} \qquad \gamma = c_p / c_v$$

$\gamma > 1 \qquad \gamma \simeq 1{,}4$ pt. gaze biatomice

$$\ln p + \gamma \ln V = \ln \text{const}$$

$$\frac{dp}{p} + \gamma \frac{dV}{V} = 0$$

$$\frac{dp}{dV} = -\gamma \frac{p}{V} \quad , \quad K_{ad} = +V \gamma \frac{p}{V} = \underline{\gamma p}$$

$$c = \sqrt{\frac{K_{ad}}{\rho}} = \sqrt{\gamma \frac{p}{\rho}} = \boxed{\sqrt{\gamma \frac{RT}{\mu}}}$$

$$c = \sqrt{\gamma \frac{RT}{\mu}} = c_0\sqrt{1+\alpha t}\,; \quad \alpha = \frac{1}{273{,}15}\,K^{-1}$$

$t = 20°C \qquad c = 340\,m/s$ aer apă – 1480 m/s

nu dep.

de p!

	m/s
aer	331,46
N_2	333,64
H_2	1286
O_2	315

0°C

! $\boxed{\Delta p = c^2 \Delta \rho}$

$$\frac{dV}{V} = \frac{d\left(\frac{m}{\rho}\right)}{\frac{m}{\rho}} = -\frac{\frac{1}{\rho^2}d\rho}{\frac{1}{\rho}} = -\frac{d\rho}{\rho}$$

$$\frac{\rho-\rho_0}{\rho_0} = -\frac{\partial \xi}{\partial x} = -\frac{\partial \xi}{\partial t}\cdot\frac{\partial t}{\partial x} = -\frac{v}{c}$$

∂x

adiab

$$\frac{\Delta p}{p} = \gamma\frac{\Delta \rho}{\rho} = \gamma\frac{v}{c}$$

$$v_T = \sqrt{\overline{v^2}} \qquad \frac{m\overline{v^2}}{2} = \frac{3}{2}k_BT$$

$$\overline{v^2} = \frac{3kT}{m} = \frac{3kT}{\mu/N_A} = \frac{3RT}{\mu}$$

$$c = \sqrt{\gamma\frac{RT}{\mu}} = \sqrt{\frac{\gamma}{3}}\,v_T \quad < v_T\,, \quad c > \frac{1}{\sqrt{3}}v_T = 0{,}58\,v_T$$

mono $\gamma = \frac{5}{3}$; $c \simeq 0{,}75\,v_T$

bi $\gamma = \frac{7}{5}$ $c \simeq 0{,}68\,v_T$

poli $c \simeq 0{,}6\,v_T$

$$\boxed{0{,}58\,v_T < c < 0{,}75\,v_T}$$

Potențialul de viteze

propagarea undei acustice – mișcare irotațională – vectorul vârtej = 0 pt. fiecare punct → viteza derivă dintr-un potențial

$$\phi = \phi(x,y,z,t)\,; \quad v_x = \frac{\partial \phi}{\partial x},\ v_y,\ v_z = \frac{\partial \phi}{\partial z}$$

pt. mișc. Ox

ec. de continuitate:

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v_x}{\partial x} = 0 \quad \Big| \quad \frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial^2 \phi}{\partial x^2} = 0$$

analog $\frac{\partial p}{\partial t} = \frac{k}{\rho_0}\frac{\partial \rho}{\partial t}$

$$\frac{\partial p}{\partial t} = -\rho_0 \frac{\partial^2 \phi}{\partial t^2}$$

$$\boxed{\frac{\partial^2 \phi}{\partial t^2} = \frac{k}{\rho_0}\frac{\partial^2 \phi}{\partial x^2}}$$

$$\frac{\partial p}{\partial x}\cdot dx = -\rho_0 \frac{\partial v_x}{\partial t}dx = -\rho_0 \frac{d}{dx}\left(\frac{\partial \phi}{\partial t}\right)$$

$$dp = -\rho_0\, d\left(\frac{\partial \phi}{\partial t}\right)$$

relația dintre presiunea acustică și ϕ

pt. unde armonice

$$\tilde{\phi} = \underbrace{A e^{ik(ct-x)}}_{\phi_+} + \underbrace{B e^{ik(ct+x)}}_{\phi_-}$$

$$p = -\rho_0 \frac{\partial \phi}{\partial t} + C$$

C – din cond. ca în lipsa perturb. $p = p_0$

$$\boxed{p_i = -\rho_0 \frac{\partial \phi}{\partial t}}$$

presiunea acustică și $\frac{\partial \phi}{\partial t} = 0$

presiunea acustică (p = pi+po)

$$-A\,dp = \rho_0 A\,dx \cdot \frac{\partial^2 \xi}{\partial t^2}$$

$$\frac{\partial p}{dx} = -\rho_0 \frac{\partial v}{\partial t}$$

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v}{\partial t} = -\rho_0 \frac{d}{dt}\frac{\partial \phi}{\partial x} = -\rho_0 \frac{\partial}{\partial x}\frac{d\phi}{dt}$$

$$\partial p = -\rho\, d\left(\frac{d\phi}{dt}\right)$$

$$\frac{\partial p}{\partial t} = -\rho \frac{d^2 \phi}{dt^2}$$

$$\tilde{p}_i = -\rho_0 \frac{\partial \tilde{\phi}}{\partial t} = -ikc\rho_0 (\phi_+ + \phi_-)$$

$$\tilde{v}_x = \frac{\partial \tilde{\phi}}{\partial x} = -ik(\phi_+ - \phi_-)$$

partea reală a lui ϕ pt undele ce se propagă în sens pozitiv

$$\phi = A\cos k(ct - x)$$

$$p_i = -\rho_0 \frac{\partial \phi}{\partial t} = \rho_0 A c k \sin k(ct-x) = p_{i0} \sin k(ct-x)$$

viteza particulelor

$$v = \frac{\partial \phi}{\partial x} = +Ak \sin(ckt - kx) = +V_0 \sin k(ct-x)$$

pt. unde plane armonice

presiunea acustică și viteza de oscilație a particulelor → în fază