Oscilații și unde   Oscilaii i unde

1. Introducere. Noțiuni fundamentale

1.1. Def. Clasificări. Unit de măs.

echilibru

cauzele
- efecte vedere
- -||- dorite

F_{ortfe}

ex.

E_{pot}

2012 — 2014

mișcări periodice / aperiodice

parametri se repetă identic după un interval de timp T

mișc. intr. periodic → ciclu al vibr. periodice (parametri geometrici identici)

$$x(t) = x(t+T), \quad q(t) = q(t+T)$$

coord. generalizate

grafic

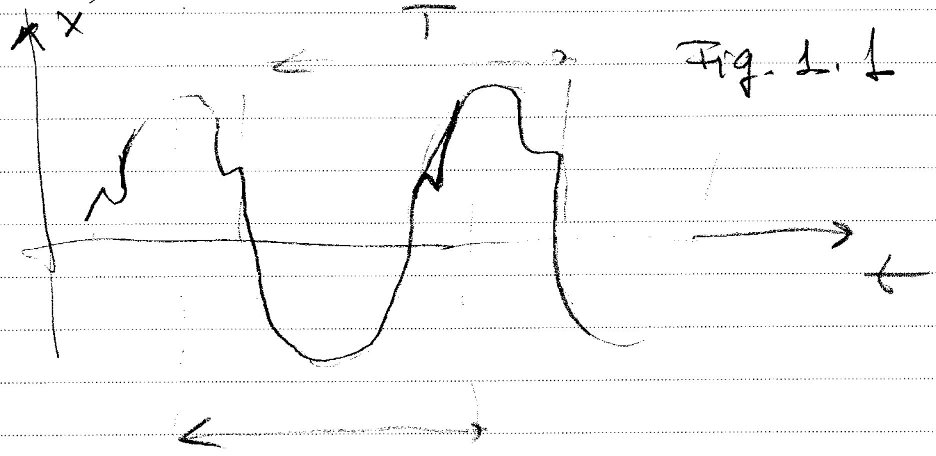


Fig. 1.1

Exemple

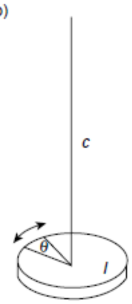
(a)

$$m\ddot{x} + mg\frac{x}{l} = 0$$
$$ml\ddot{\theta} + mg\,\theta = 0$$
$$\omega^2 = g/l$$

$$mg\sin\theta \sim mg\,\theta \approx mg\frac{x}{l}$$

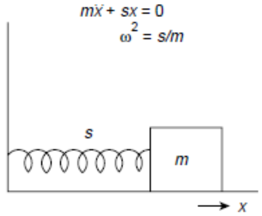
(b)

$$I\ddot{\theta} + c\theta = 0$$
$$\omega^2 = \frac{c}{I}$$



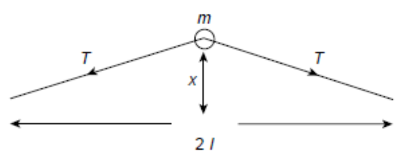
(c)

$$m\ddot{x} + sx = 0$$
$$\omega^2 = s/m$$



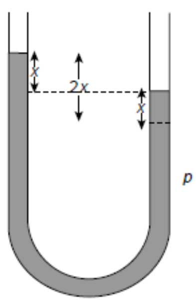
(d)

$$m\ddot{x} + 2T\frac{x}{l} = 0$$
$$\omega^2 = \frac{2T}{l\,m}$$



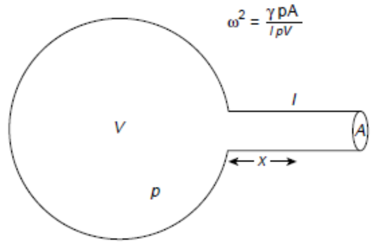
(e)

$$p\,l\ddot{x} + 2\,pg\,x = 0$$
$$\omega^2 = 2g/l$$

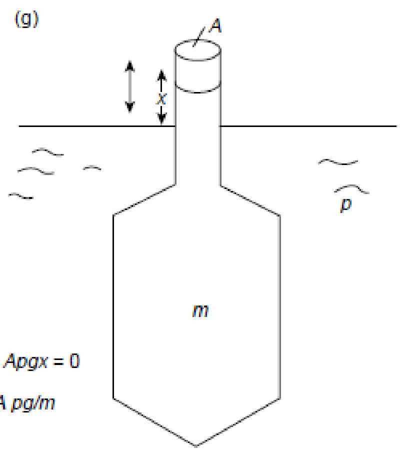


(f)

$$p\,Al\ddot{x} + \frac{\gamma\,pxA^2}{V} = 0$$
$$\omega^2 = \frac{\gamma\,pA}{l\,pV}$$

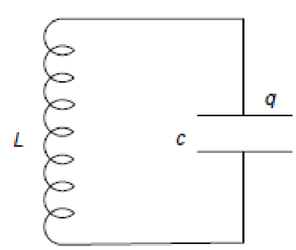


(g)



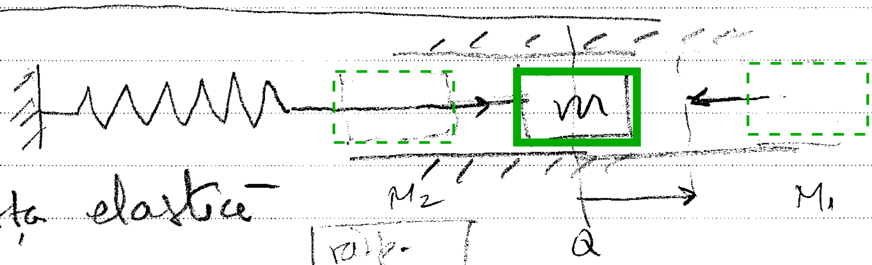
$$m\ddot{x} + Apgx = 0$$
$$\omega^2 = A\,pg/m$$

(h)



$$L\ddot{q} + \frac{q}{c} = 0$$
$$\omega^2 = \frac{1}{Lc}$$

Mişcarea osc. armonică



forța elastică

$F = -kX$ | resp. liniar | $m \cdot a = -kX$

$$\boxed{m\ddot{X} + kX = 0}$$

$$\ddot{X} + \frac{k}{m} X = 0 \qquad \omega^2 = \frac{k}{m}$$

$$x = e^{rt}$$

$$(r^2 + \omega^2)e^{rt} = 0 \qquad r^2 + \omega_0^2 = 0 \qquad r = \pm i\omega_0$$

sol. partic. $x = e^{i\omega t}$ sau ; $x = e^{-i\omega t}$

sol gen. $X = A_1 e^{i\omega_0 t} + A_2 e^{-i\omega_0 t}$

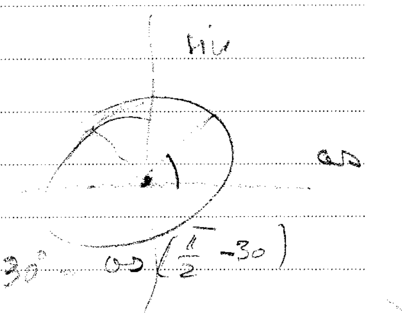
$$e^{i\omega_0 t} = \cos\omega_0 t + i\sin\omega_0 t$$

$$e^{-i\omega_0 t} = \cos\omega_0 t - i\sin\omega_0 t$$

$$X = A_1\cos\omega_0 t + A_2\cos\omega_0 t + i(A_1\sin\omega_0 t - A_2\sin\omega_0 t)$$

$$= (A_1 + A_2)\cos\omega_0 t + i(A_1 - A_2)\sin\omega_0 t$$

$$= B_1\cos\omega_0 t + B_2\sin\omega_0 t = A e^{i(\omega_0 t + \varphi)}$$



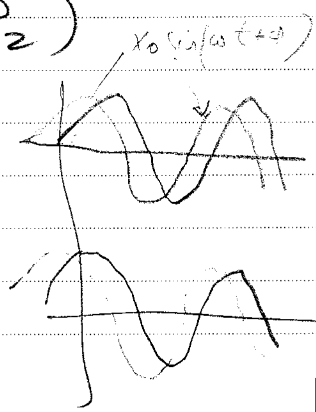
$$X_1 = X_{01}\sin(\omega_0 t + \varphi_1)$$

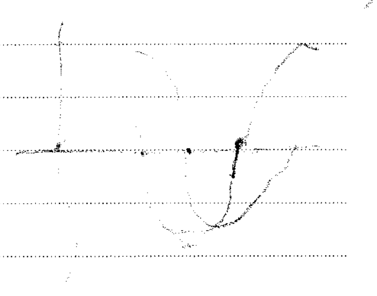
$\sin 30^\circ = \cos\left(\frac{\pi}{2} - 30\right)$

$$X_2 = X_{02}\cos(\omega_0 t + \varphi_2)$$

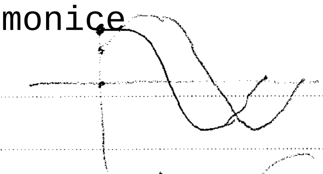
$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$

$$\boxed{\begin{aligned} X &= X_0\sin(\omega_0 t + \varphi) \\ X &= X_0\cos(\omega_0 t + \varphi) \end{aligned}}$$





Parametrii micrii oscilatorii armonice



1. Pendul elastic

$$\omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} \; ; \; \nu = \frac{\omega}{2\pi} = \frac{1}{T} \quad \text{Hz}$$

ω – frecv. unghiulară (pulsația)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

k – const. elastică $[k] = N/kg$

F – forța el.

elongația x

amplitudinea x_0

perioada –

frecv. –

faza $\theta = \omega t + \varphi$

φ – faza inițială

$$x_0 \sin(\omega t + \varphi) = x_0 \sin(\omega t + \omega T + \varphi)$$

$$= x_0 \sin(\omega t + \varphi + 2\pi)$$

$$\omega T = 2\pi \qquad T = \frac{2\pi}{\omega}$$

$$x = x_0 \sin(\omega t + \varphi) = x_0 \sin \omega\left(t + \frac{\varphi}{\omega}\right)$$

2. ~~Pendulul elastic~~ matematic

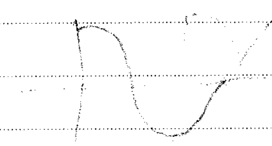
$$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$$

$$\cos\theta \cdot \cos\frac{\pi}{2} - \sin\theta \cdot \sin\frac{\pi}{2}$$

$$x = x_0 \sin(\omega t + \varphi)$$

$$v = \dot{x} = \frac{dx}{dt} = \omega x_0 \cos(\omega t + \varphi)$$

$$a = \ddot{x} = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 x_0 \sin(\omega t + \varphi)$$

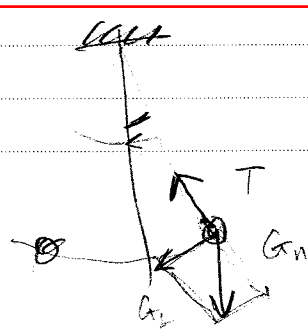


$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$$

Pendulul elastic $T = 2\pi\sqrt{\frac{m}{k}}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Pendulul matematic



$$\vec{G} + \vec{T} = m\vec{a}$$

$$T - G_n = m a_{cp}$$

$$G_t = G \sin\theta$$

$$x \sim l\theta$$

$$ml\frac{d^2\theta}{dt^2} + mg\sin\theta = 0$$

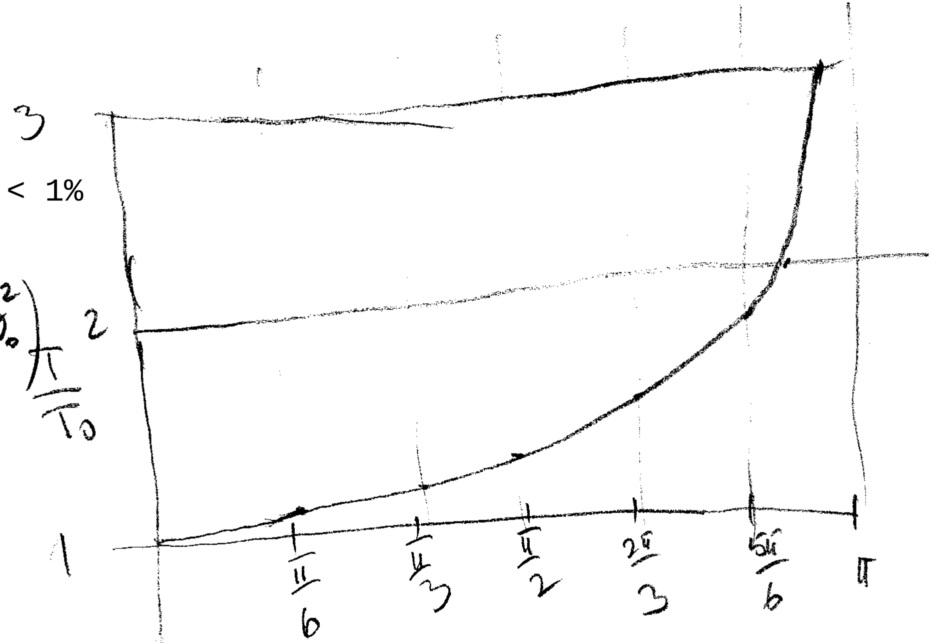
$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

Leiden Cryogenics b.v.

pt. 23°

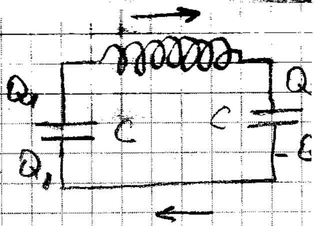
corecția < 1%

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{16}\theta_0^2\right)$$



Circuitul LC

DATUM:



$$u = L\frac{dI}{dt}$$

$$Q_1 > 0 \rightarrow u_1 = \frac{Q_1}{C_1}$$

$$Q_2 > 0 \rightarrow u_2 = \frac{Q_2}{C_2}$$

$$L\frac{dI}{dt} = \frac{Q_1}{C} - \frac{Q_2}{C}$$

la echilibru $Q = 0$

$$\boxed{Q_1 = -Q_2 \qquad \frac{dQ_2}{dt} = I}$$

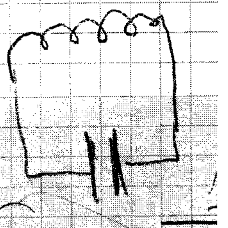
$$L\frac{dI}{dt} = \frac{Q_1}{C} - \frac{Q_2}{C} = -2\frac{Q_2}{C}$$

$$\cancel{L}\frac{d^2I}{dt^2} = -2\frac{1}{C}\frac{dQ_2}{dt} \quad \Big| \quad L\frac{d^2I}{dt^2} + \frac{2}{C}I = 0$$

$$\frac{d^2I}{dt^2} + \frac{2}{LC}I = 0 \quad \Big| \quad \boxed{\frac{d^2I}{dt^2} + \omega^2 I = 0}$$

$$\omega^2 = \frac{2}{LC} \qquad \boxed{T = 2\pi\sqrt{\frac{LC}{2}}}$$

$$\boxed{i(t) = I_0 \cos(\omega t + \varphi)}$$



$$\frac{q}{C} = U = -L\frac{dI}{dt} = -L\frac{d^2q}{dt^2}$$

$$L\ddot{q} + \omega^2 q = 0 \qquad \omega^2 = \frac{1}{LC}$$

$$E_L = \int u I \cdot dt = \int L \frac{dI}{dt} \cdot I \cdot dt = \int_0^I L I \, dI = \frac{1}{2}$$

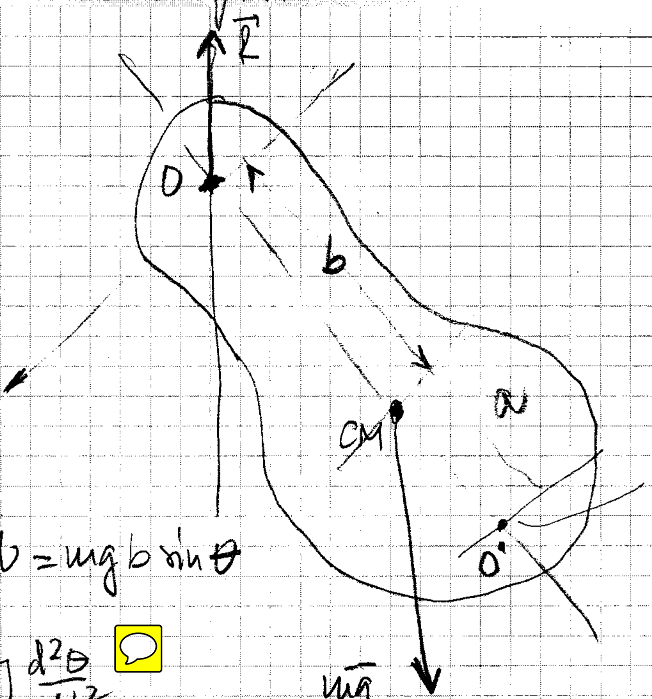
=1/2L*I^2

$$\frac{1}{2} C u^2 = \frac{q^2}{2C}$$

$$E = \frac{1}{2} L \dot{q}^2 + \frac{1}{2} \frac{q^2}{C} = E$$

Pendulul fizic

DATUM:



centrul de oscilatie

$M = mgb \sin\theta$

$= -J \dfrac{d^2\theta}{dt^2}$

unghiuri mici $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} + \frac{mgb}{J}\sin\theta = 0$$

$$\boxed{\frac{d^2\theta}{dt^2} + \omega^2 \sin\theta = 0}$$

$$\omega^2 = \frac{mgb}{J}$$

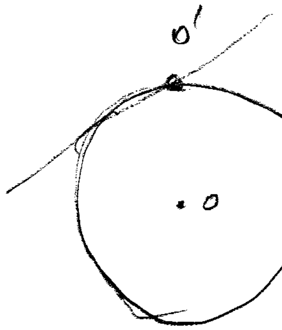
$$\boxed{T = 2\pi\sqrt{\frac{J}{mbg}}}$$

pendulul matematic

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$l_{redusă} = \dfrac{J}{mb}$

ex.



$$J_O = mR^2$$

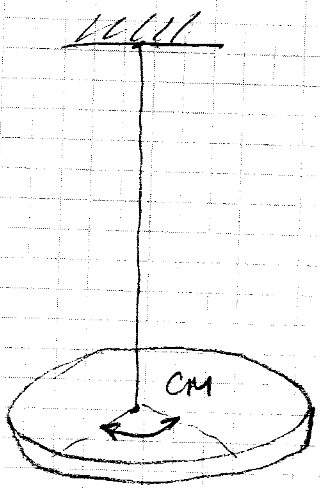
$$J_{O'} = mR^2 + mR^2 = 2mR^2$$

$$T = 2\pi \sqrt{\frac{2\cancel{m}R^{\cancel{2}}}{\cancel{m}g\cancel{R}}} = 2\pi\sqrt{\frac{2R}{g}}$$

$$l_r = \frac{2\cancel{m}R^{\cancel{2}}}{\cancel{m}\cancel{R}} = 2R$$

Pendulul de torsiune

DATUM:



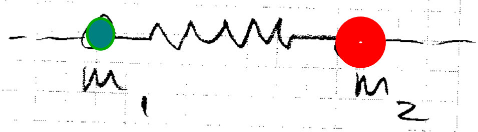
$$\mathcal{M} = -C\theta$$

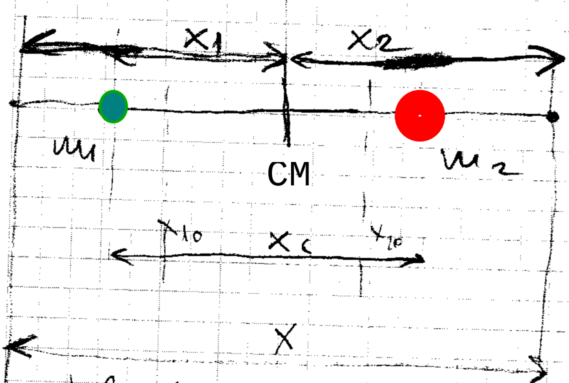
$$J \cdot \frac{d^2\theta}{dt^2} = -C\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{C}{J}\cdot\theta = 0}$$

$$\boxed{T = 2\pi\sqrt{\frac{J}{C}}}$$

Molecula biatomică





$$m_1 \frac{d^2x_1}{dt^2} = -k(X - X_c)$$

$$m_2 \cdot \frac{d^2x_2}{dt^2} = k(X - X_c)$$

pozițiile d.t. față de centrul de masă

$$X_1 = \frac{m_2}{m_1+m_2} X \qquad X_2 = ---$$

$$\frac{m_1 m_2}{m_1+m_2} \frac{d^2X}{dt^2} = -k(X - X_c)$$

$$\frac{m_1 m_2}{m_1+m_2}\frac{d^2(x-x_c)}{dt^2}+k(x-x_c)=0$$

$$x_1+x_2=x$$
$$x_{10}+x_{20}=x_c$$

alungirea resortului este $x-x_c$

$$\omega^2=\frac{k}{\frac{m_1 m_2}{m_1+m_2}}=\frac{k}{\mu}$$

o singură frecvență

Legea izocronismului micilor oscilații:

perioada nu depinde de amplitudine

- amplitudinea depinde de cond. inițiale

$$x_0=A\cos\alpha \qquad v_0=-\omega A\sin\alpha$$

$$A=\sqrt{x_0^2+v_0^2/\omega^2} \qquad \operatorname{tg}\alpha=-\frac{v_0}{\omega x_0}$$

Energia în mișcarea armonică simplă

$$E_p = -\int F\,dx = +\int kx\,dx = \frac{1}{2}kx^2 + C$$

alegem $C = 0$ la echilibru

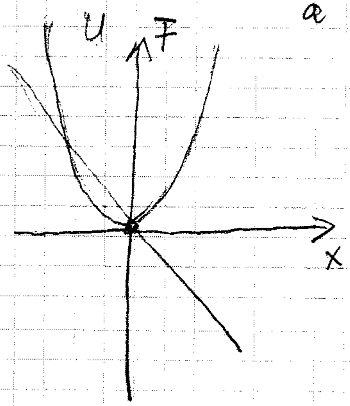
$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x_0^2 \sin^2(\omega t + \varphi)$$

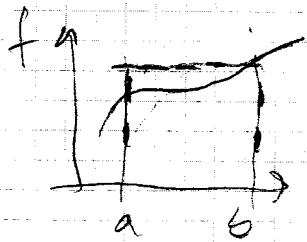
$$E_c = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_0^2 \cos^2(\omega t + \varphi)$$

$$E = E_c + E_p = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}kx_0^2$$

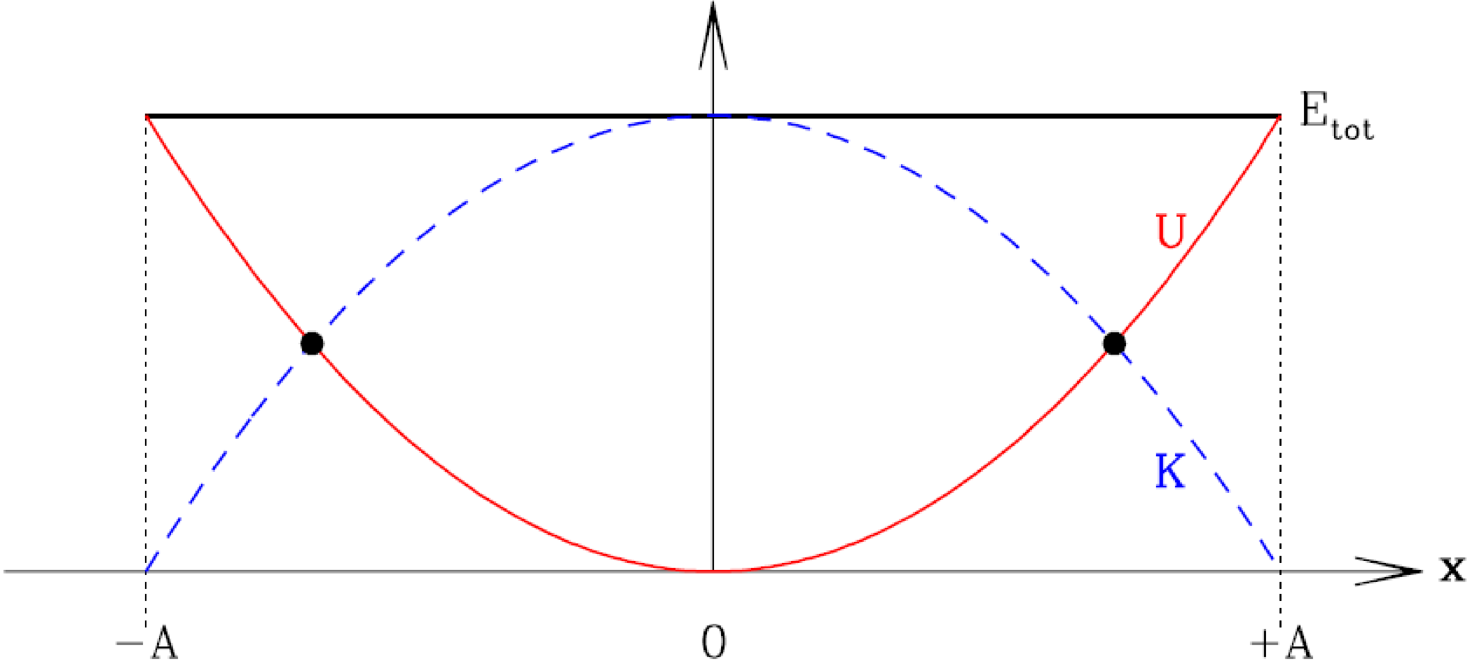
valoarea medie

$$\bar{x} = \frac{1}{b-a}\int_a^b f(t)\,dt, \quad \bar{x}(b-a) = \int_a^b f(t)\,dt$$





$$\bar{f}\cdot(a-b) = S$$



E_tot
U
K
x
−A
0
+A

DATUM:

valori medii

$$\overline{f+g} = \bar{f} + \bar{g} \quad \Big| \quad \overline{const \cdot f} = const \cdot \bar{f}$$

$$\overline{f \cdot g} \neq \bar{f} \cdot \bar{g}$$

valoarea medie a lui sin(x) sau cos(x) pe o

$$\overline{\sin^2\varphi} = \overline{\frac{1}{2}(1-\cos 2\varphi)} = \frac{1}{2}; \quad \overline{\cos^2\varphi} = \overline{\frac{1}{2}(1+\cos 2\varphi)} = \frac{1}{2}$$

$$\overline{\sin(\varphi+\alpha)\cdot\sin(\varphi+\beta)} = \overline{\cos(\varphi+\alpha)\cdot\cos(\varphi+\beta)}$$

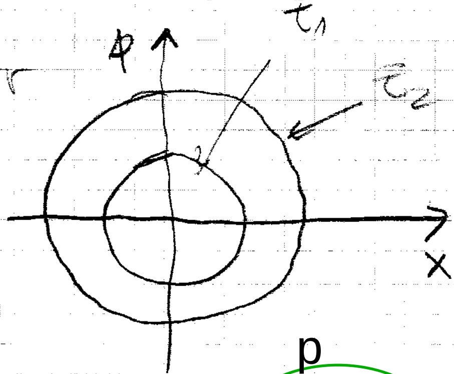
$$= \frac{1}{2}\cos(\alpha-\beta)$$

$$\overline{\sin(\varphi+\alpha)\cdot\cos(\varphi+\beta)} = \frac{1}{2}\sin(\alpha-\beta)$$

$$\overline{E_c} = \frac{1}{2} m\omega^2 X_0^2 \, \overline{\sin^2(\omega t+\varphi)} = \frac{1}{4} m\omega^2 X_0^2 = \frac{E}{2}$$

$$\overline{E_p} = \text{-||-} \quad \overline{\cos^2(\omega t+\varphi)} = \frac{1}{4} m\omega^2 X_0^2 = \frac{E}{2}$$

În spațiul fazelor

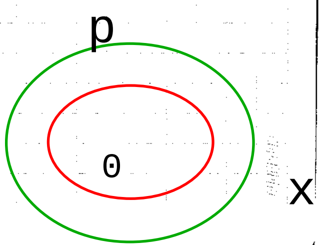


$$\ddot{x} + \omega_0^2 x = 0$$

$$x = X_0 \sin(\omega t+\varphi)$$

$$p = mv = m X_0 \omega \cos(\omega t+\varphi)$$

$$\frac{x^2}{X_0^2} + \frac{p^2}{m^2 X_0^2 \omega_0^2} = 1$$



ecuaia unei elipse

MQ în mecanica cuantică DATUM:

$$\boxed{E = h\nu\left(n+\frac{1}{2}\right)} \qquad h = 6{,}6256\cdot 10^{-34}\ J\cdot s$$

$$\Delta x \cdot \Delta p \sim \frac{\hbar}{2} \;;\quad \Delta x \sim x \;;\quad \Delta p \sim p$$

$$\overline{x^2}\cdot\overline{p^2} \geq \frac{\hbar^2}{4} = \frac{h^2}{16\pi^2} \qquad \overline{x^2} = \frac{h^2}{16\pi^2}\cdot\frac{1}{\overline{p^2}}$$

$$\overline{E} = \frac{1}{2m}\overline{p^2} + \frac{k}{2}\overline{x^2} = \frac{1}{2m}\overline{p^2} + \frac{k h^2}{32\pi^2}\frac{1}{\overline{p^2}} \qquad \overline{p^2} = y^2$$

$$= A y^2 + B\frac{1}{y^2} \qquad \frac{dE}{dy} = 0 \qquad A = \frac{1}{2m} \quad B = \frac{kh^2}{32\pi^2}$$

$$2Ay - 2B\frac{1}{y^3} = 0$$

$$2Ay^4 - 2B = 0 \qquad y^4 = \frac{B}{A}$$

$$p^2 = \left(\frac{B}{A}\right)^{1/2} = \left(\frac{\frac{kh^2}{32\pi^2}}{\frac{1}{2m}}\right)^{1/2} = \frac{\sqrt{k}\,h}{\cancel{2}\sqrt{m}\,\pi} = \frac{h\sqrt{\omega}}{2\pi}$$

$$E = A\left(\frac{B}{A}\right)^{1/2} + B\left(\frac{A}{B}\right)^{1/2} = 2(AB)^{1/2}$$

$$= 2\left(\frac{1}{2m}\cdot\frac{kh^2}{32\pi^2}\right)^{1/2} = \frac{\cancel{2}}{8\pi}h\omega = \frac{1}{2}\hbar\omega$$

$$= \frac{1}{2}h\nu$$