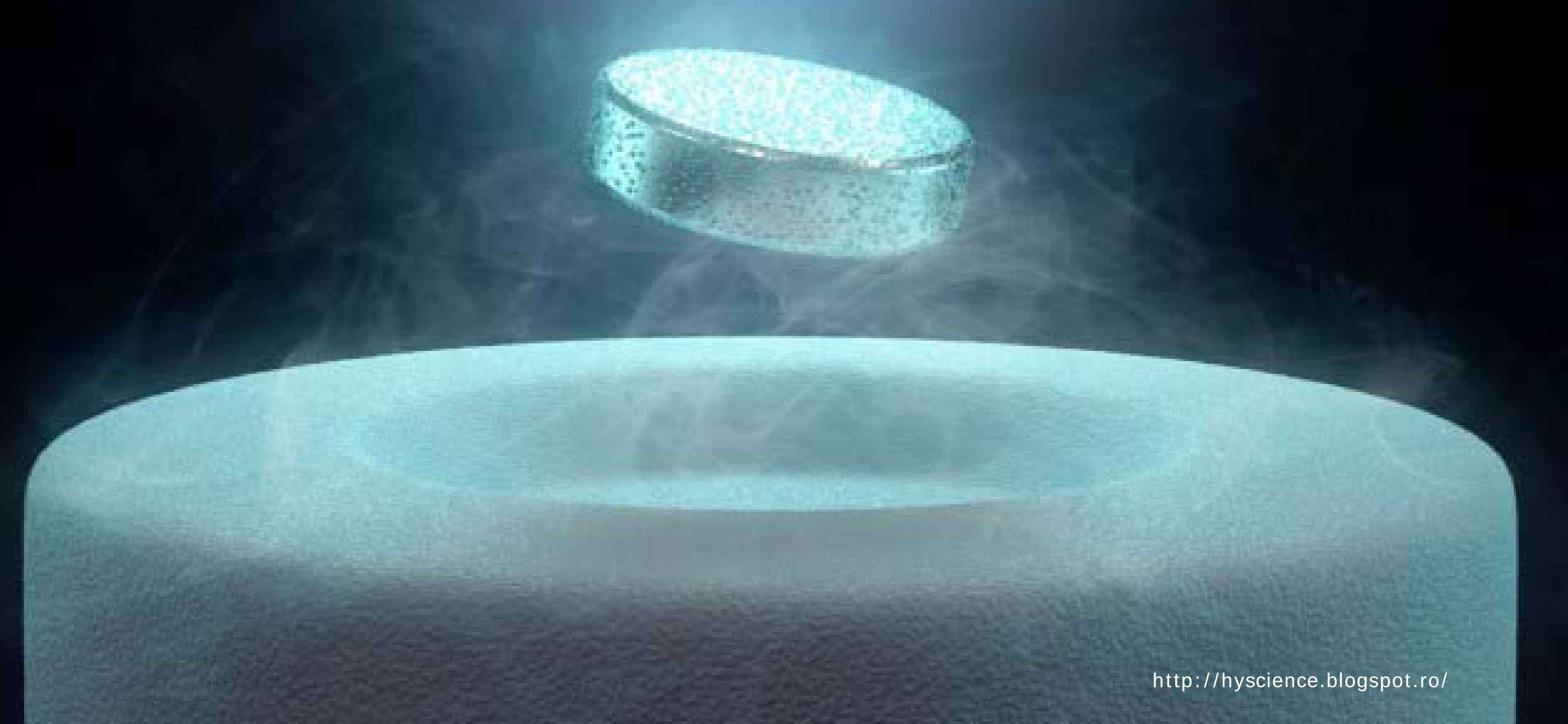
Advanced solid state physics

Introduction to superconductivity



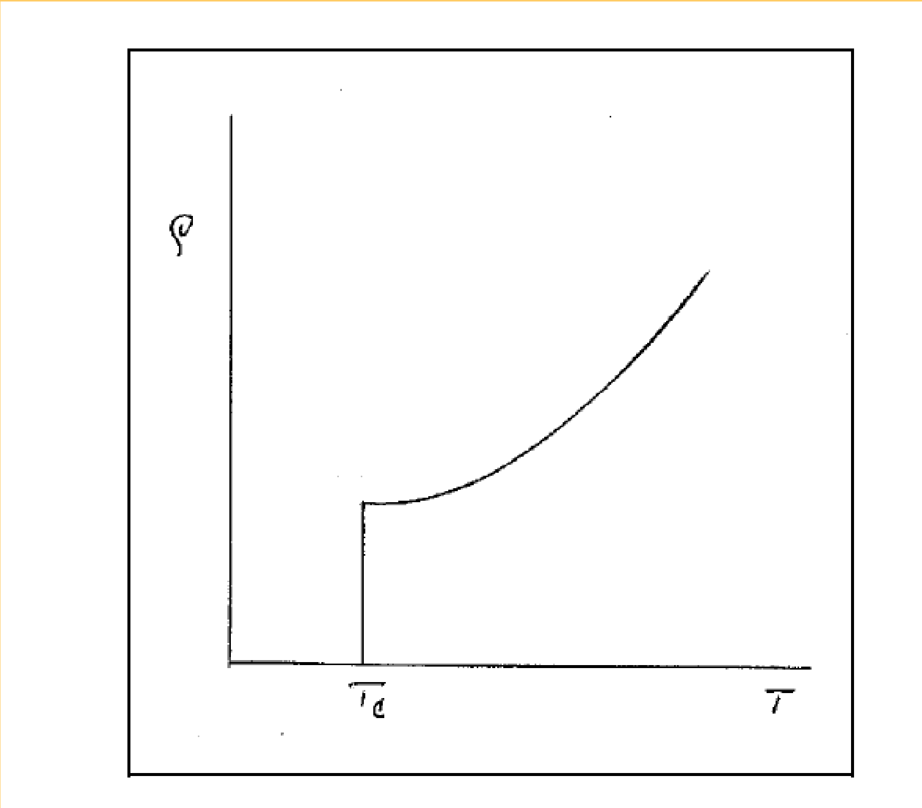
Outline

- Introduction to superconductors
- Kamerlingh Onnes
- Evidence of a phase transition
- MEISSNER EFFECT
- Characteristic lengths in SC
- Categories of SC
- Magnetic properties
- Critical current density

Introduction to superconductors

SUPERCONDUCTIVITY

- property of complete disappearance of electrical resistance in solids when they are cooled below a characteristic temperature. This temperature is called transition temperature or critical temperature.



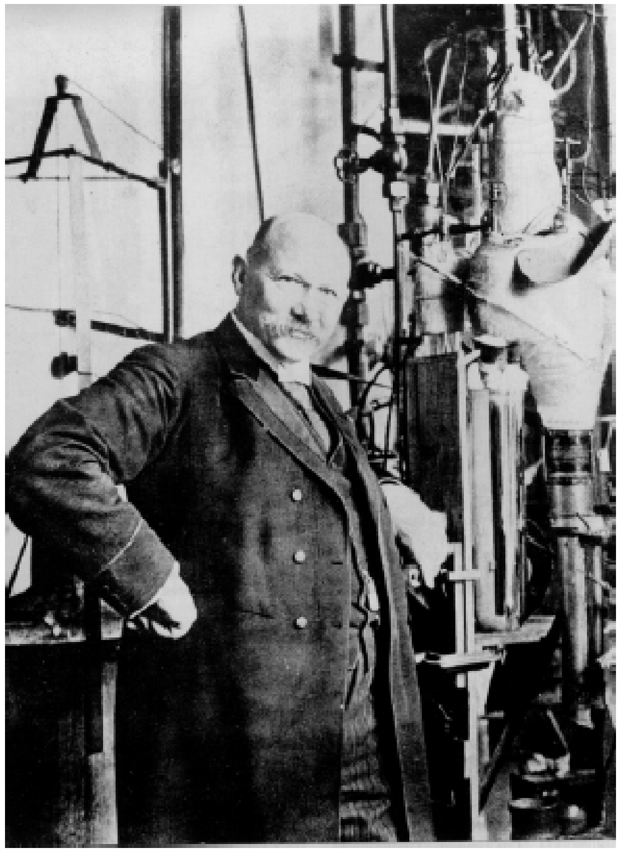
Superconductive state of mercury (T_C=4.15 K) was discovered by the Dutch physicist Heike Kamerlingh Onnes in 1911, several years after the discovery of liquid helium (1908).

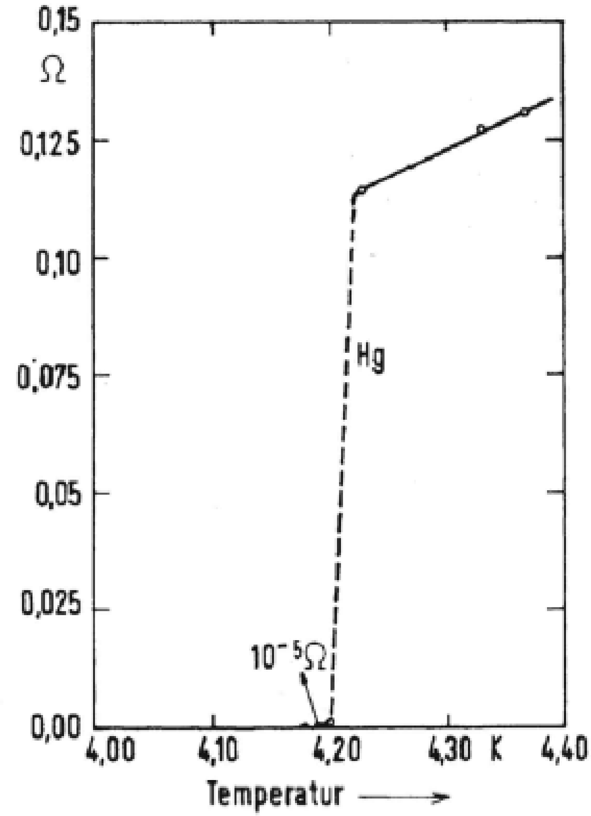
April 8, 1911

Dirk van Delft, *Freezing physics. Heike Kamerlingh Onnes and the quest for cold*, Koninklijke Nederlandse Akademie van Wetenschappen, Amsterdam 2007

Superconductivity was discovered by Kamerlingh-Onnes in 1911 in mercury (Hg), having $T_c \approx 4$ K

".. Mercury has passed into a new state, which on account of its extraordinary electrical properties may be called the superconductive state"

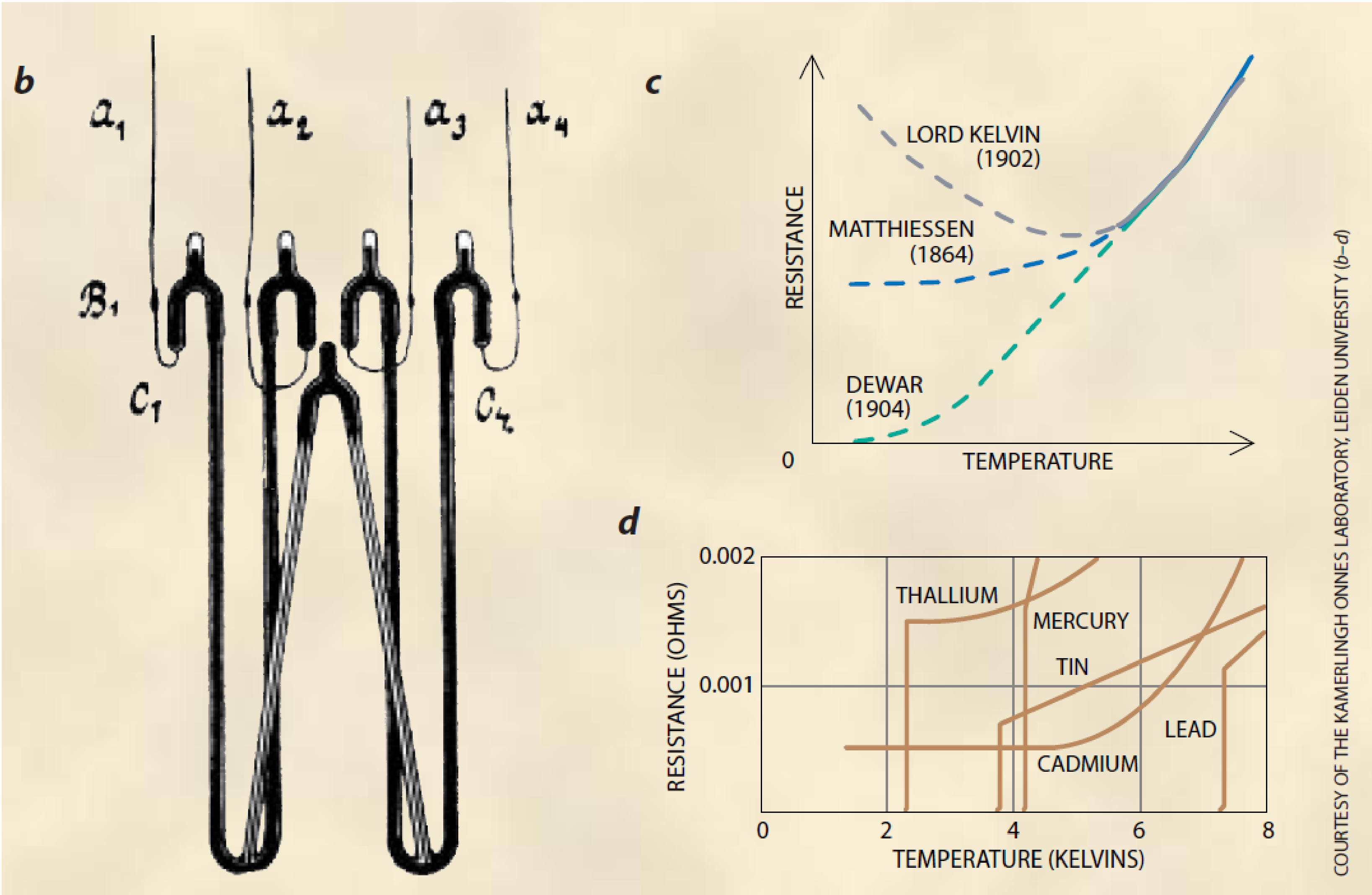




Dependence of the resistance on temperature

Nobel prize 1913

'for his investigations on the properties of matter at low temperatures which led, *inter alia, to the production of liquid helium*'



From: Rudolf de Bruyn Ouboter, "*Heike Kamerlingh Onnes's Discovery of Superconductivity*", Scientific American March 1997

Heike Kamerlingh Onnes (*far right*) shows his helium liquefactor to three theoretical physicists: **Niels Bohr** (visiting from Kopenhagen), **Hendrik Lorentz**, and **Paul Ehrenfest** (*far left*).

Dirk van Delft, *Freezing physics. Heike Kamerlingh Onnes and the quest for cold*, Koninklijke Nederlandse Akademie van Wetenschappen, Amsterdam 2007

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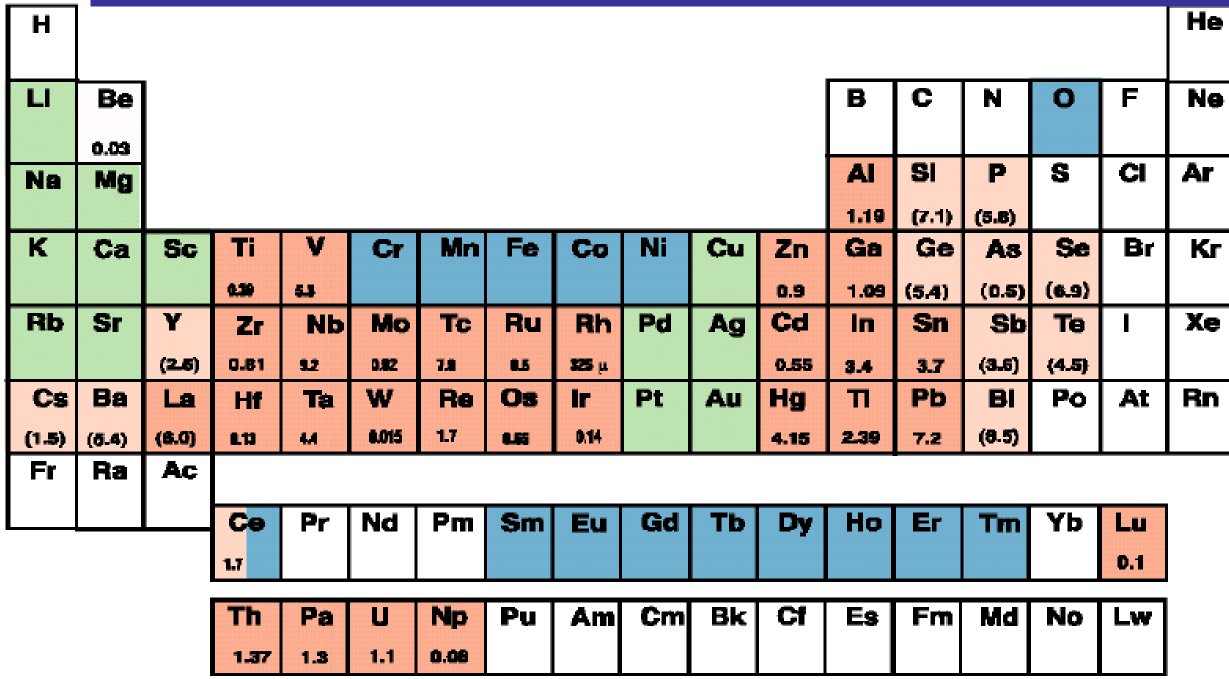
Kamerlingh Onnes Liquefied He 1908

THE CLASSIC SUPERCONDUCTORS



Good metals → Not Superconductors

Superconductors → poor metals

Superconductors under pressure

Magnetic metals

Classical elemental superconductors

Element	Transition temperature, K
Zinc	0.88
Aluminum	1.20
Indium	3.41
Tin	3.72
Mercury	4.15
Lead	7.19

Table I. Transition temperatures and crystal-structure data of superconducting elements.

	Superconductivity data	Crystal-structure data				
Element	Transition temperature[a] °K	Structure type	System	Lattice a, Å	Constants b, Å	c, Å
Al	1.171-1.196	A1–Cu	Cubic	4.0496		
β-La	5.4-*6.3*	A1–Cu	Cubic	5.296		
Ir	0.14	A1–Cu	Cubic	3.8389		
Pb	7.175-*7.193*-7.23	A1–Cu	Cubic	4.9502		
α-Th	1.37	A1–Cu	Cubic	5.0843		
V	*5.03*-5.13	A2–W	Cubic	3.0282		
Nb	*9.09*-9.465	A2–W	Cubic	3.3007		
Mo	0.92	A2–W	Cubic	3.1468		
Ta	4.39-*4.482*	A2–W	Cubic	3.298		
				3.8389		
Ti	*0.387*-0.49	A3–Mg	Hexagonal	2.9504		4.6833
Zn	0.825-*0.855*-0.875	A3–Mg	Hexagonal	2.6649		4.9468
Zr	0.546-0.565	A3–Mg	Hexagonal	3.2312		5.1477
Tc	8.22-9.3-11.2	A3–Mg	Hexagonal	2.735		4.388
Ru	0.47-*0.493*	A3–Mg	Hexagonal	2.7058		4.2816
Cd	*0.52*-0.602	A3–Mg	Hexagonal	2.9788		5.6167
Hf	0.165	A3–Mg	Hexagonal	3.1946		5.0511
Re	*1.699*-2.42	A3–Mg	Hexagonal	2.760		4.458
Os	0.58-*0.655*-0.71	A3–Mg	Hexagonal	2.7353		4.3191
Tl	2.36-2.39	A3–Mg	Hexagonal	3.4566		5.5248
α-La	4.8-*4.9*-5.0	A3′–La	Hexagonal	3.770		12.159
Sn	3.701-*3.722*	A5–White Sn	Tetragonal	5.8314		3.1814
In	3.396-*3.408*	A6–In	Tetragonal	4.5979		4.9467
α-Hg	4.153	A10–Hg	Rhombohedral	2.9863	$\alpha = 70°44.6'$	
Ga	*1.087*-1.103	A11–Ga	Orthorhombic	4.5198	7.6602	4.5258
α-U	0.68-0.7	A20–α–U	Orthorhombic	2.8536	5.8698	4.9555
β-Hg	3.949		Tetragonal	3.995		2.825

REVIEWS OF
MODERN PHYSICS
Volume 35, Number 1 January 1963

Superconductivity

B. T. Matthias,* T. H. Geballe, and V. B. Compton
Bell Telephone Laboratories, Murray Hill, New Jersey

- Non transition elements
- Transition elements
- Intermetallic compounds
- Alloys

REVIEWS OF
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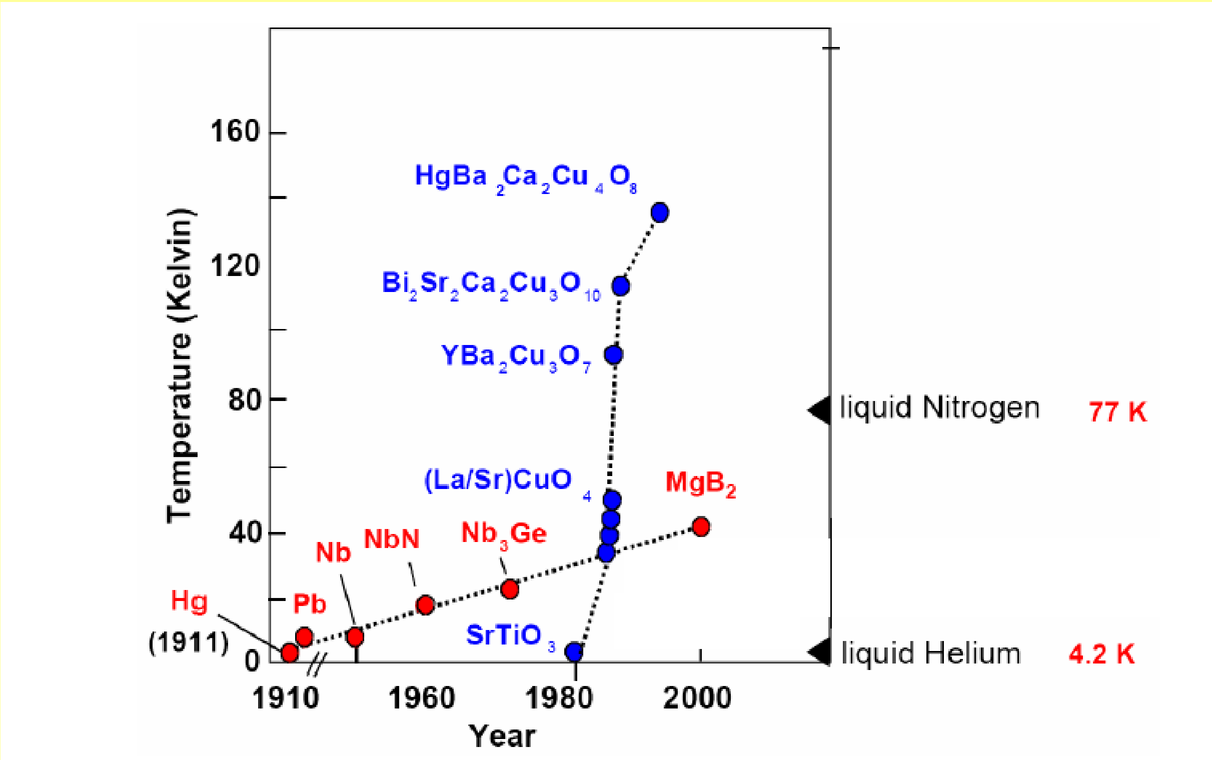
Superconductivity

B. T. MATTHIAS,* T. H. GEBALLE, AND V. B. COMPTON
Bell Telephone Laboratories, Murray Hill, New Jersey

Until 1983 record T_c=23.3 K was that of Nb_3Ge alloy.

High-Temperature SC



Until **1986**, the highest known critical temperature of any SC was 23.3 K. Various theories predicted that it would be impossible to have much higher critical temperatures. They were wrong. In **1987**, a new category of *type-II* SC, called ***HT* SC**, were discovered with critical temperatures significantly above that of liquid nitrogen (77K).

Since then scientists have experimented with many different forms of perovskites producing compounds that SC at temperatures over 130K.

High temperature superconductors discovered in1986: T_c=80-93 K,

parent structure $YBa_2Cu_3O_7$.

At present the record transition temperature (HBCCO)is now at T_C= 134 K.

SUPERCONDUCTIVITY MILESTONES

1908 Making liquid helium (4.2 K)

- **1911** Dutch physicist Heike Kamerlingh Onnes discovers superconductivity in mercury at temperature of 4 K.
- **1913** Kamerlingh Onnes is awarded the Nobel Prize in Physics for his research on the properties of matter at low temperature.
- **1933** W. Meissner and R. Ochsenfeld discover the Meissner Effect.
- **1941** Scientists report superconductivity in niobium nitride at 16 K.
- **1953** Vanadium-3 Silicon found to superconduct at 17.5 K.

•**1962** Westinghouse scientists develop the first commercial niobium- titanium superconducting wire.

•**1972** John Bardeen, Leon Cooper, and John Schrieffer win the **Nobel Prize in Physics** for the first successful **theory** of how superconductivity works.

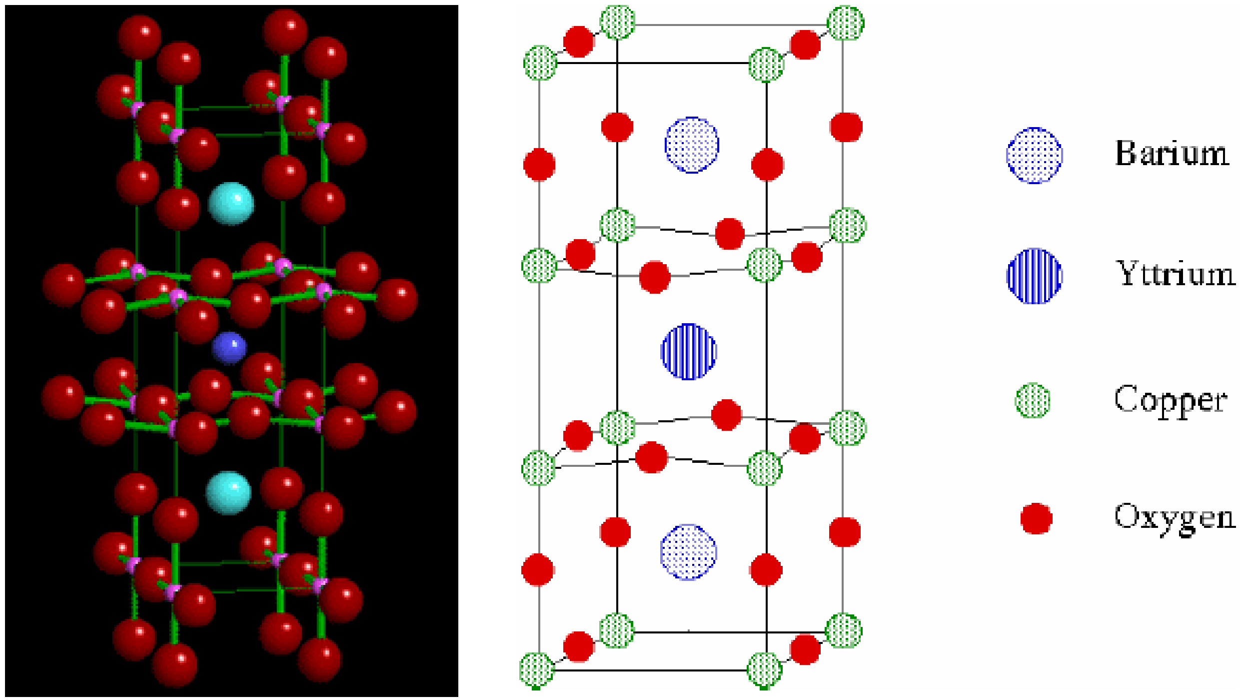
•**1986** IBM researchers Alex Müller and Georg Bednorz make a ceramic compound of lanthanum, barium, copper, and oxygen that superconducts at 35 K.

•**1987** Scientific groups at the University of Houston and the University of Alabama at Huntsville substitute yttrium for lanthanum and make a ceramic that superconducts at 92 K, bringing superconductivity into the liquid nitrogen range, **YBCO**.

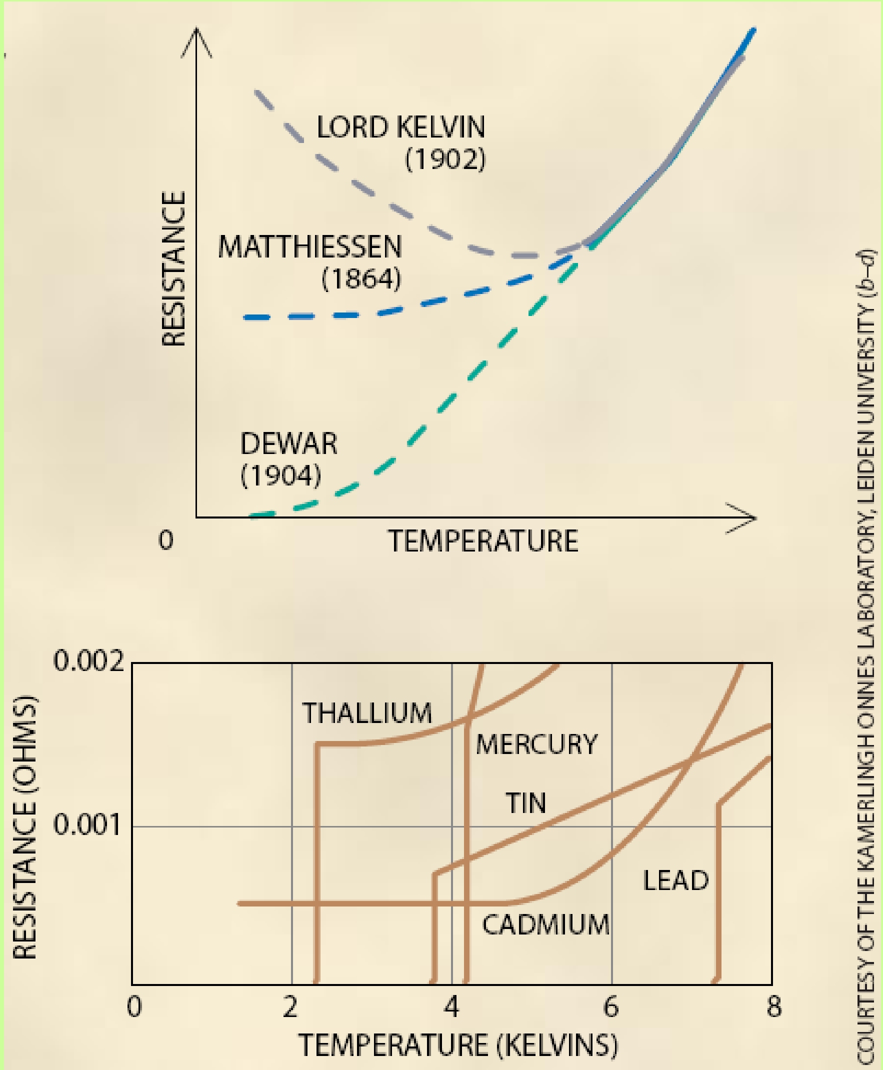
•**1988** Allen Hermann of the University of Arkansas makes a superconducting ceramic containing calcium and thallium that superconducts at **120 K**. Soon after, IBM and AT&T Bell Labs scientists produce a ceramic that superconducts at **125 K**.

•**1993** A. Schilling, M. Cantoni, J. D. Guo, and H. R. Ott from Zurich, Switzerland, produces a superconductor from mercury, barium and copper, ($HgBa_2Ca_2Cu_3O_8$) with maximum transition temperature of **133K**.

YBCO



Evidence of a phase transition



Dirk van Delft, *Freezing physics. Heike Kamerlingh Onnes and the quest for cold*, Koninklijke Nederlandse Akademie van Wetenschappen, Amsterdam 2007

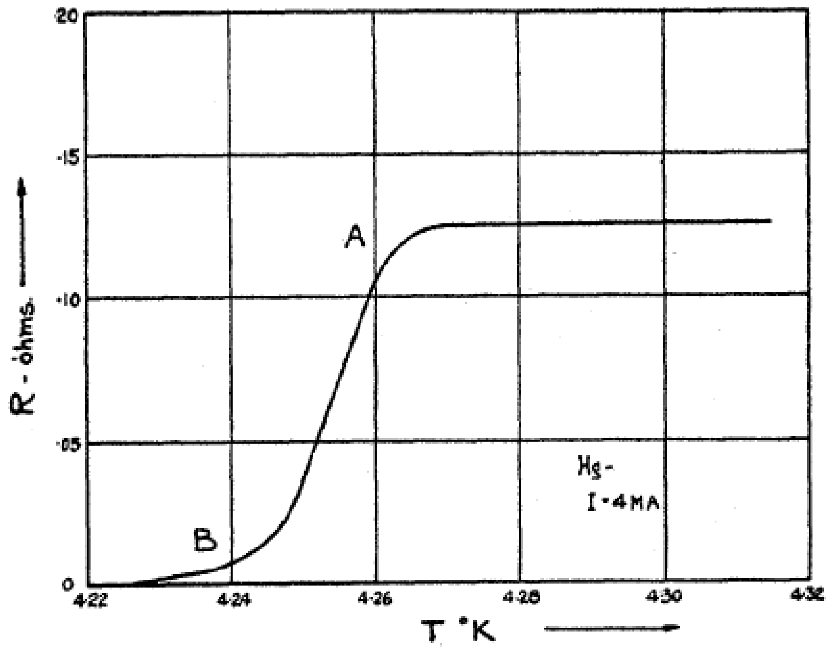


FIG. 2. Onnes' curve for mercury.

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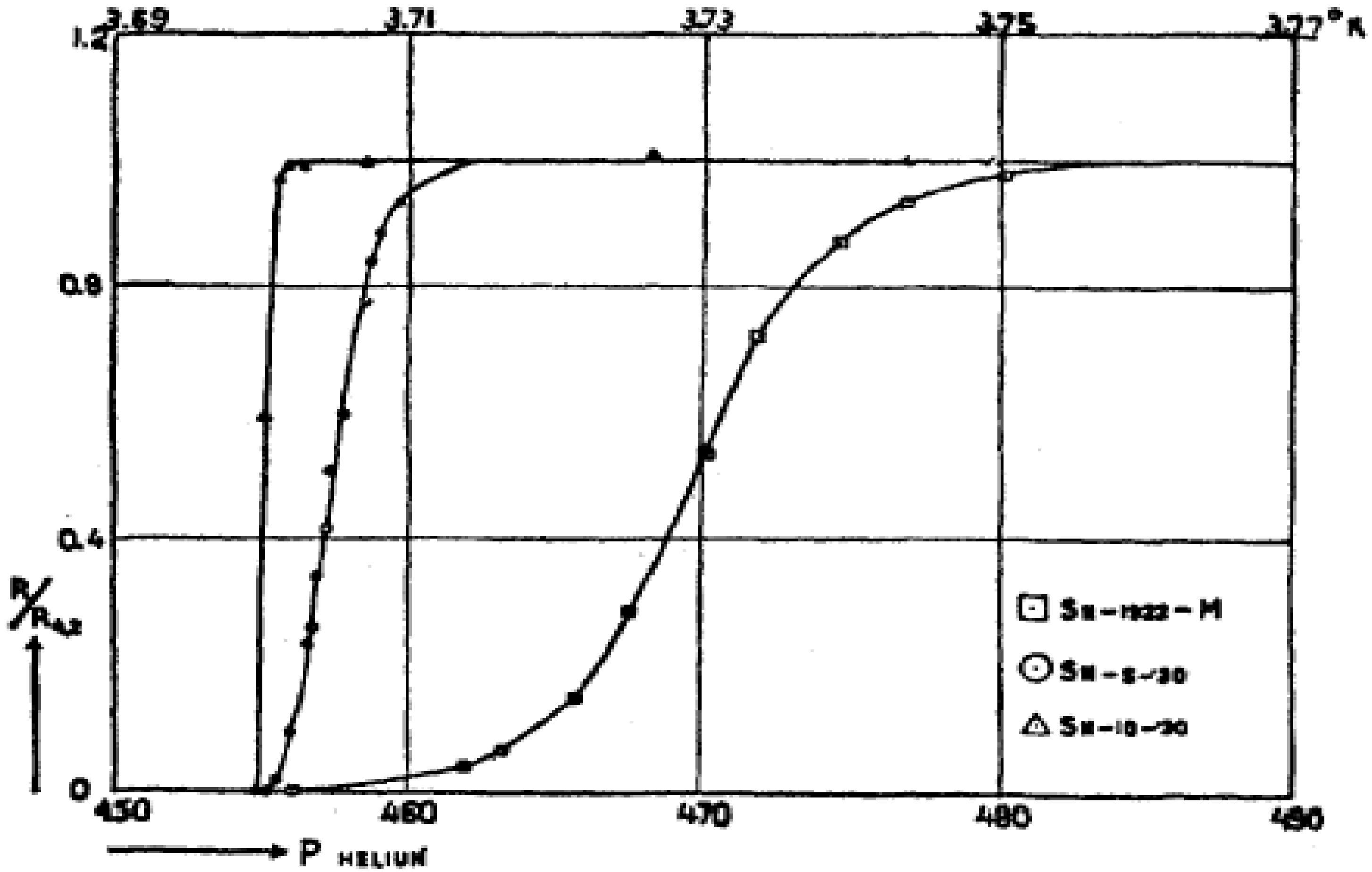

FIG. 4. Transition curves for monocrystalline and polycrystalline tin.

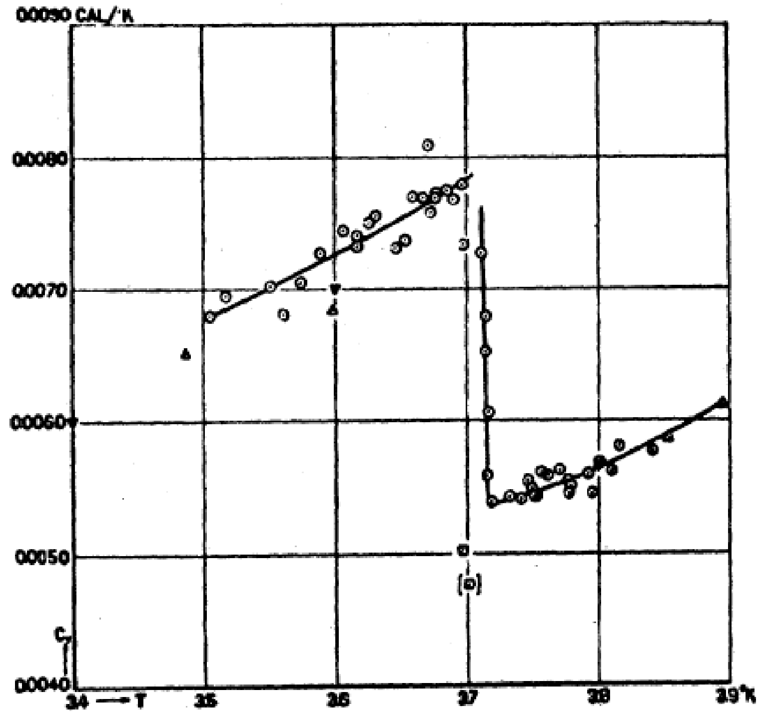

FIG. 16. Undisturbed (no external field) specific heat of tin.

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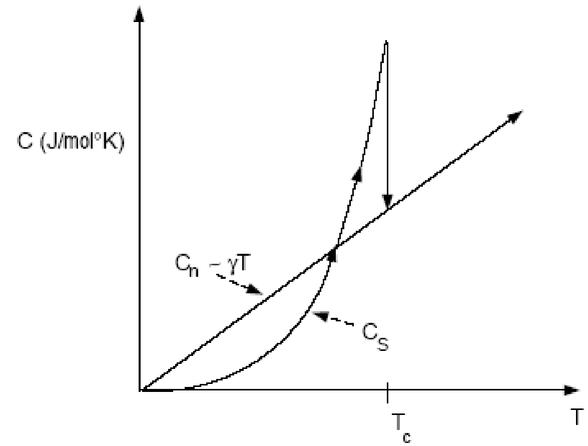


Figure 1: *The specific heat of a superconductor C_S and and normal metal C_n. Below the transition, the superconductor specific heat shows activated behavior, as if there is a minimum energy for thermal excitations.*

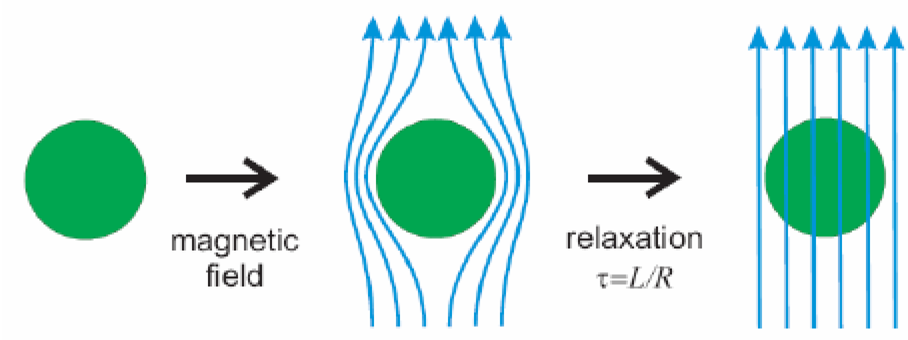
$$C_s \sim e^{-\beta\Delta}$$

Meissner effect

Discovered by Walther Meissner and Robert Ochsenfeld in 1933.

Example 1: a conductor

Magnetic field induces a screening current (Lentz' rule) which generates the opposite field.



A real conductor in magnetic field.

In an *ideal* conductor: $\vec{E} = \vec{j}\rho = 0$

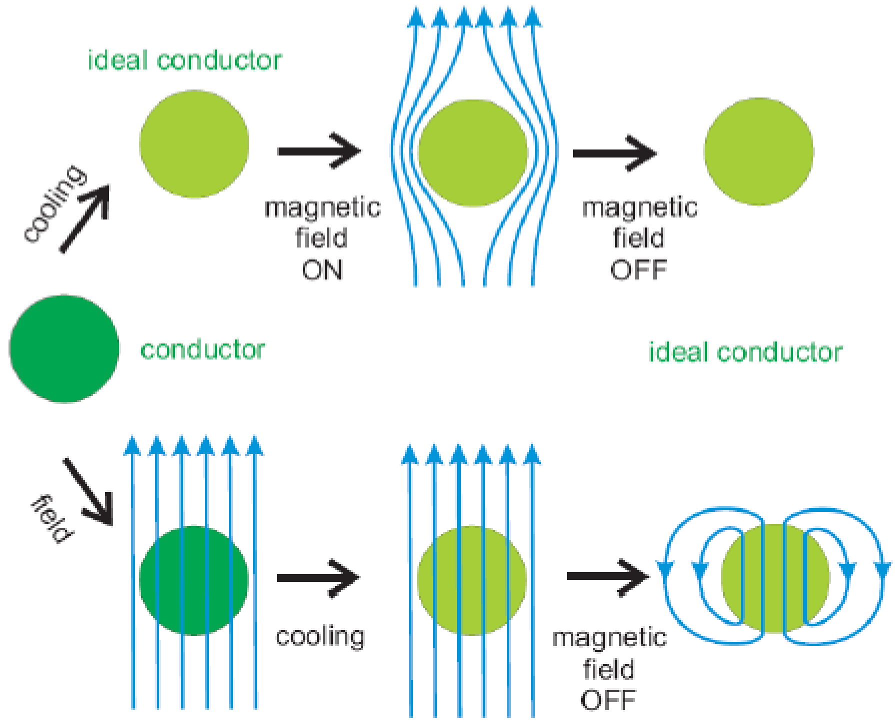
According to Maxwell's equation $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$

inside the conductor we have $\vec{B} = \mathrm{const}$

http://www.physics.buffalo.edu/phy514/w11/index.html

Example 2: an ideal conductor

Magnetic field induces a screening current (Lentz' rule) which generates an opposite magnetic field inside the sample



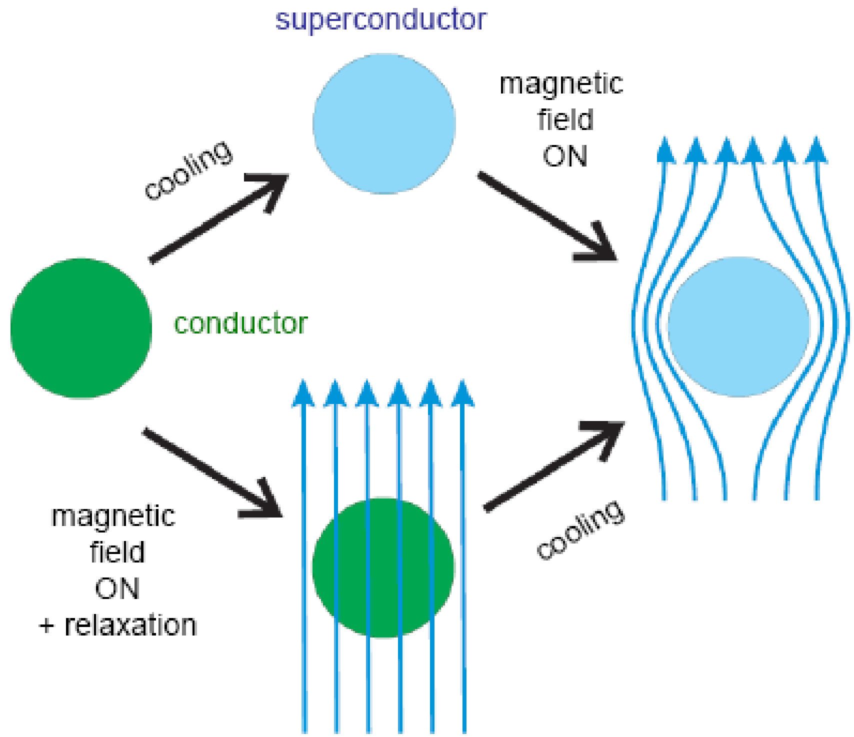
An ideal conductor in magnetic field

http://www.physics.buffalo.edu/phy514/w11/index.html

Example 3: finally, a superconductor

Meissner effect:

Superconductor always expels the magnetic flux

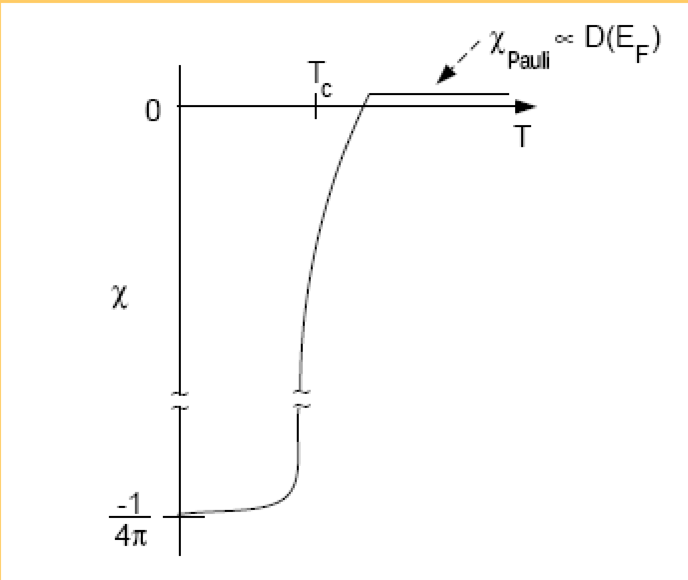


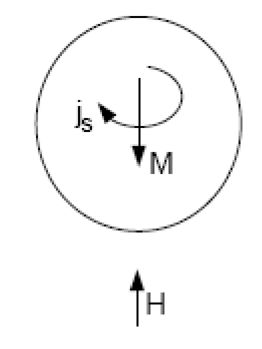
A superconductor in magnetic field

A superconductor is an ideal diamagnet

$$\mathbf{B} = \mu H = 0 \Rightarrow \quad \mu = 0 \qquad \mathbf{M} = \chi H = \frac{\mu - 1}{4\pi} H$$

$$\chi_{SC} = -\frac{1}{4\pi}$$





$$\mathbf{M} = -\frac{1}{4\pi} H_{\text{ext}}$$

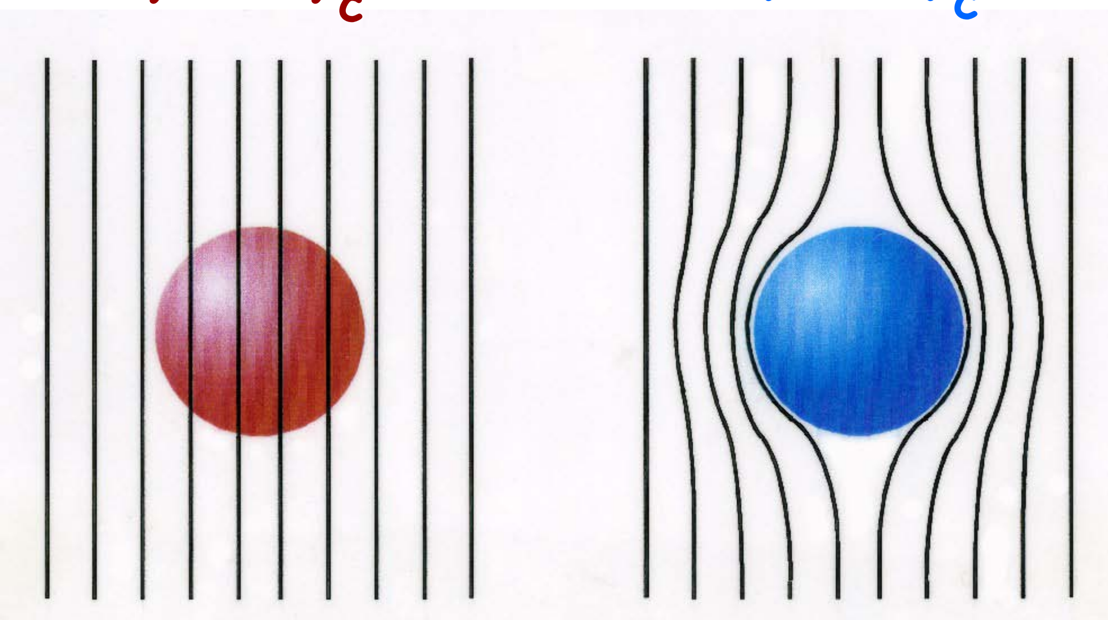
http://randomwallpapers.net/meissner-effect-magnet-photography_w497559

The Meissner effect is used to levitate a SC above a magnet

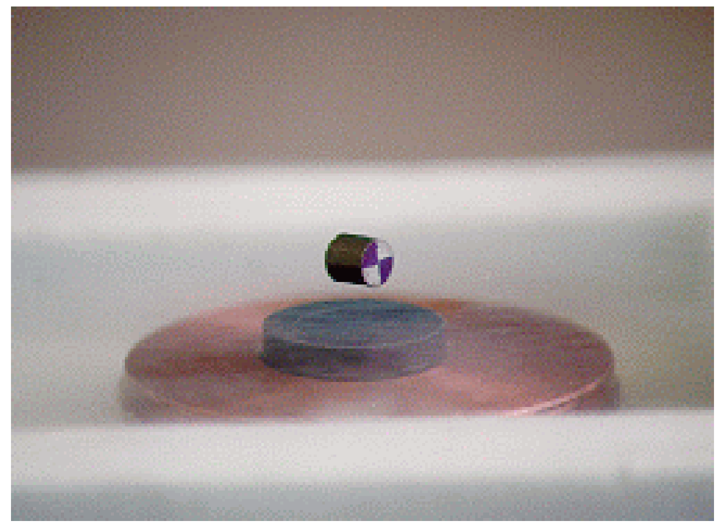
The Meissner (and Ochsenfeld) Effect

superconductors push out magnetic fields

$T > T_c$ $T < T_c$



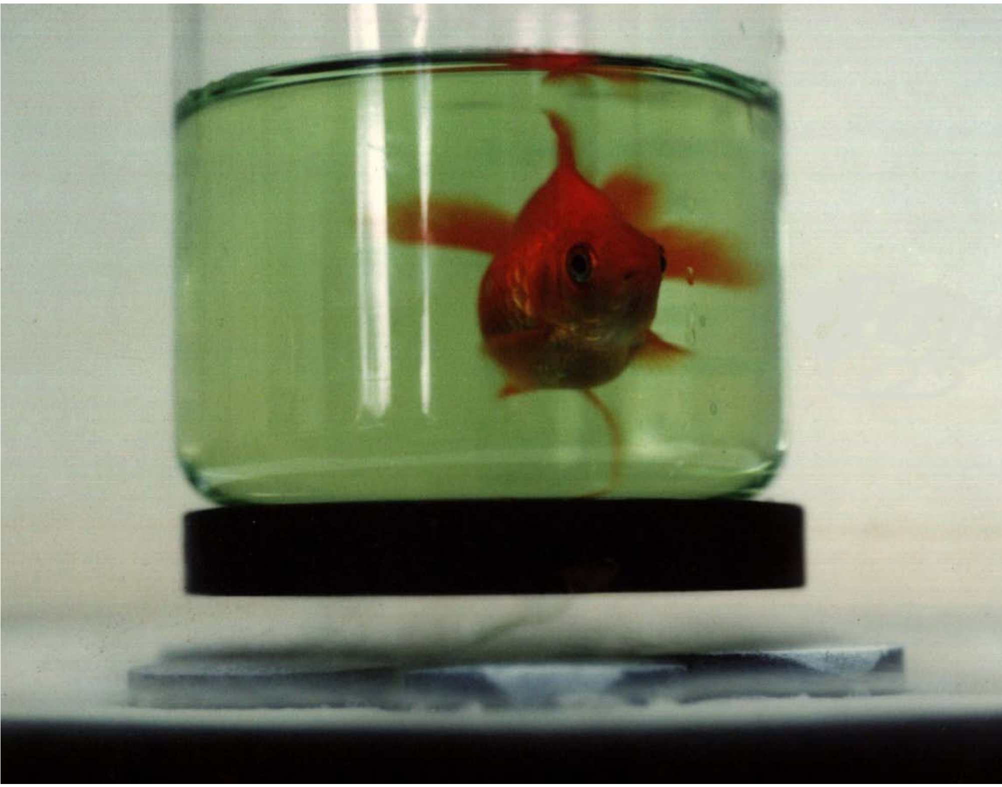
- and keep them out with constantly- flowing resistanceless currents



http://www.physics.ubc.ca/~outreach/phys420/p420_96/bruce/ybco.html

this 'diamagnetic' property is more fundamental than zero resistance

Levitation with High-Tc Superconductors



A 1 kg or so of fish bowl

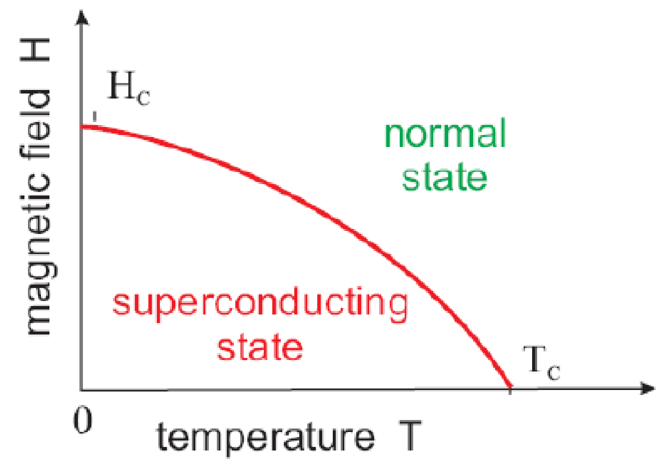
up to 202 kg of sumo wrestler!

Superconductivity is destroyed:

- by increasing temperature at $T > T_c$
- by large magnetic field $H > H_c$

Phase diagram of a superconductor in the $H - T$ plane is described by an empirically found formula:

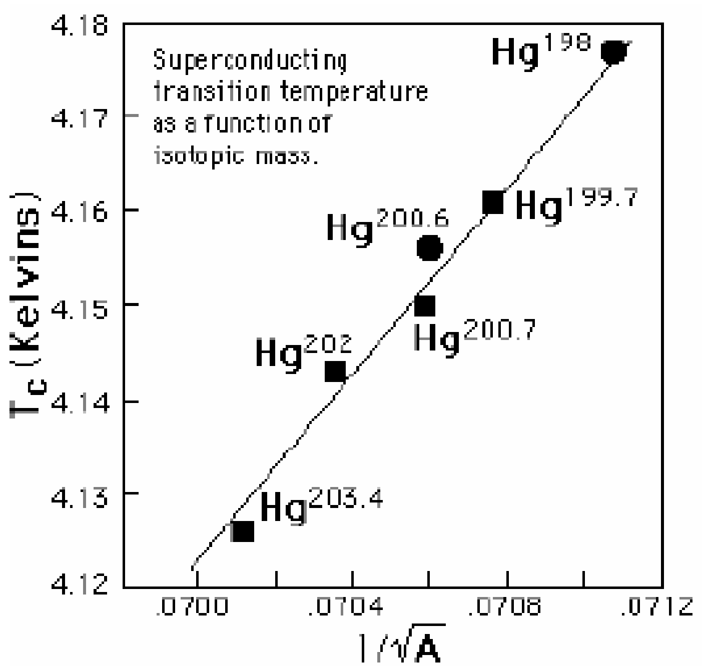
$$\frac{H_c(T)}{H_c(0)} = \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$



$H - T$ diagram for the superconducting state

Isotope Effect

Isotope	Natural abundance (atom %)
^{196}Hg	0.15
^{198}Hg	9.97
^{199}Hg	16.87
^{200}Hg	23.10
^{201}Hg	13.18
^{202}Hg	29.86
^{204}Hg	6.87



SC transition temperature depends on atomic mass

If electrical conduction in mercury were purely electronic, there should be no dependence upon the nuclear masses.

⇩

the first direct evidence for interaction between the electrons and the lattice.

Characteristic lengths in SC

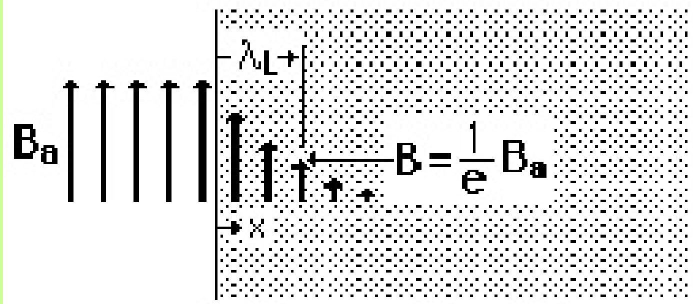
The **London equation** shows that the magnetic field exponentially decays to zero inside a SC (Meissner effect)

$$\vec{\nabla} \times \vec{J} = -\frac{1}{\mu_0 \lambda_L^2} \vec{B}$$

$$\lambda_L = \sqrt{\frac{\varepsilon_0 m c^2}{n e^2}}$$

λ_L = London penetration depth

n = superconducting electron density



$$B_{inside} = B_a e^{-x/\lambda_L}$$

The London penetration depth is the distance required to fall to 1/e times the externally applied field B_a.

The Pippard coherence length:

$$\xi_0 = \frac{2\hbar v_F}{\pi E_g}$$

v_F = Fermi velocity

E_g = Superconducting band gap

for pure SC far from T_c temperature-dependent Ginzburg-Landau coherence length is approximately equal to Pippard coherence length

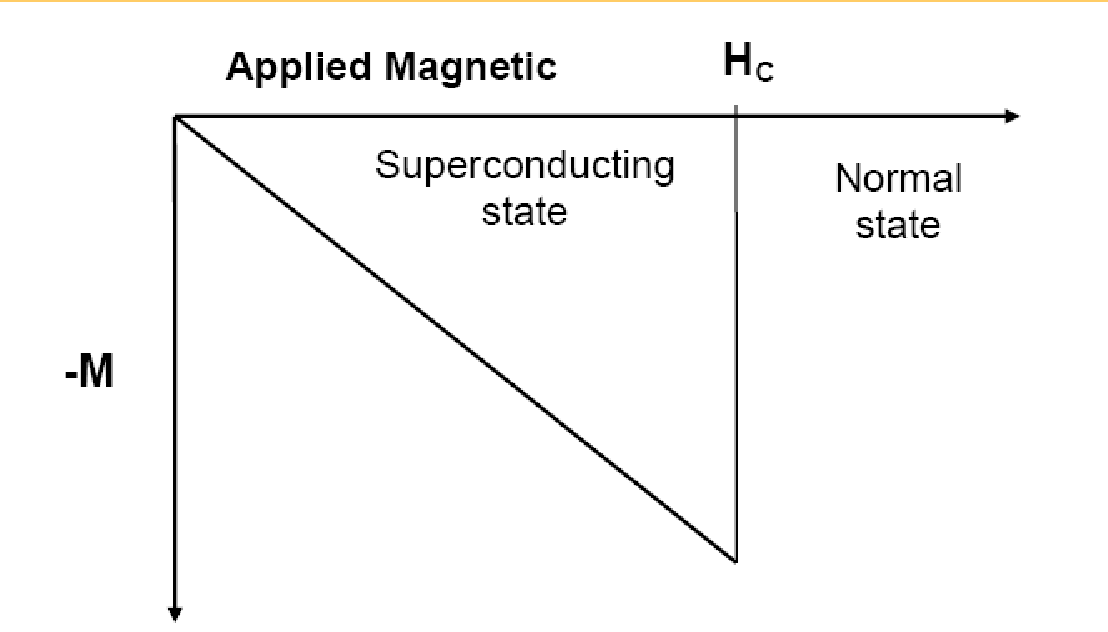
$$\xi(T) = \xi_0$$

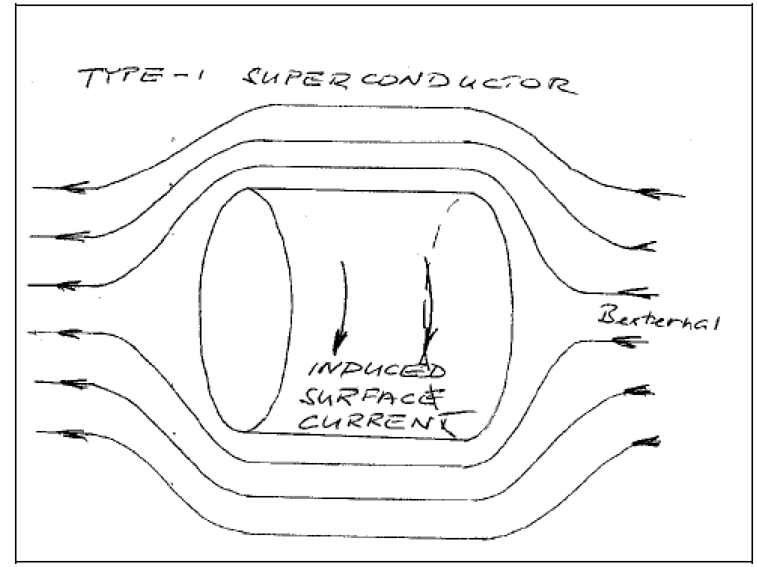
$$k = \frac{\lambda}{\xi}$$

Coherence length is a measure of the shortest distance over which superconductivity may be established

Categories of SC

Type I





- Very pure samples of lead, mercury, and tin are examples of Type I superconductors
- very pure metals that typically have critical fields too low for use in superconducting magnets
- The strongest type-I superconductor, pure lead has a critical field of about 800 gauss.

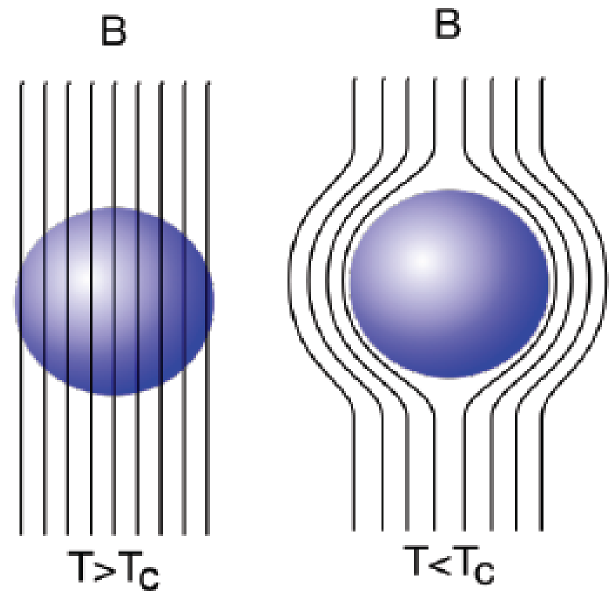
In **Type I superconductor** the magnetic field is completely expelled from the interior for $B<B_C$.

Type I

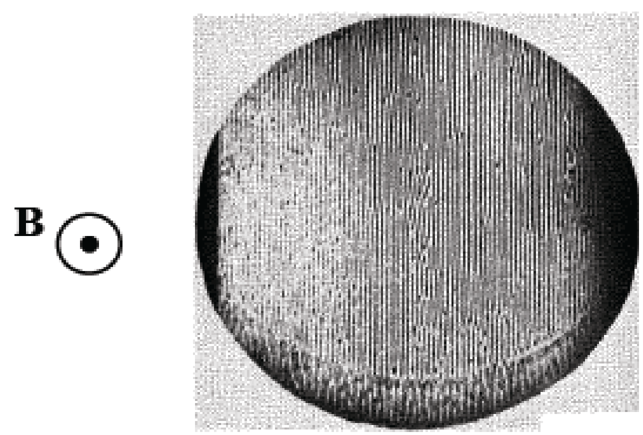
In reality:
– Meissner state due to circulating currents, so, at least in a thin layer on the surface:

$$B \neq 0 \text{ as } \nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

length λ (thickness o the layer)



– depending on the geometry, B will penetrate in the sample for H_i<H<H_c :
"intermediate state"



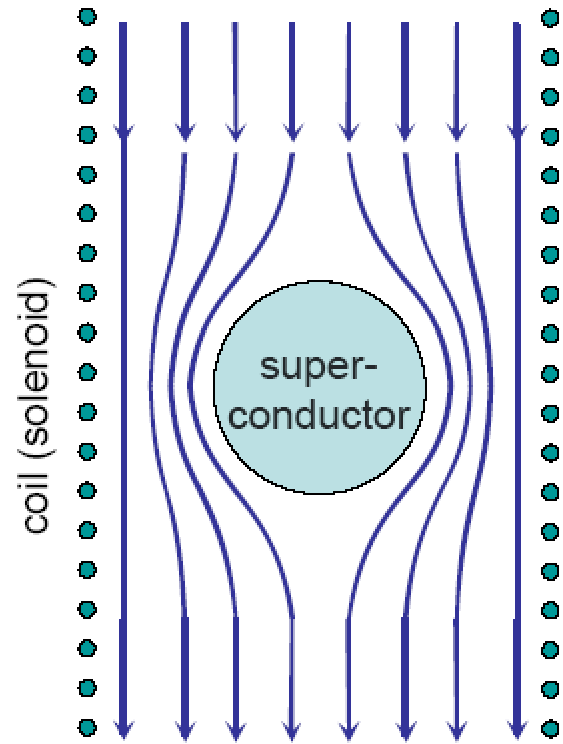
https://en.wikipedia.org/wiki/Meissner_effect

Dependence on sample shape

The superconductivity is destroyed when the field reaches the critical value H_{cm}.

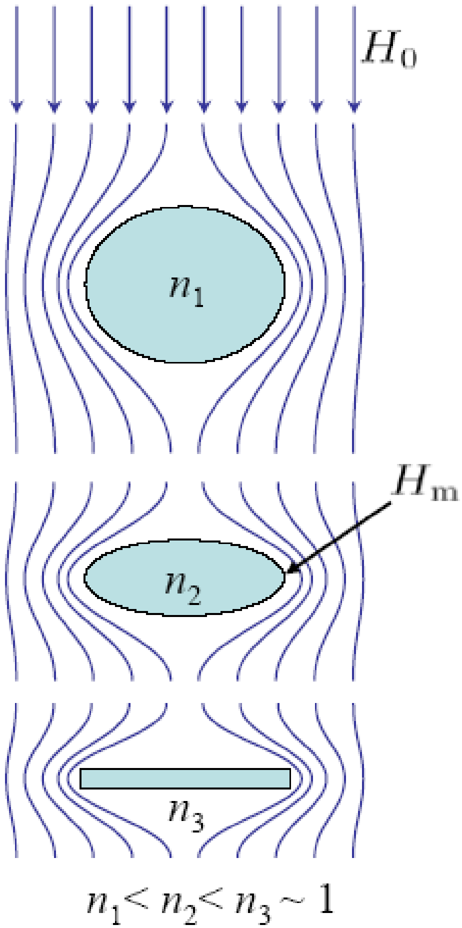
The field lines will have a higher density at the 'equator' and there will be a local increase in magnetic field.

So what happens when the equatorial field reaches the critical value H_{cm} ?



Superconducting sphere in a homogeneous field of a solenoid

Demagnetizing factor



H_m the maximum field at the surface

H_0 the external field far away from the body

$$H_m = \frac{H_0}{1-n}$$

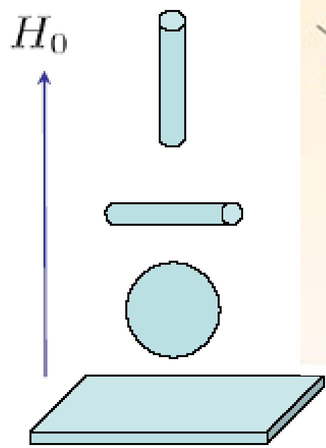
the factor n depends on geometry and is called demagnetizing factor

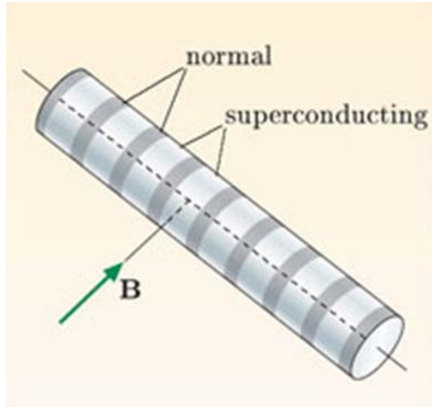
cylinder in parallel field $n = 0$

cylinder in transverse field $n = 1/2$

sphere $n = 1/3$

thin plate in perpendicular field $n = 1$

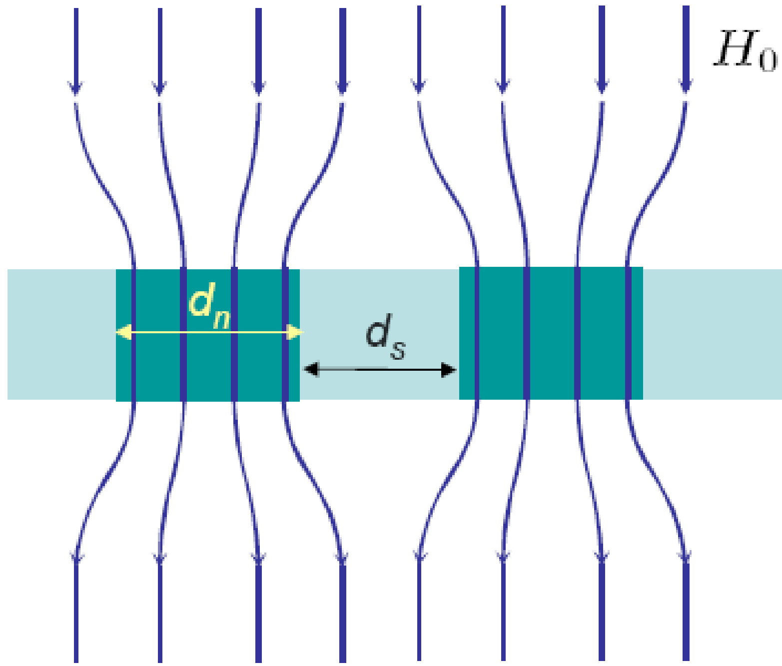


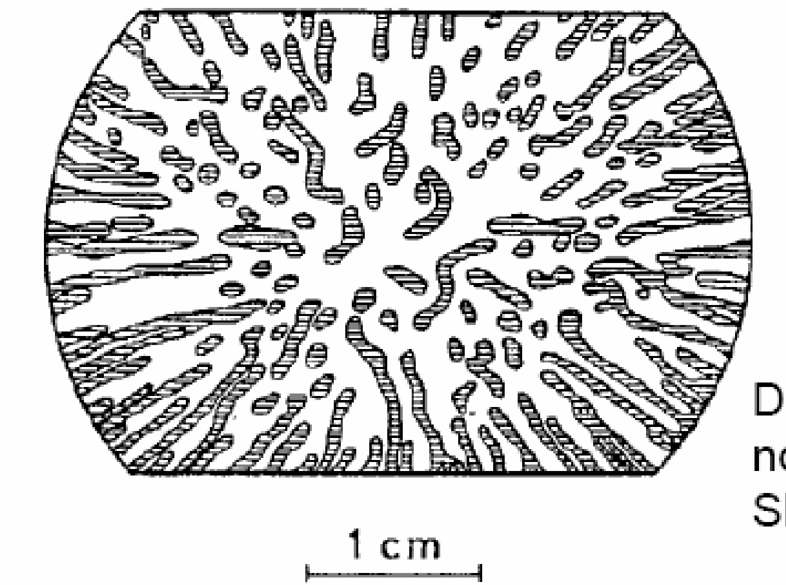


Intermediate state in type I superconductors

For a disk of the infinite radius, the transition to the *intermediate state* occurs in an infinitesimally small field H_0

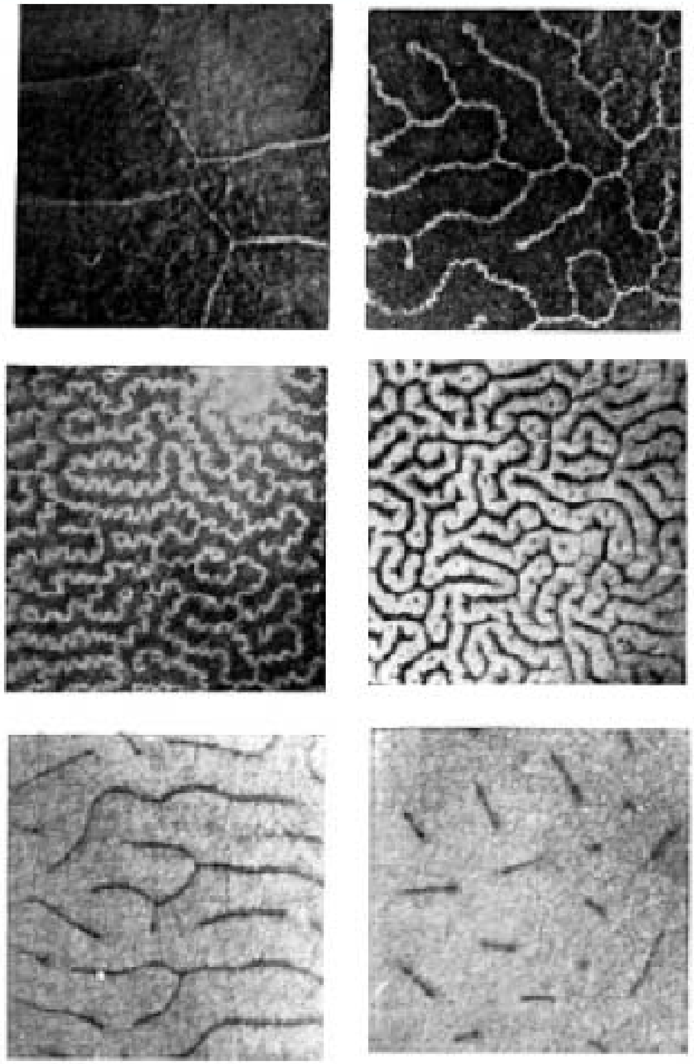
Intermediate state consists of regions of normal material carrying a magnetic field mixed with regions of superconducting material containing no field.





Distribution of the superconducting and normal regions in a tin sphere.
Shaded regions are superconducting

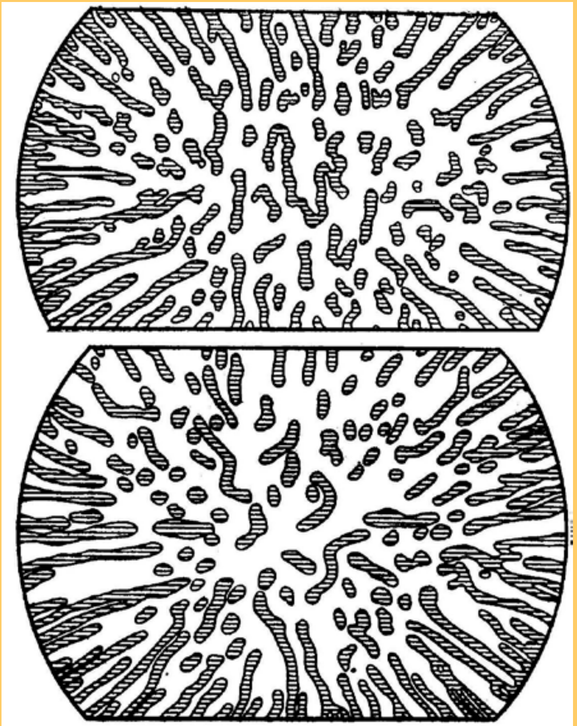
Intermediate state in type I superconductors



Meandering laminar structure with alternating normal and superconducting regions, typical for type I superconductors with the magnetic field applied normal to a flat slab.

T. E. Faber
Proc. Roy. Soc. **A248**, 460 (1958)

Intermediate state (SC of *type I*)



A distribution of superconducting and normal states in tin sphere (supercondacting ranges are shaded)

Intermediate state of a monocrystalline tin foil of 29 μm thickness in perpendicular magnetic field (normal regions are dark)

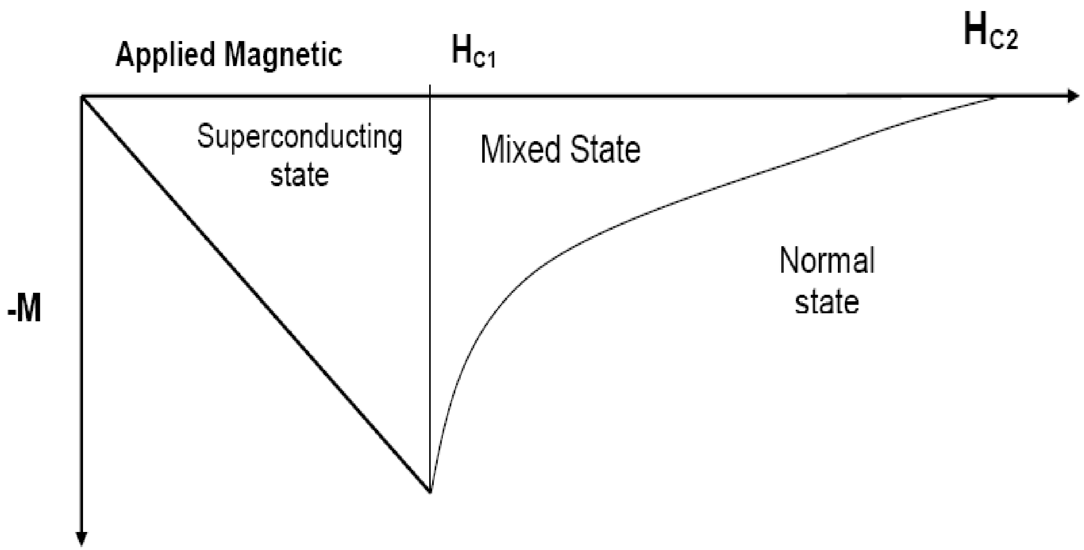
Type I

http://www.superconductors.org/Type1.htm

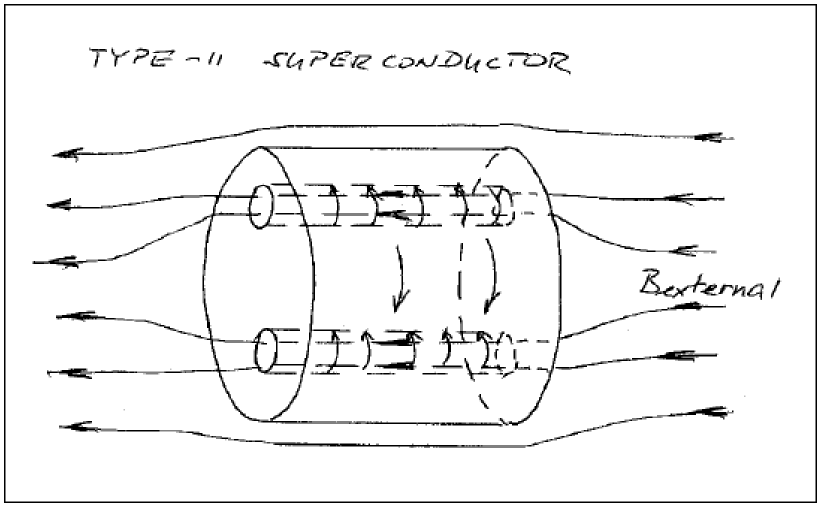
Lead (Pb)	7.196 K	FCC
Lanthanum (La)	4.88 K	HEX
Tantalum (Ta)	4.47 K	BCC
Mercury (Hg)	4.15 K	RHL
Tin (Sn)	3.72 K	TET
Indium (In)	3.41 K	TET
Palladium (Pd)*	3.3 K	
Chromium (Cr)*	3 K	
Thallium (Tl)	2.38 K	HEX
Rhenium (Re)	1.697 K	HEX
Protactinium (Pa)	1.40 K	TET
Thorium (Th)	1.38 K	FCC
Aluminum (Al)	1.175 K	FCC
Gallium (Ga)	1.083 K	ORC
Molybdenum (Mo)	0.915 K	BCC
Zinc (Zn)	0.85 K	HEX
Osmium (Os)	0.66 K	HEX
Zirconium (Zr)	0.61 K	HEX
Americium (Am)	0.60 K	HEX
Cadmium (Cd)	0.517 K	HEX
Ruthenium (Ru)	0.49 K	HEX
Titanium (Ti)	0.40 K	HEX
Uranium (U)	0.20 K	ORC
Hafnium (Hf)	0.128 K	HEX
Iridium (Ir)	0.1125 K	FCC
Beryllium (Be)	0.023 K (SRM 768)	HEX
Tungsten (W)	0.0154 K	BCC
Platinum (Pt)*	0.0019 K	
Lithium (Li)	0.0004 K	BCC
Rhodium (Rh)	0.000325 K	FCC

Type II

http://www.superconductors.org/Type2.htm



High temperature ceramic superconductors such as $YBa_2Cu_3O_7$ (YBCO) and $Bi_2CaSr_2Cu_2O_9$ are examples of Type II superconductors, and most superconducting metals (niobium (*Nb*), vanadium(*V*), Technetium (*Tc*), metallic compounds and alloys.



Type II superconductors have two values of critical magnetic field, for $B<B_{C1}$ the magnetic field is completely expelled (Type-I behavior), whereas for $B_{C1}<B<B_{C2}$ the magnetic field partially penetrates through the material.

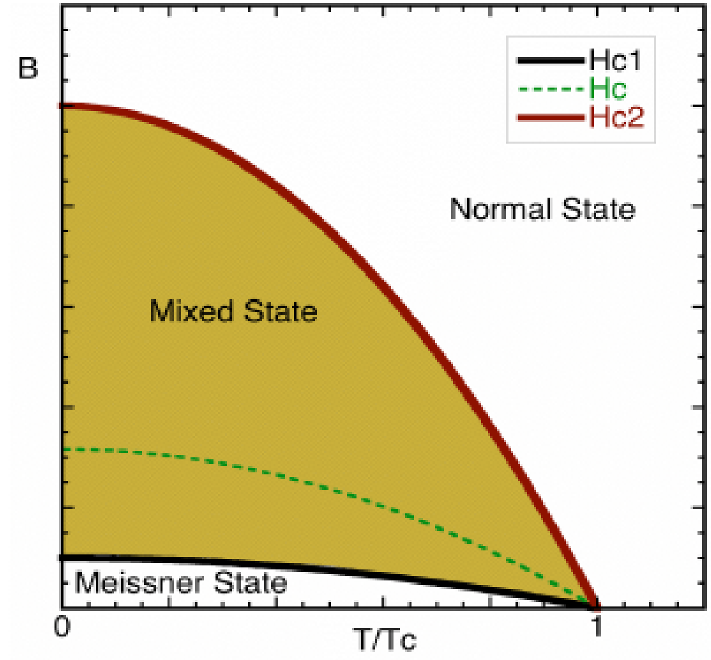
Many superconductors support large fields
CERN-LHC, H>8T over kms…

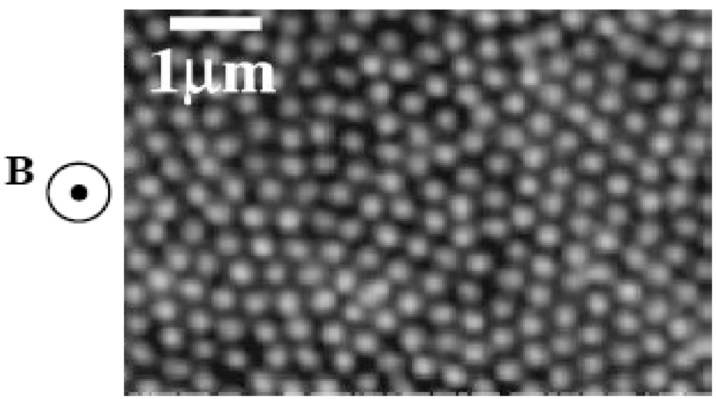
Nb, (pure), T_c~9K and H_c~0.1T
NbTi alloy, T_c~9K and H_{c2}>10T

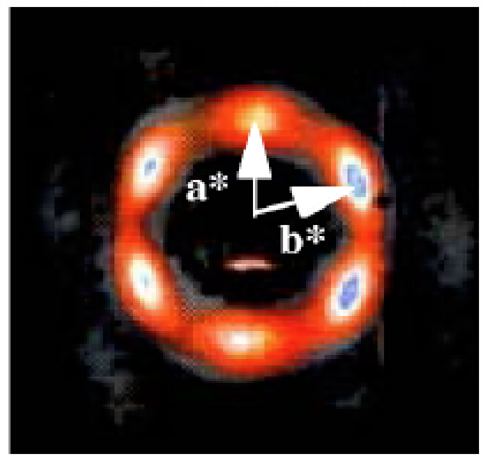
$CeCoIn_5$, T_c~2K, H_{c2}~12T…

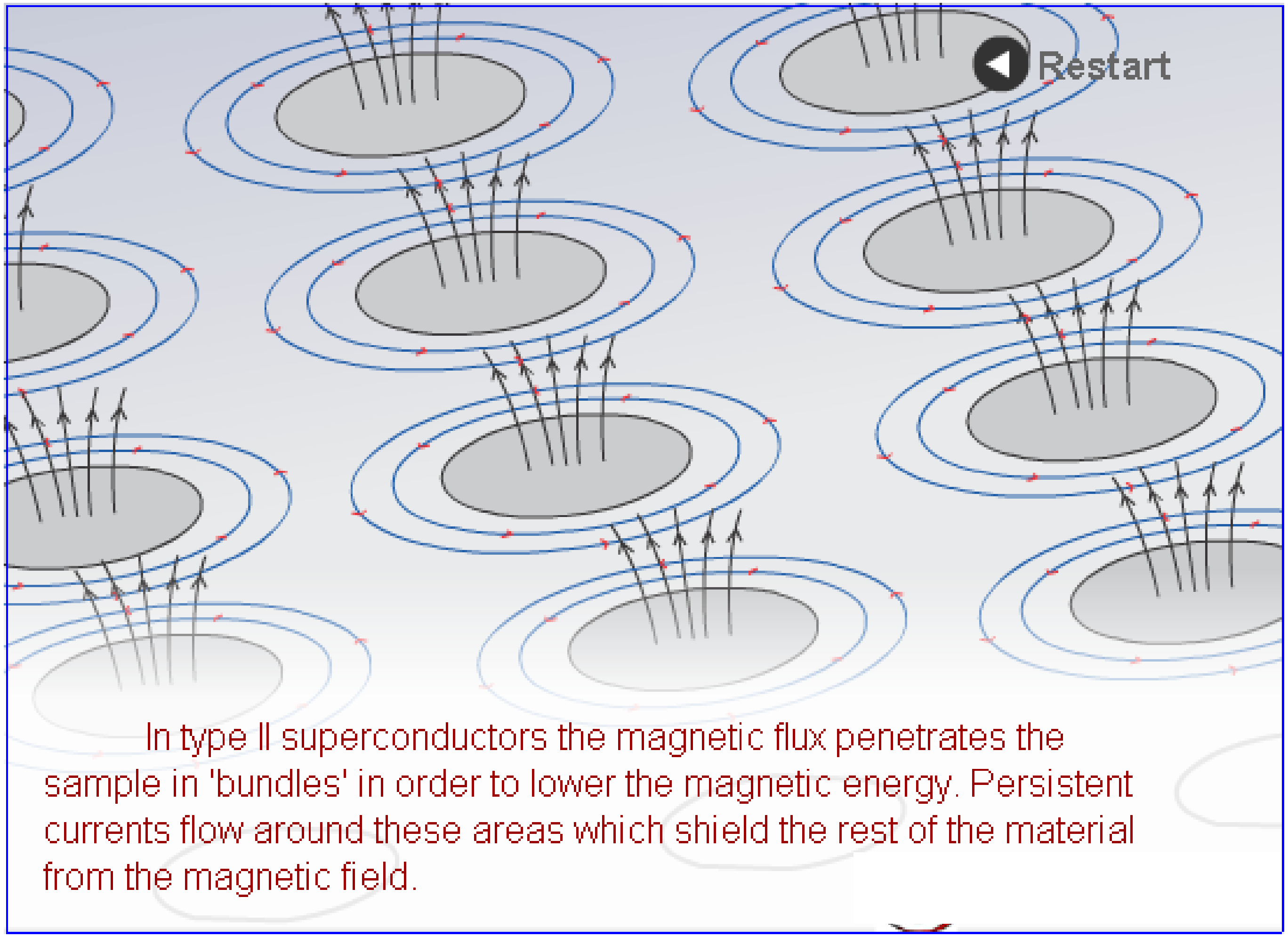
But they show very little Meissner effect !
Type II superconductors

In the mixed state, B penetrates as flux lines, forming an ordered lattice, seen by decoration, neutrons, STM…









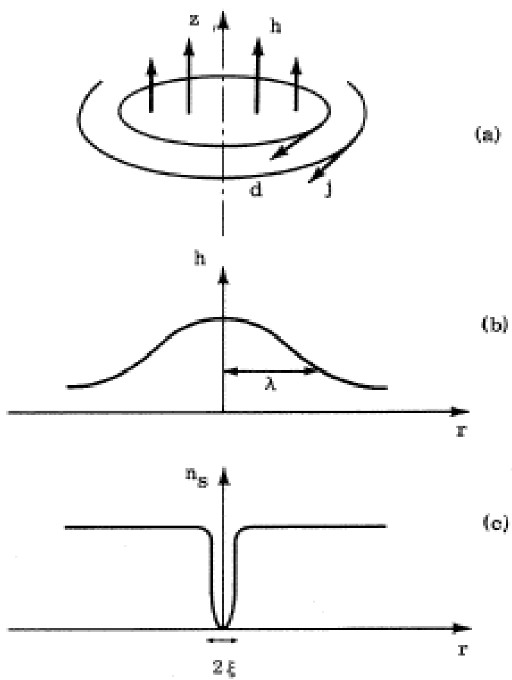
source: Wikipedia

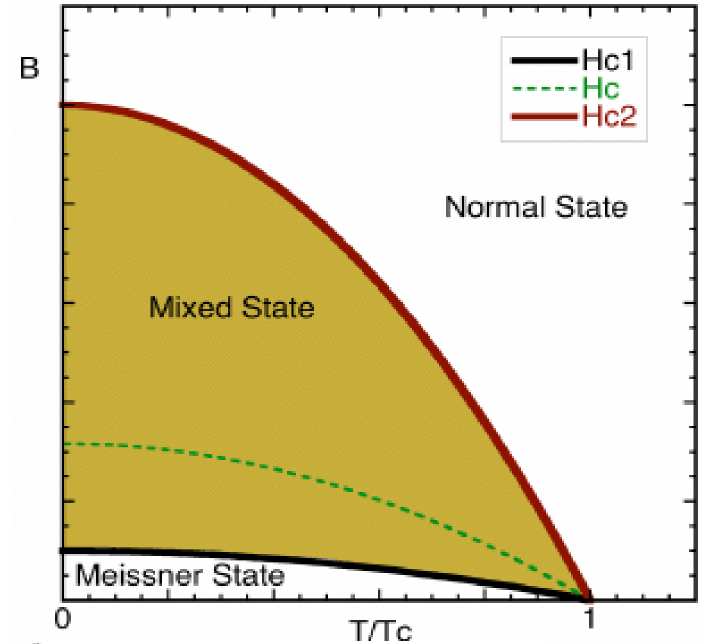
1. Type I and type II superconductors: general phenomena-Vortices

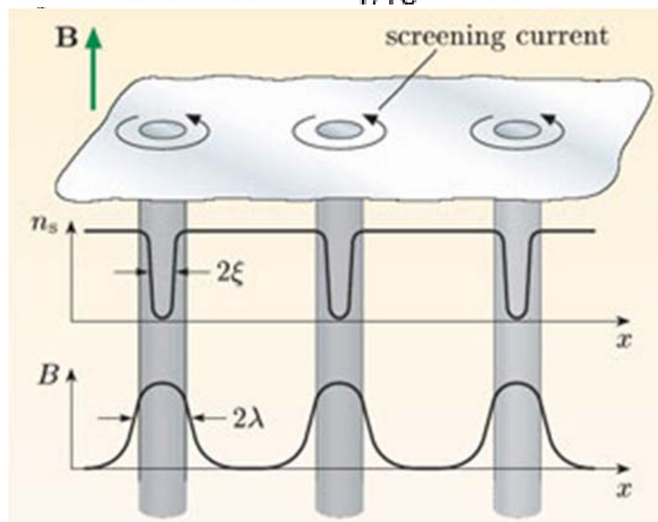
In the mixed state, B penetrates as flux lines…

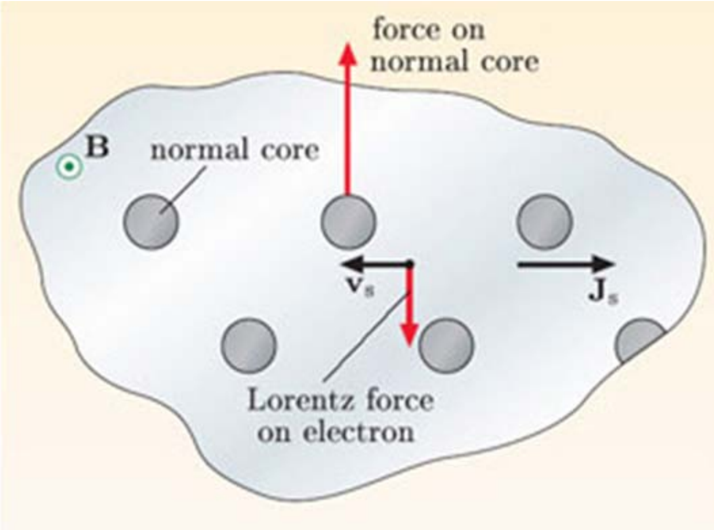
Diameter of the flux "tubes" ~λ (created by supercurrents)

The superconducting state is destroyed in the vortex cores, of size ~ ξ << λ



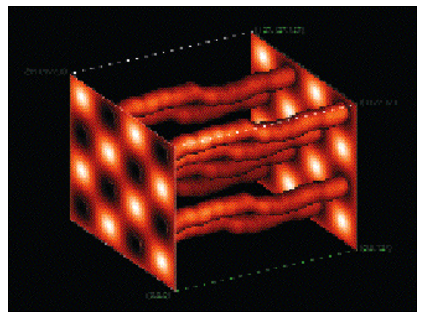






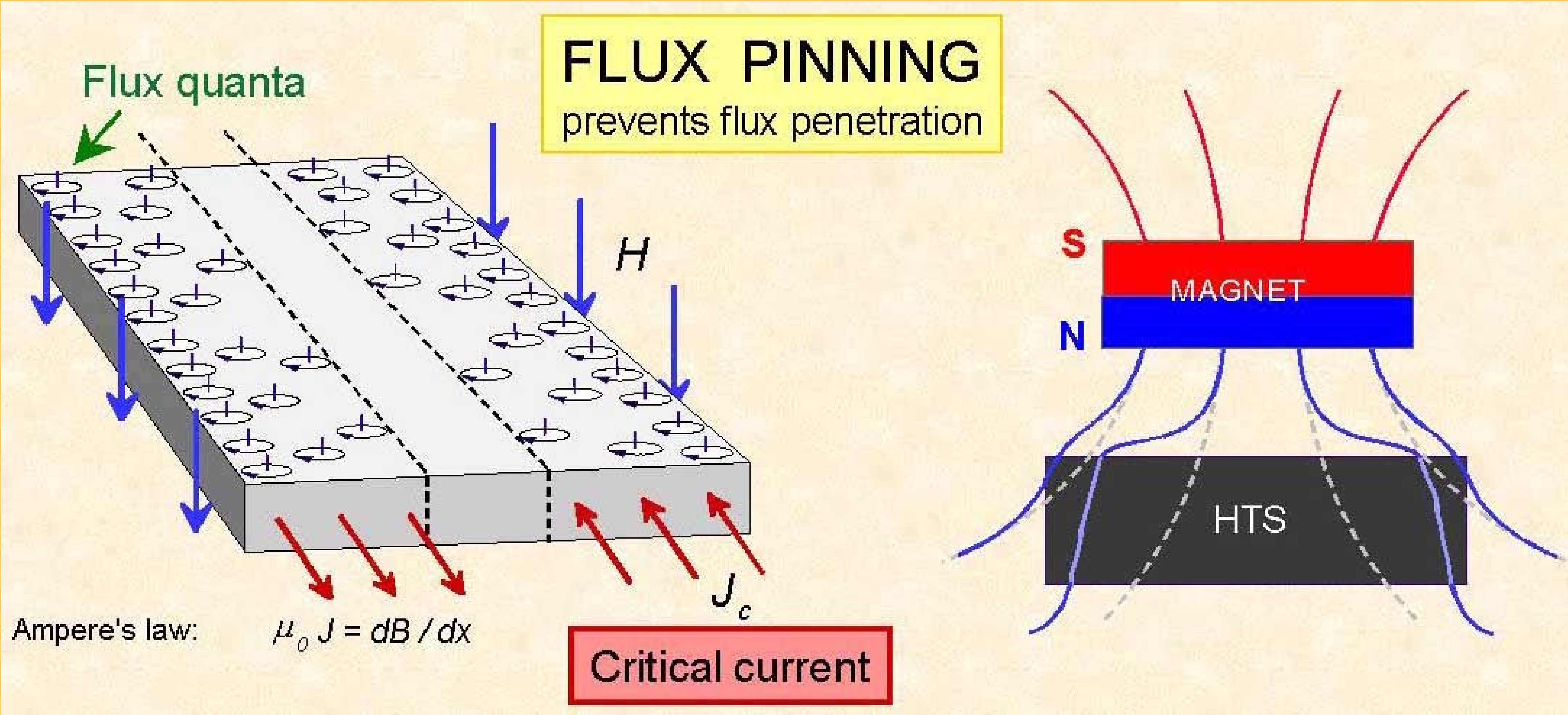
Lorentz force can cause the cores and their associated magnetic flux to move, and the flux motion will induce an emf that drives a current through the normal cores, somewhat like an eddy current. Energy is therefore dissipated in the normal cores, and this energy must come from the power supply. The energy dissipation means that the flow of electrons is impeded, and therefore there is a resistance to the flow of the current.

$$\vec{F} = q\vec{v} \ x \ \vec{B}$$



defects effectively pin the normal cores in position – they provide a potential barrier to motion of the cores, so that the force on the cores must exceed a certain value before the cores can move.

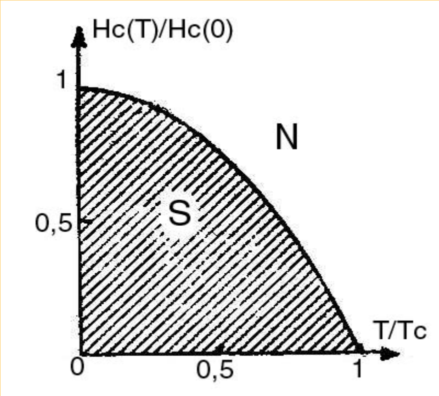
Alexey Ustinov ; http://www.pi.uni-karlsruhe.de/lehre/superconductivity/folien/Superconductivity-2008-02.pdf



CRITICAL CURRENT DENSITY (J_c): The maximum value of electrical current per unit cross sectional area that a superconductor can carry without measurable resistance.

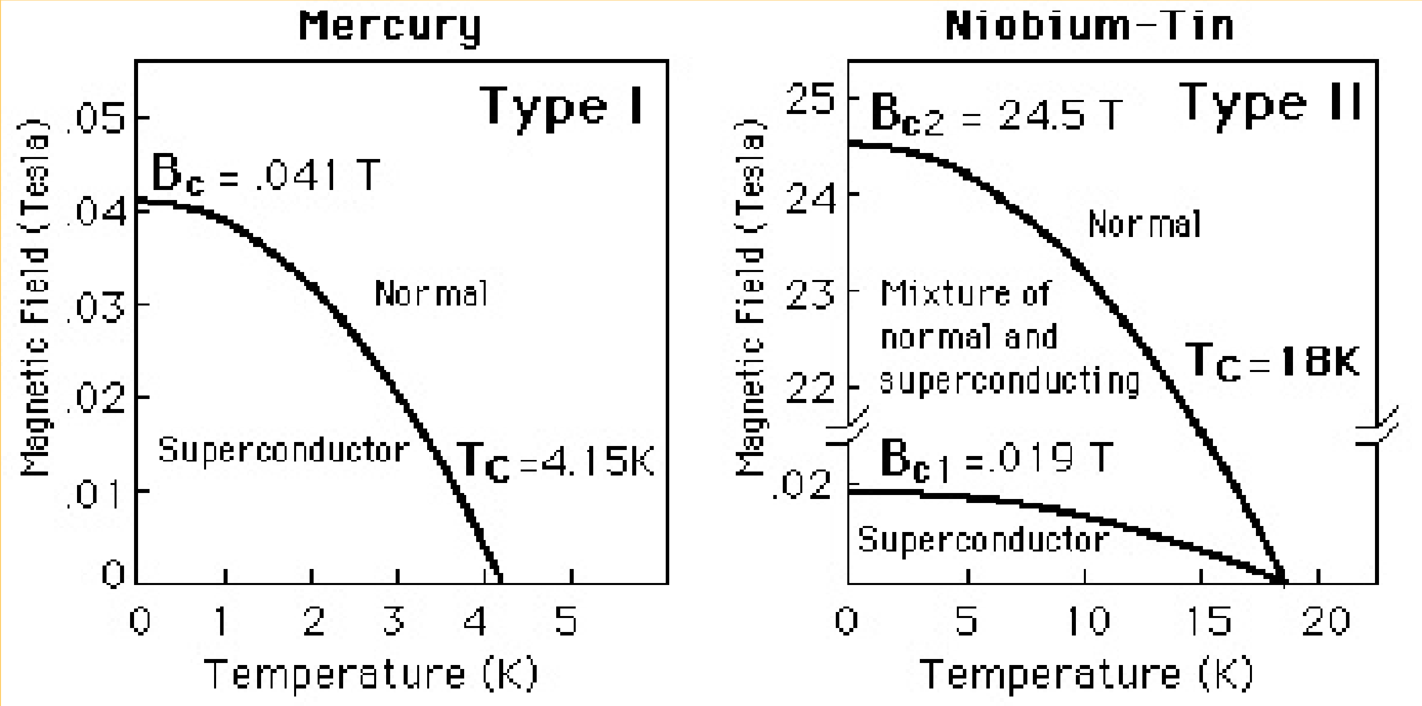
•For some applications Jc can be in excess of $1000 Amm^{-2}$.

Magnetic properties



$$H_c(T) = H_c(0)[1 - (\frac{T}{T_c})^2]$$

Phase boundaries between superconducting, mixed and normal states of I and II type SC.



http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/scbc.html

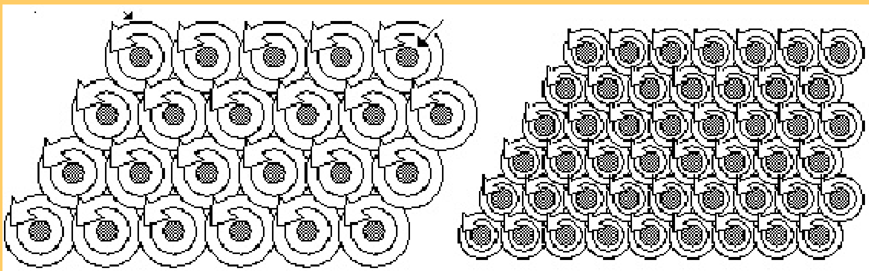
Mixed state (SC of type II)

Abrikosov theory [1957]

A.A. Abrikosov "*On the magnetic properties of ...* ", Soviet Physics JETP 5, 1174 (1957)

Supercurrent

Normal core



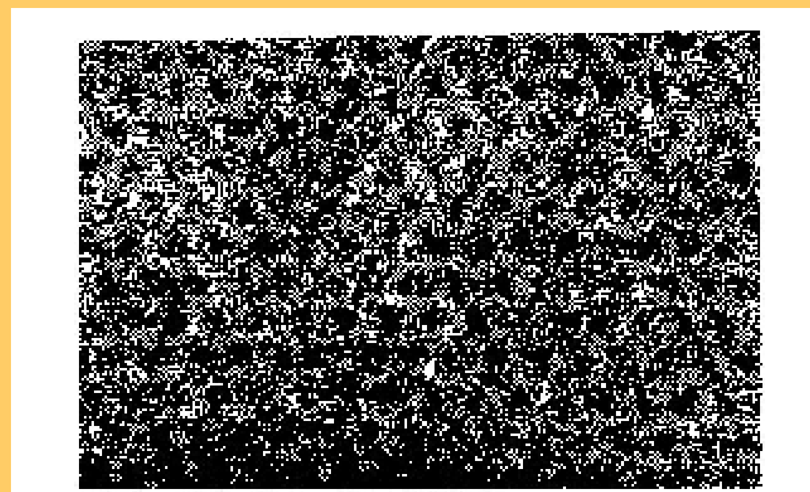
$$\Phi_0 = \frac{2\pi\hbar}{2e} \cong 2.0678 x10^{-15} tesla \cdot m^2$$

$$\Phi = n\left(\frac{h}{2e}\right) = n\Phi_0$$

Normal regions are approximately 300nm

Closer packing of normal regions occurs at higher temperatures or higher external magnetic fields

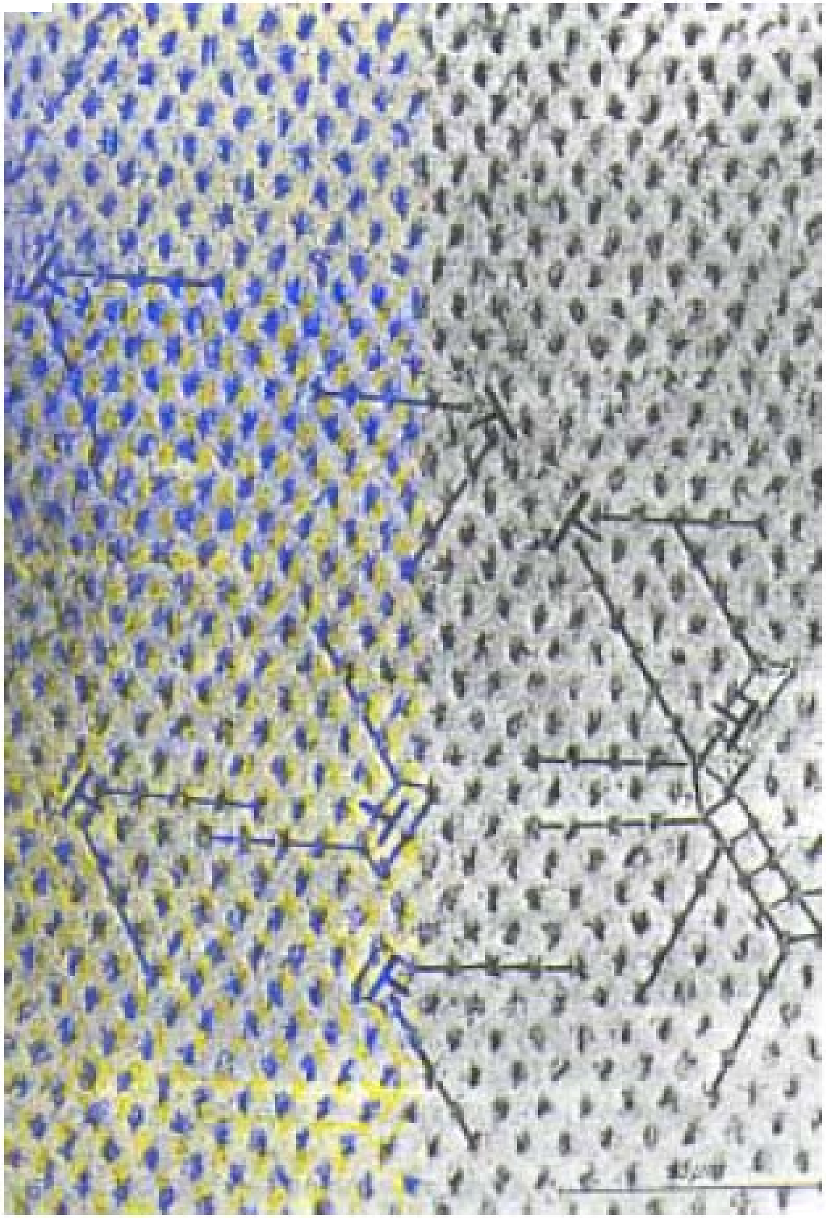
Triangular lattice of vortex lines going out to the surface of SC $Pb_{0.98}In_{0.02}$ foil in perpendicular to the surface magnetic field



Vortex Imaging by Decoration

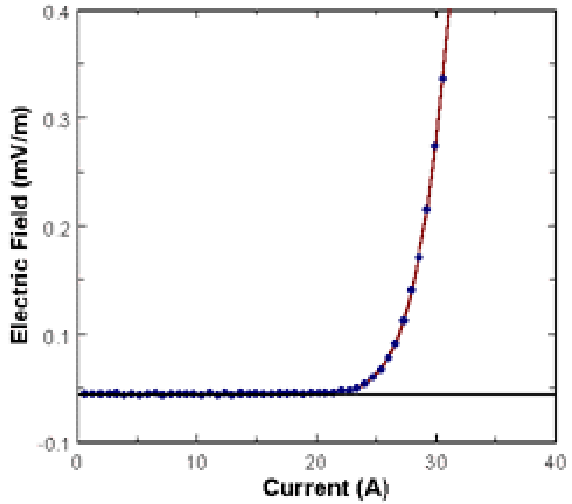
Vortex state can be imaged in several ways
- Magnetic decoration
- Small angle neutron scattering
- Hall probes

Lattice structure confirmed *and* defects in lattice seen

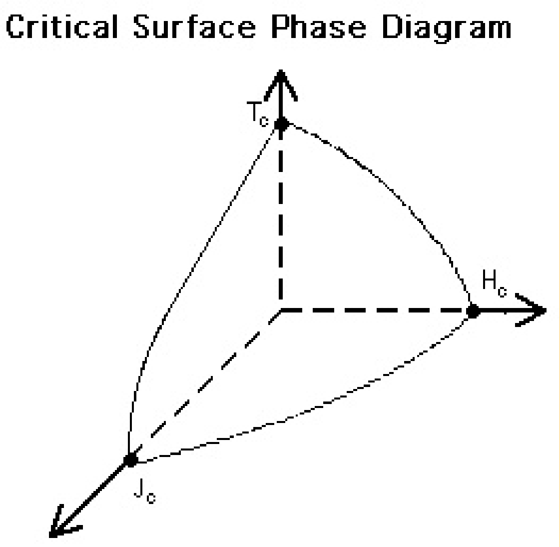


Trauble and Essmann 1967

Critical current density



Above a critical current a superconductor has "resistance"



Three critical parameters T_c, H_c and J_c define the boundaries of the environment within which a SC can operate.

Critical current is certain maximum current that SC materials can be made to carry, above which they stop being SCs. If too much current is pushed through a SC, it will revert to the normal state even though it may be below its T_c. The colder you keep the SC the more current it can carry.

TABLE 21.7 Critical Temperatures and Magnetic Fluxes for Selected Superconducting Materials

	Material	*Critical Temperature* T_C *(K)*	*Critical Magnetic Flux Density* B_C *(tesla)*[a]	
		Elements		
	Aluminum	1.18	0.0105	
	Lead	7.19	0.0803	
Type I	Mercury (α)	4.15	0.0411	B_c
	Tin	3.72	0.0305	
	Titanium	0.40	0.0056	
	Tungsten	0.02	0.0001	
		Compounds and Alloys		
	Nb–Ti alloy	10.2	12	
	Nb–Zr alloy	10.8	11	B_{c2}
Type II	Nb_3Sn	18.3	22	
	Nb_3Al	18.9	32	
	Nb_3Ge	23.0	30	
	V_3Ga	16.5	22	
	$PbMo_6S_8$	14.0	45	

The old

•Looking for an explanation: a series of failures and a triumph

- During 46 years, from 1911 to 1957, superconductivity remains unexplained.

- In 1950 it is the most important problem in theoretical Physics.

- Richard Feynman: *« No one is smart enough to explain it »*

Superconductivity: the old and the new *A one hundred year voyage of discoveries*
André-Marie Tremblay
Département de Physique Université de Sherbrooke
http://www.physique.usherb.ca/~tremblay

2. Main physical concepts: $\boxed{\text{two fluids model}}$

E. Silva, http://webusers.fis.uniroma3.it/~silva/disp/twofluid.pdf

Superconductors sustain current without dissipation: zero resistance (see also Meissner state, vortices…), persistent current in a ring…

Simple idea:

- The electrons are "locked" together to build the current flow =>
- Condensate of electrons, like the coherent condensate of photons in a laser, or the condensate of He^4 atoms in superfluid He^4…

  but electrons are fermions =>

- Condensate of electron pairs, so called Cooper pairs -> builds the $\boxed{\text{first fluid}}$

Electron pairs are all in the same state => no entropy, But properties of a superconductor change with T =>

- There also exist thermal excitations: "normal electrons", they build the $\boxed{\text{second fluid}}$

Consequence on transport properties:

- resistivity: zero resistance because the Cooper pair condensate short-circuits thermal excitations ($\sigma = \sigma_{condensate} + \sigma_{thermal}$)
- thermal conductivity: bad thermal conductor, as $\kappa = \kappa_{condensate} + \kappa_{thermal}$,

  but $\kappa_{condensate} = 0$ (no entropy, no energy to exchange)

  $\kappa_{thermal}$ disappears with the number of thermal excitations when T->0

- BCS theory (1957)

Quantum behavior at the macroscopic scale

Leon Cooper

Nobel Prize : 1972

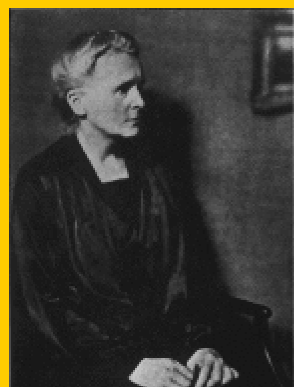
John Bardeen* Robert Schrieffer

- John Bardeen :
- The only person to receive two Nobel prizes in Physics !!!

Invention : TRANSISTOR!

W. Shockley, J. Bardeen, W.H. Brattain

Superconductivity: the old and the new *A one hundred year voyage of discoveries*
André-Marie Tremblay
Département de Physique Université de Sherbrooke
http://www.physique.usherb.ca/~tremblay

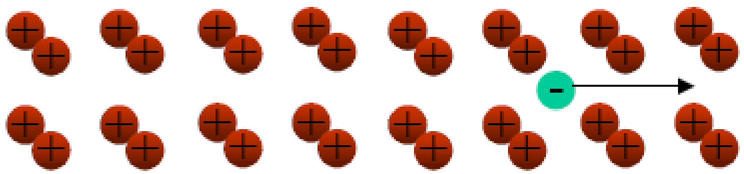


Marie Curie:

1903 Physics with H.A. Becquerel

1911 Chemistry (alone)

Resistance of a normal metal



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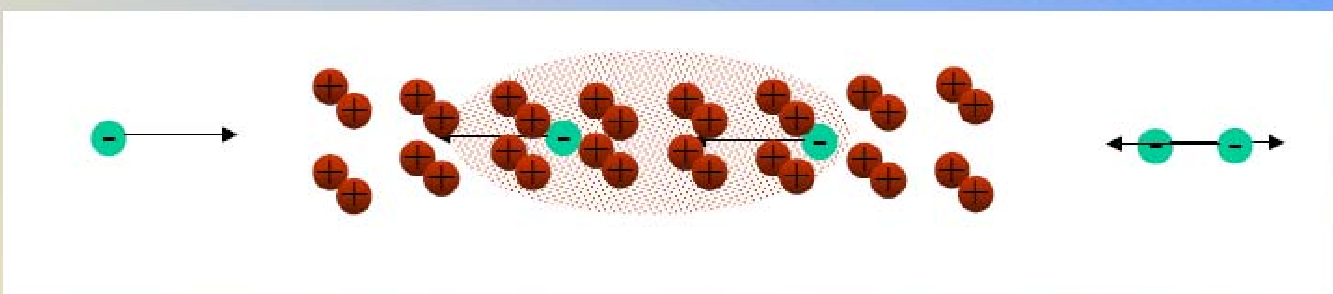
Ingredient #1

Attraction and formation of Cooper pairs

Cooper pairs
or
Prelude to supraconductivity



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http://www.physique.usherb.ca/~tremblay

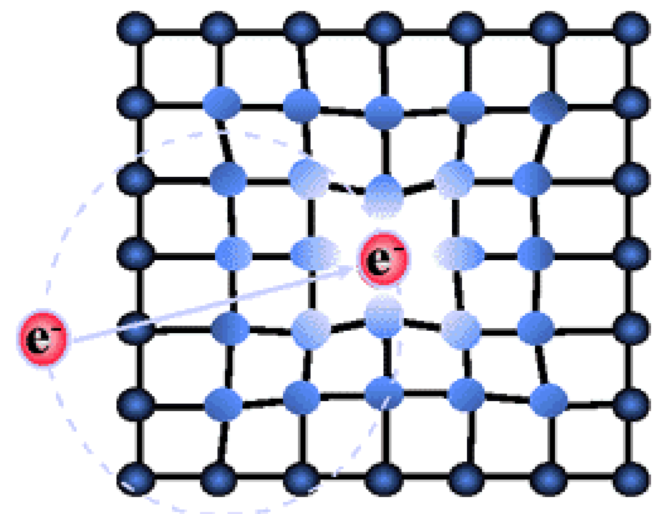
Ingredient #2
Coherence

pairs

Cooper form "bosons" that all "condense" in the same quantum state. Their wave functions all have the same quantum phase. Electrons are in a single coherent state (Like in a big atom!).

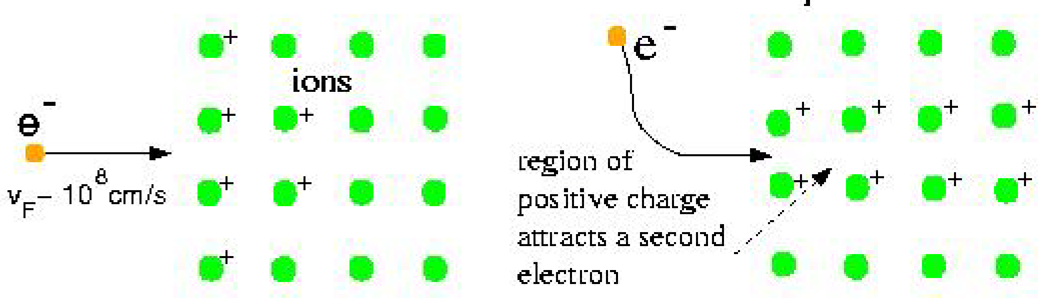
Cooper pair formation BCS theory [1957]

http://superconductors.org/history.htm#resist


The Formation Of "Cooper Pairs"

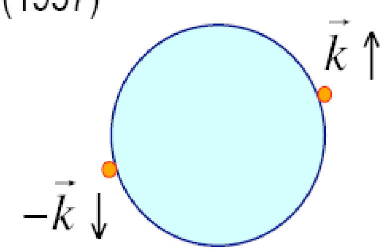
The electron-electron attraction of the Cooper pairs caused the electrons near the Fermi level to be redistributed above or below the Fermi level. Because the number of electrons remains constant, the energy densities increase around the Fermi level and energy gap results.

Electronic Correlations in Conventional superconductors



http://www.chm.bris.ac.uk/webprojects2000/igrant/theory.html

Bardeen-Cooper-Schrieffer
(1957)



pairs of electrons
diametral on Fermi surface;
vanishing total momentum

Superconductivity: the old and the new *A one hundred year voyage of discoveries*

André-Marie Tremblay

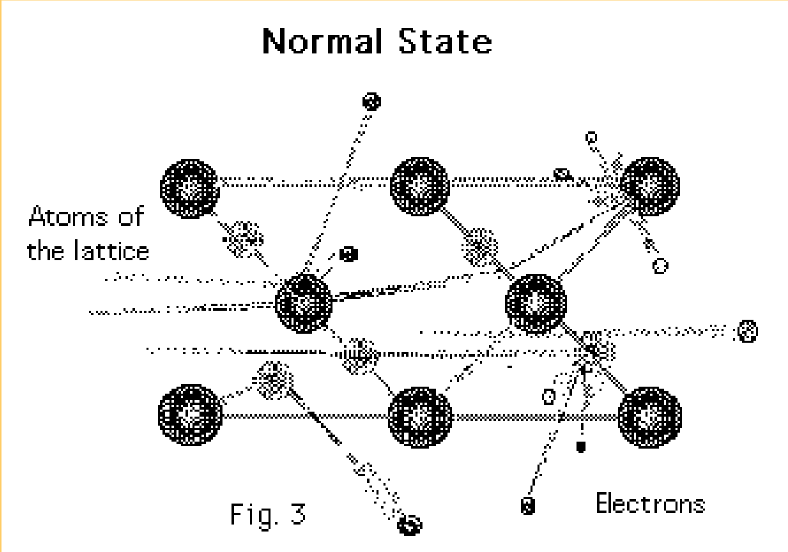


Fig. 3

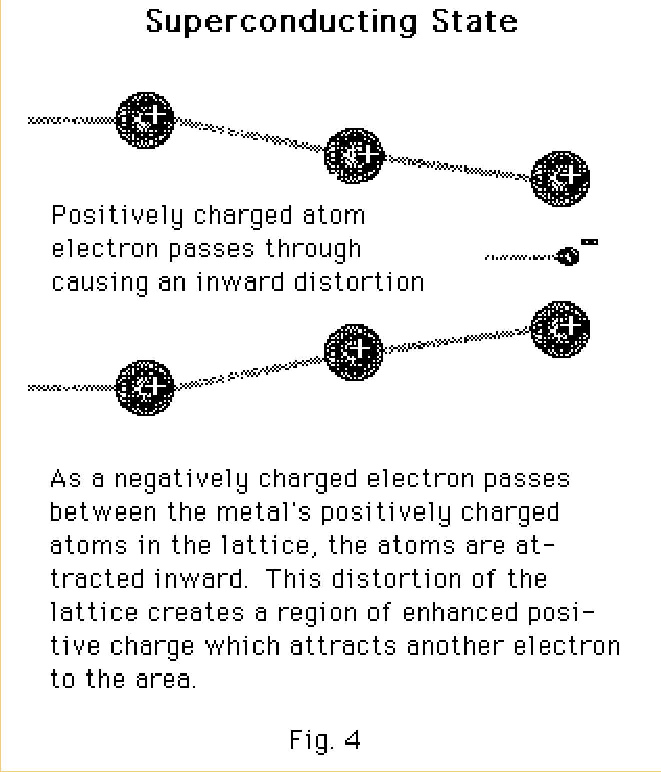


Fig. 4

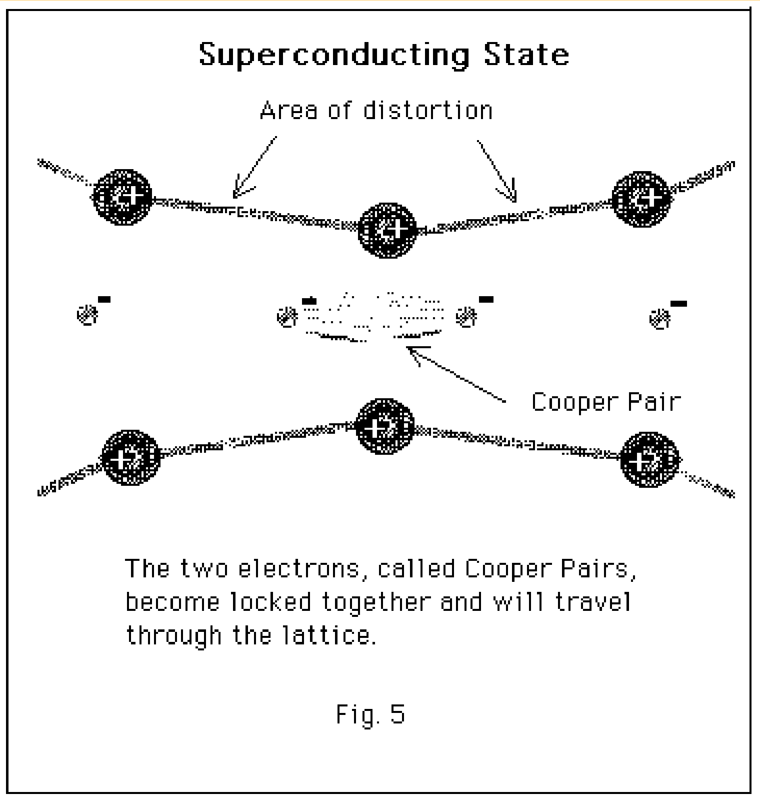
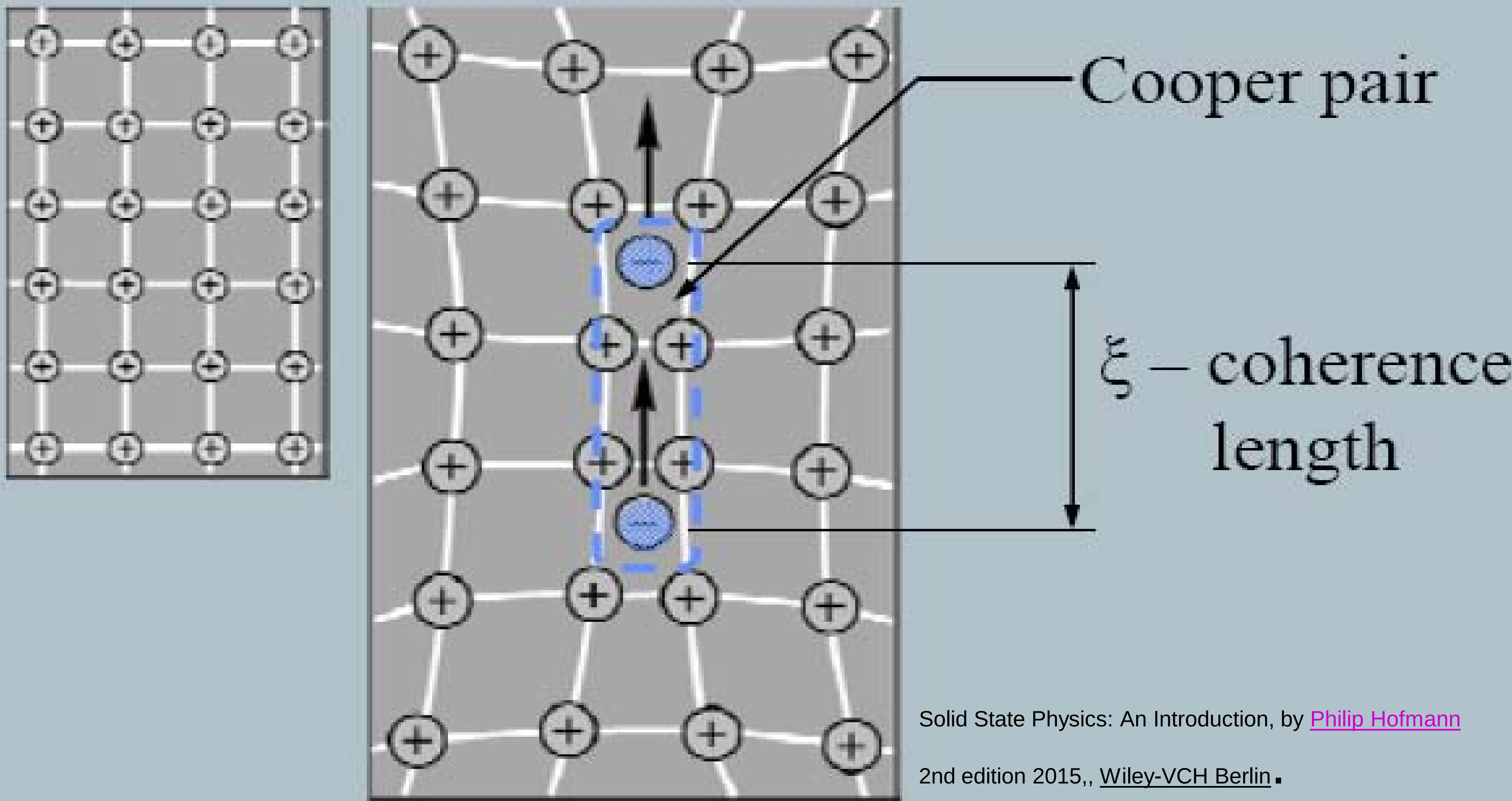


Fig. 5

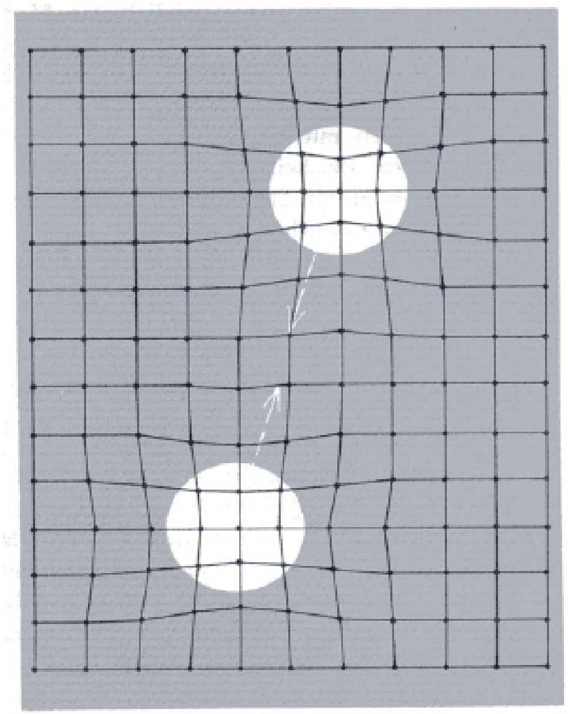
Electron-phonon interaction. Cooper pai

Bardeen – Cooper – Schrieffer (**BCS**) theory:



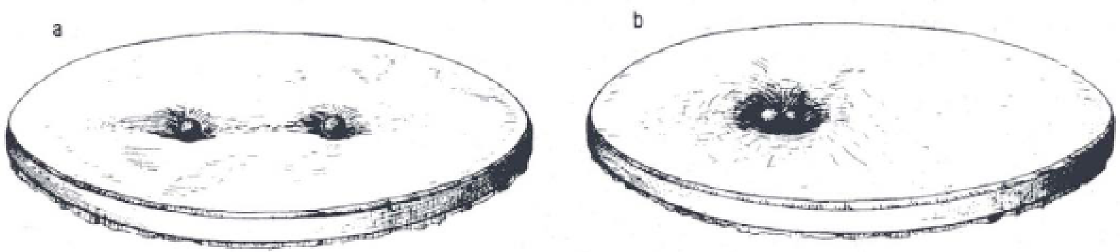
Solid State Physics: An Introduction, by Philip Hofmann

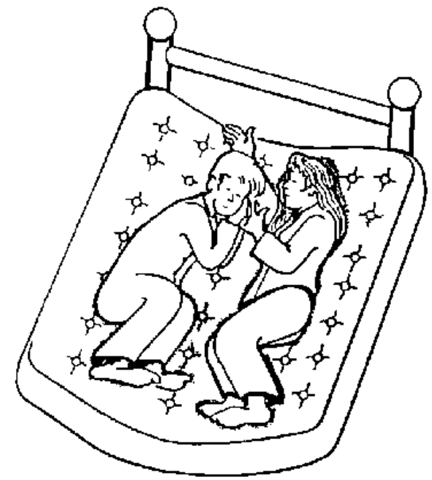
2nd edition 2015,, Wiley-VCH Berlin.



Formation of Cooper pairs requires an interaction between the electrons and the lattice.

•An electron attracts the positive atomic ions causing a dimple. This dimple attracts other electrons because it forms a potential well. As a result, two electrons are attracted instead of repelled.





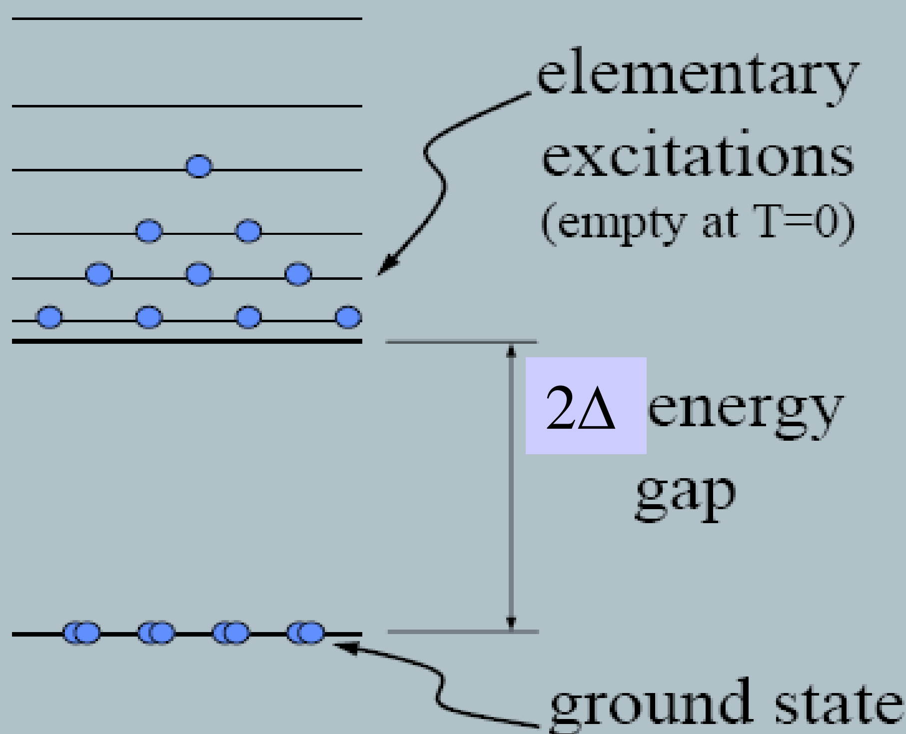
A sagging mattress draws two sleepers together.

Randy Simon, Andy Smith, Superconductors

Energy gap

Superconductors:

(Bardeen – Cooper – Schrieffer (**BCS**) theory)



$$\Delta(0) = \frac{3.52}{2} \underbrace{k_B}_{1.38\cdot 10^{-23}\ \mathrm{J/K}} T_c$$

$$\Delta(T) = \Delta(0)\sqrt{1-(T/T_c)}$$

$$\Delta(T_c/2) = 0.71\,\Delta(0);$$

$$\Delta(2T_c/3) = 0.58\,\Delta(0);$$

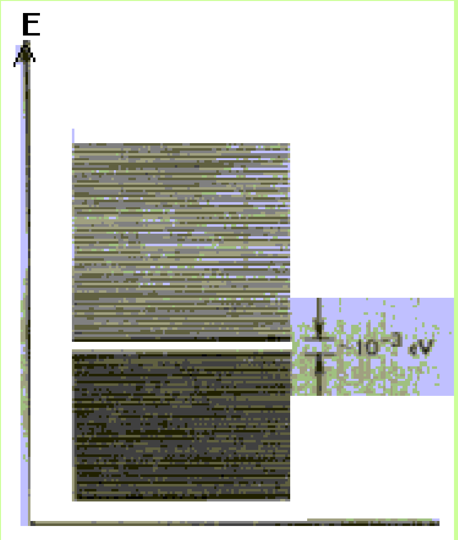
$$\Delta(3T_c/4) = \Delta(0)/2$$

Estimate the value of energy gap in niobium at liquid He temperature (4.2 K) if $T_c^{Nb} \approx 9.25$ K

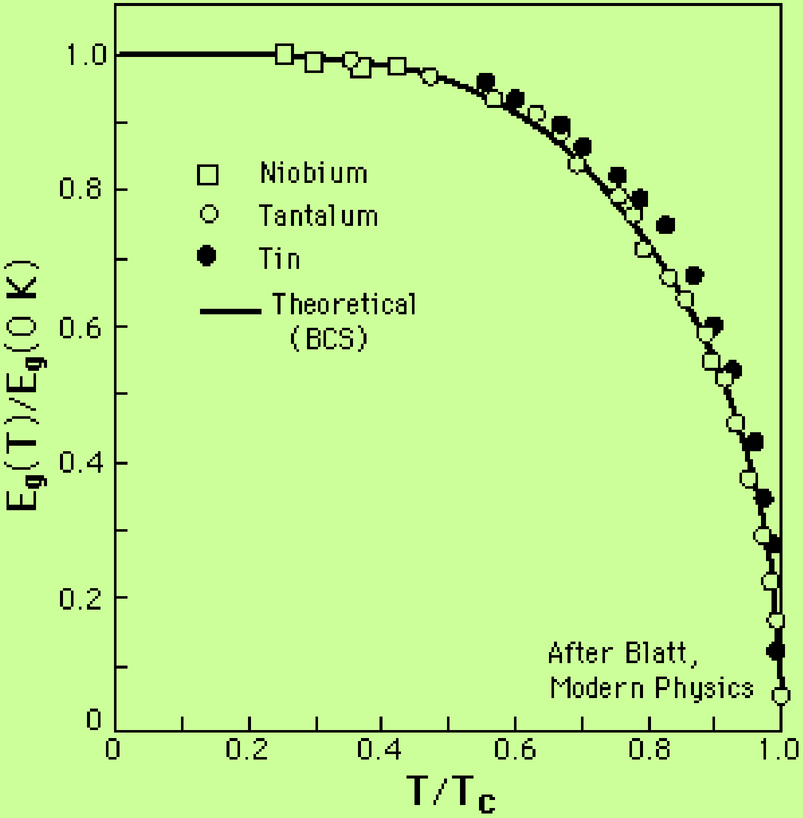
$k_B = 1.38\cdot 10^{-23}$ J/K

$e = 1.6\cdot 10^{-19}$ C

•http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/bcs.html

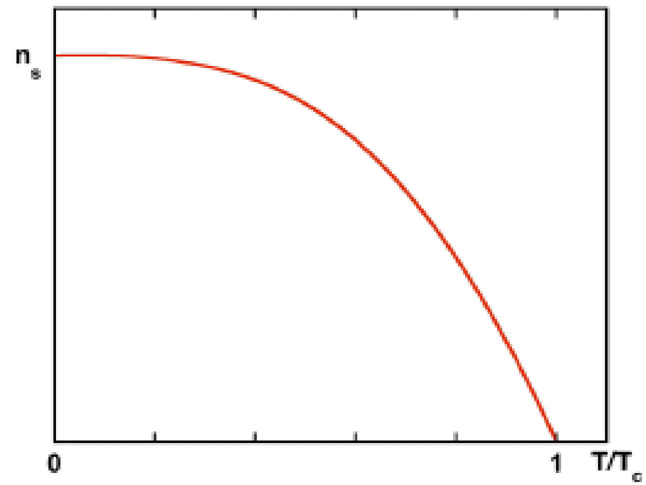


$$E_g = 2\Delta(T)$$
$$E_g(0) = 2\Delta(0) = 3,528 k_B T_c$$



•http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/bcs.html

The change that appear in a superconductor below Tc is the condensate of Cooper pairs: the order parameter could be n_s, the superfluid density



It is more than that ! Condensate of bosons => described by a complex wave function, Ψ,

Wave function of a macroscopic object:
the condensate of all electrons…

$$|\Psi|^2 = n_s,\ \text{and}\ \Psi = \sqrt{n_s}\, e^{i\varphi}$$

The phase is responsible for quantization of the magnetic flux !

$$d\varphi = \frac{1}{\hbar}(\mathbf{p} - e^*\mathbf{A}).\mathbf{dx}\ :\ \text{for pairs of electrons,}\ e^* = 2e$$

around a single vortex, at several λ from the vortex core,

$$\oint \mathrm{d}\varphi = 2\pi.n = \oint \frac{1}{\hbar}(\mathbf{p} - 2e\mathbf{A}).\mathbf{dx} = \frac{2e}{\hbar}\int B.dS$$

$$\Phi = n\Phi_0,\ \text{where}\ \boxed{\Phi_0 = \frac{h}{2e} = 2.10^{-15}\,Wb,}\ \text{is the flux quantum}$$

$$\boxed{\Phi = \int B\,dS}$$

For a vortex, $\Phi = \Phi_0$

BCS Theory of Superconductivity T. Burgener, http://katzgraber.org/teaching/ss07/files/burgener.pdf

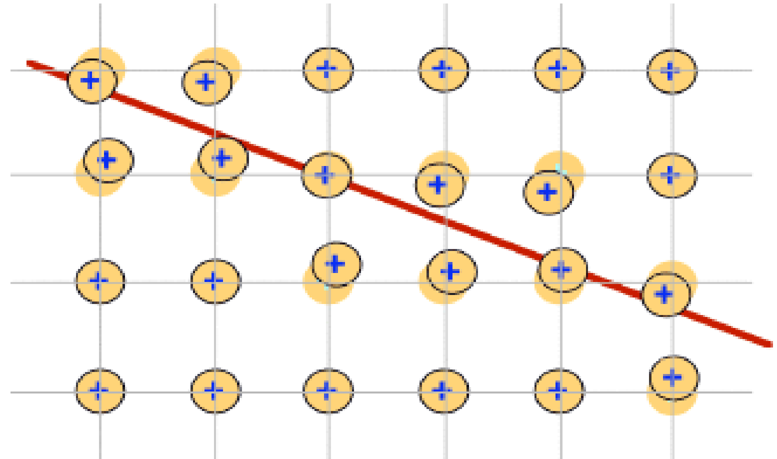
Chang; http://phy.ntnu.edu.tw/~changmc/Teach/SS/SS_note/chap10.pdf

3. Some Equations : the microscopic BCS theory

Cooper pairs: a weak interaction between electrons makes leads to a bounding state, of binding energy

$$\Delta \approx \hbar\omega_D \exp\left(-\frac{2}{\rho_d |V_0|}\right), \text{ with extension } \xi_0 \approx \frac{1}{(\Delta k)} \approx \frac{1}{(\Delta\varepsilon)}\left(\frac{\partial\varepsilon}{\partial k}\right) \approx \frac{\hbar v_F}{\Delta}$$

Mechanism for the interaction:
electron-phonon interaction, or
"polarizability of the lattice"->
favored for "time reversed states"



Bound state: classified by its "orbital momentum":

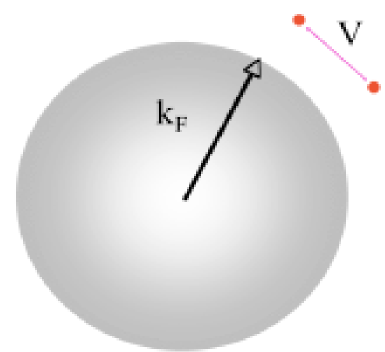
$$\psi(1,2) = -\psi(2,1), \text{ and } \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$= \varphi(\mathbf{r})\chi(\sigma_1,\sigma_2), \text{ with } \chi(\sigma_1,\sigma_2) = |S=0\rangle \text{ or } |S=1\rangle$$

$$\varphi(\mathbf{r}) = f(r)Y_l^m\left(\frac{\mathbf{r}}{r}\right), \text{ if } l=0,\ 2\ldots \text{ then } S=0 \text{ (singlet)}$$

spherical harmonic function

$$\text{if } l=1,\ 3\ldots \text{ then } S=1 \text{ (triplet)}$$



if $l=0$: s-wave superconductivity

if $l>0$, unconventional superconductivity

$l=1$: p-wave (superfluid He^3), $l=2$: d-wave (high-Tc)...

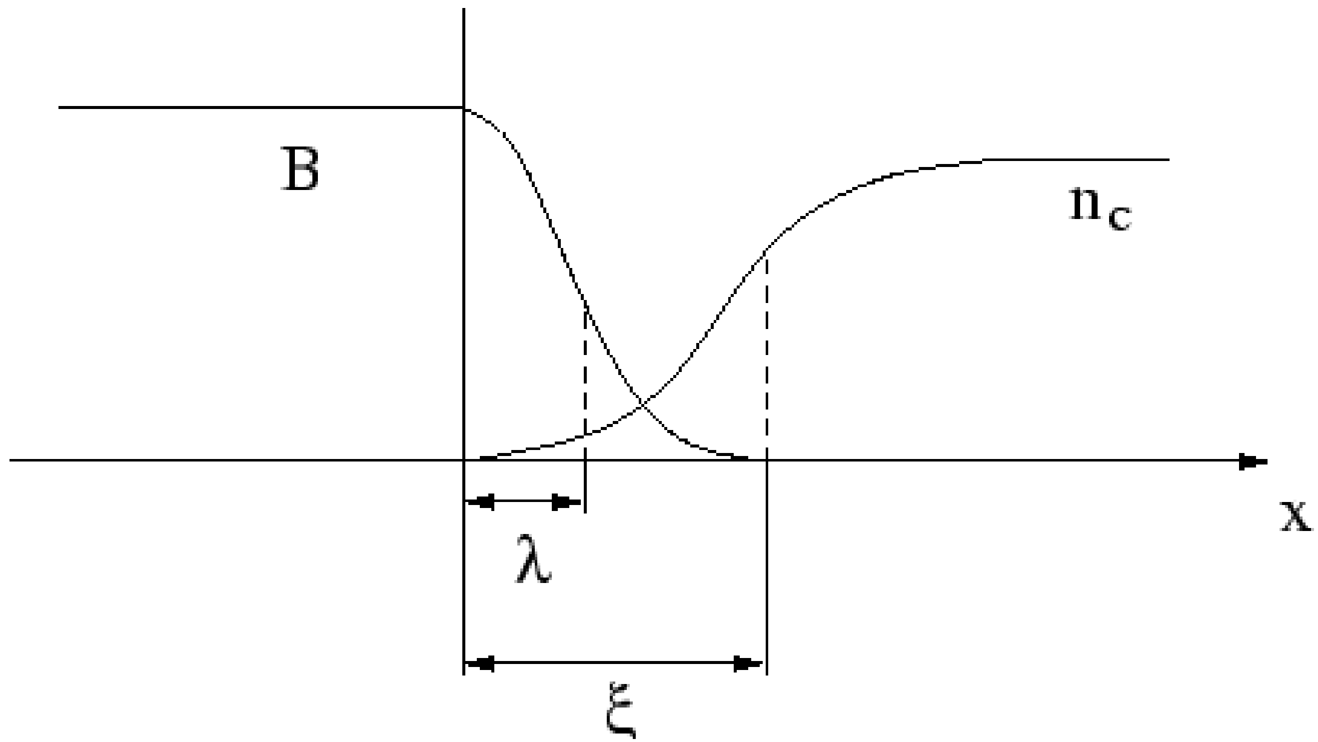


Figure 2.2: The exponential drop of the magnetic field and the rise of the Cooper-pair density at a boundary between a normal conductor and a superconductor.

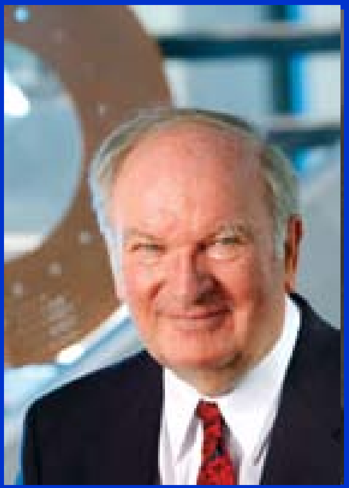
from Kittel

John Bardeen

(May 23, 1908 – January 30, 1991)

Leon Cooper

(born February 28, 1930)

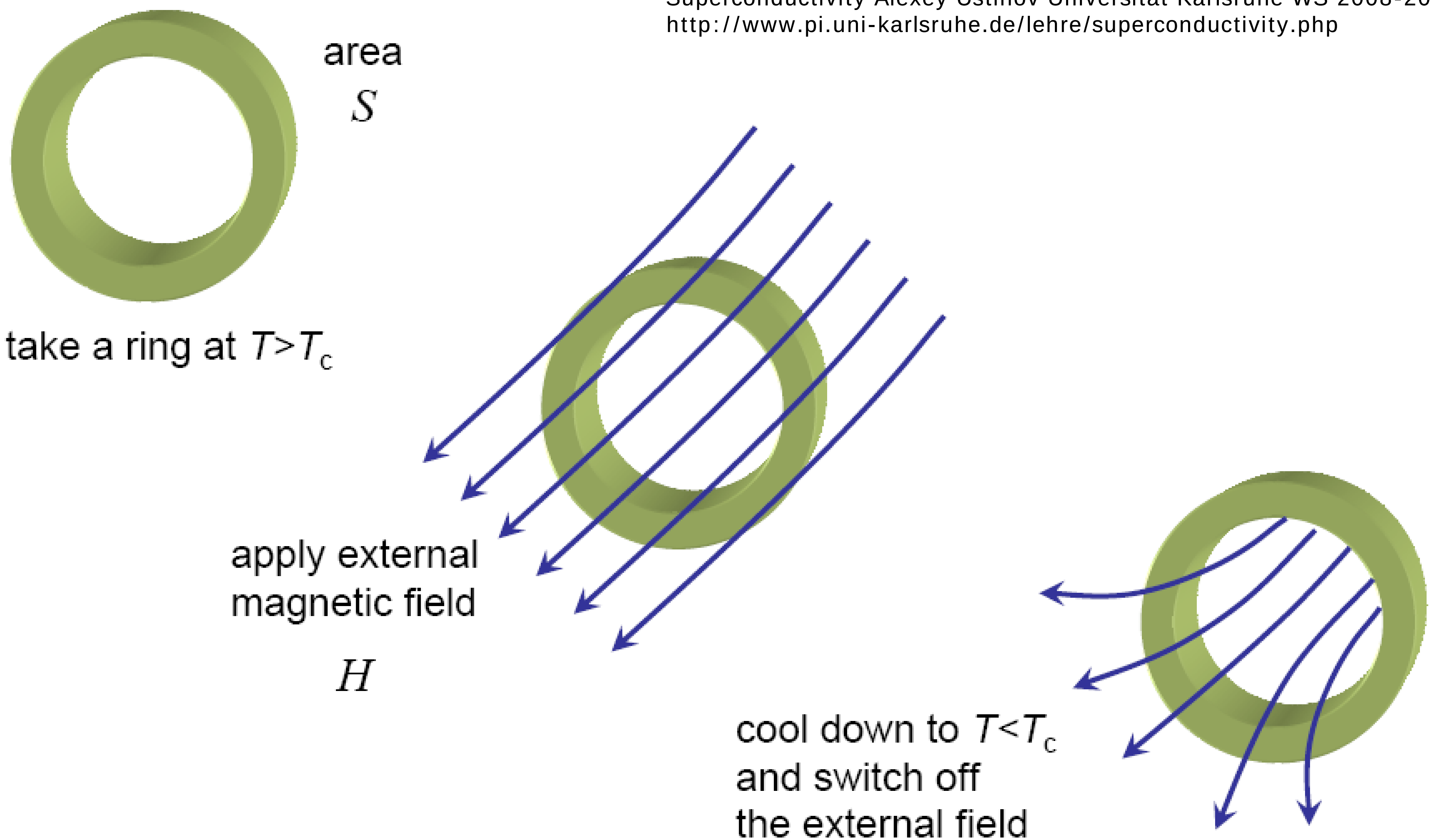


Bob Schrieffer

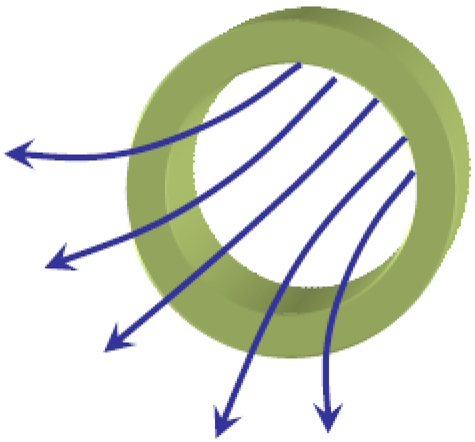
(born May 31, 1931)

Magnetic flux quantization

Superconductivity Alexey Ustinov Universität Karlsruhe WS 2008-2009
http://www.pi.uni-karlsruhe.de/lehre/superconductivity.php



Magnetic flux quantization



$$\Phi = \oint \vec{A} \cdot \mathrm{d}\vec{\ell} \approx H \cdot S$$

$$\Phi = n\left(\frac{h}{2e}\right) = n\Phi_0$$

Experimental result:

$$\boxed{\Phi = n\Phi_0}$$

where $\Phi_0 = 2.07 \times 10^{-15}\,\mathrm{V} \cdot \mathrm{s}$

$$n = 0, \pm 1, \pm 2, \ldots$$

how large is
the magnetic
flux in the ring ?

$$\Phi_0 = \frac{2\pi\hbar}{2e} \cong 2.0678 x 10^{-15} tesla \cdot m^2$$

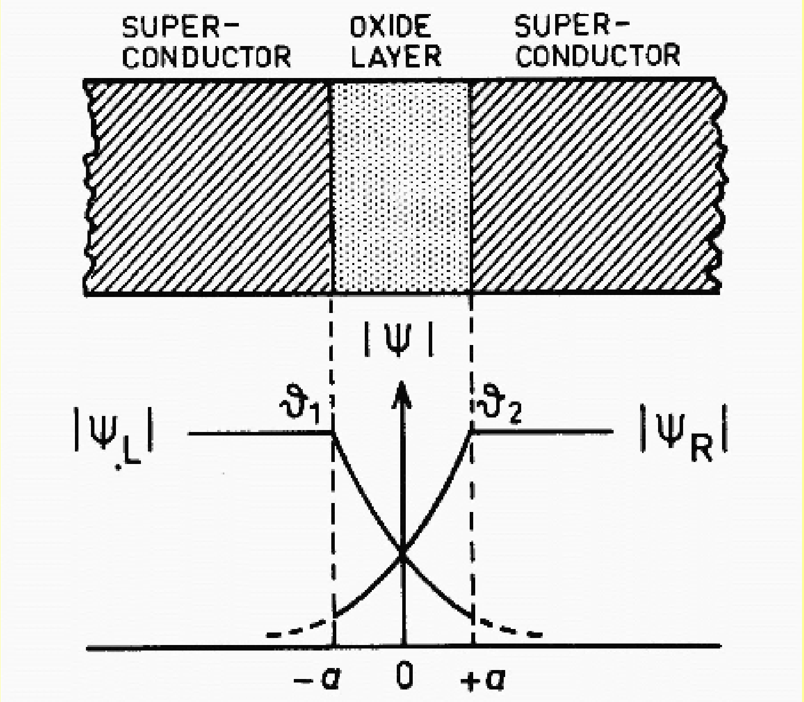
S. Deaver and W.M. Fairbank, *Phys. Rev. Lett.* **7**, 43 (1961)
R. Doll and M. Näbauer, *Phys. Rev. Lett.* **7**, 51 (1961)

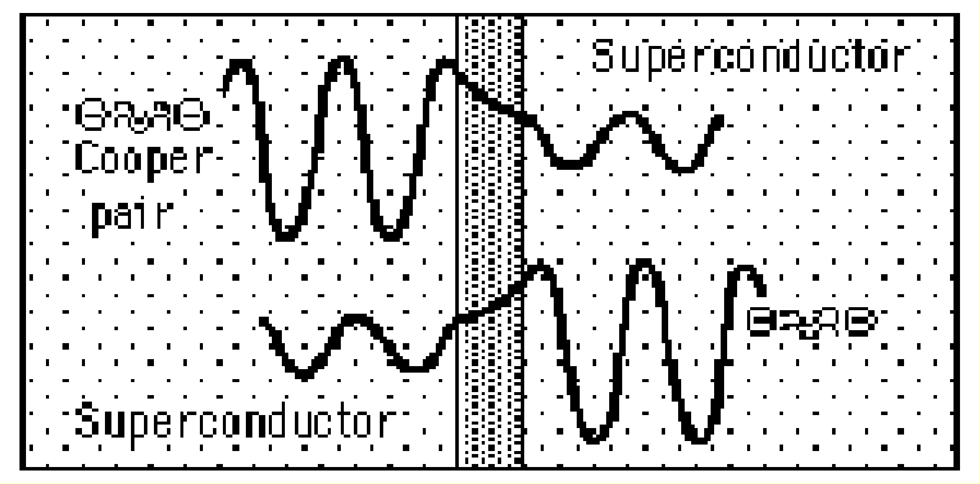
• http://phy.ntnu.edu.tw/~changmc/Teach/SS/SS_note/chap10.pdf

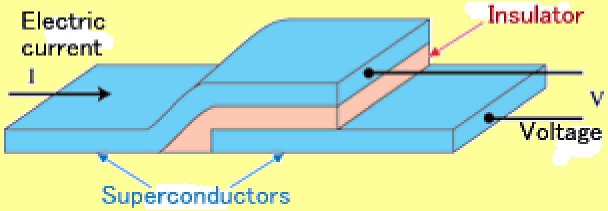
Josephson Tunnelling

•If two superconducting regions are brought together then as they come close electron-pairs will be able to tunnel across the gap and the two electron-pair waves will become coupled. As the separation decreases the strength of the coupling increases.

•The tunnelling of the electron-pairs across the gap carries with it a superconducting current as predicted by B.D. Josephson and is called ``Josephson Tunnelling'' with the junction between the two superconductors called a ``Josephson Junction''.







Like a superconductor this gap has a critical current. If a supercurrent, i_s , flows across a gap between regions with a phase difference, $\Delta\varphi$, it is related to the critical current, by:

$$i_s = i_c \cdot \sin(\Delta\varphi)$$

The current flow is controlled by the phase difference between SC1 and SC2

DC Josephson effect

There is an electrical DC current flowing through a Josephson contact even in the absence of an external voltage.

AC Josephson effect

By applying a DC voltage V > V_C (critical voltage) on the contact, an oscillating current flow is generated with the frequency: $\boxed{\omega = \frac{2eV}{\hbar}}$

$$\boxed{i_s = i_c \cdot \sin(\frac{2eV}{\hbar} t + \varphi_0)}$$

$$\frac{2e}{\hbar} = 3.04 \cdot 10^9 \,\mathrm{Hz/\mu z}$$

If V ~ 1mV → $\omega \sim 10^{12}$ Hz

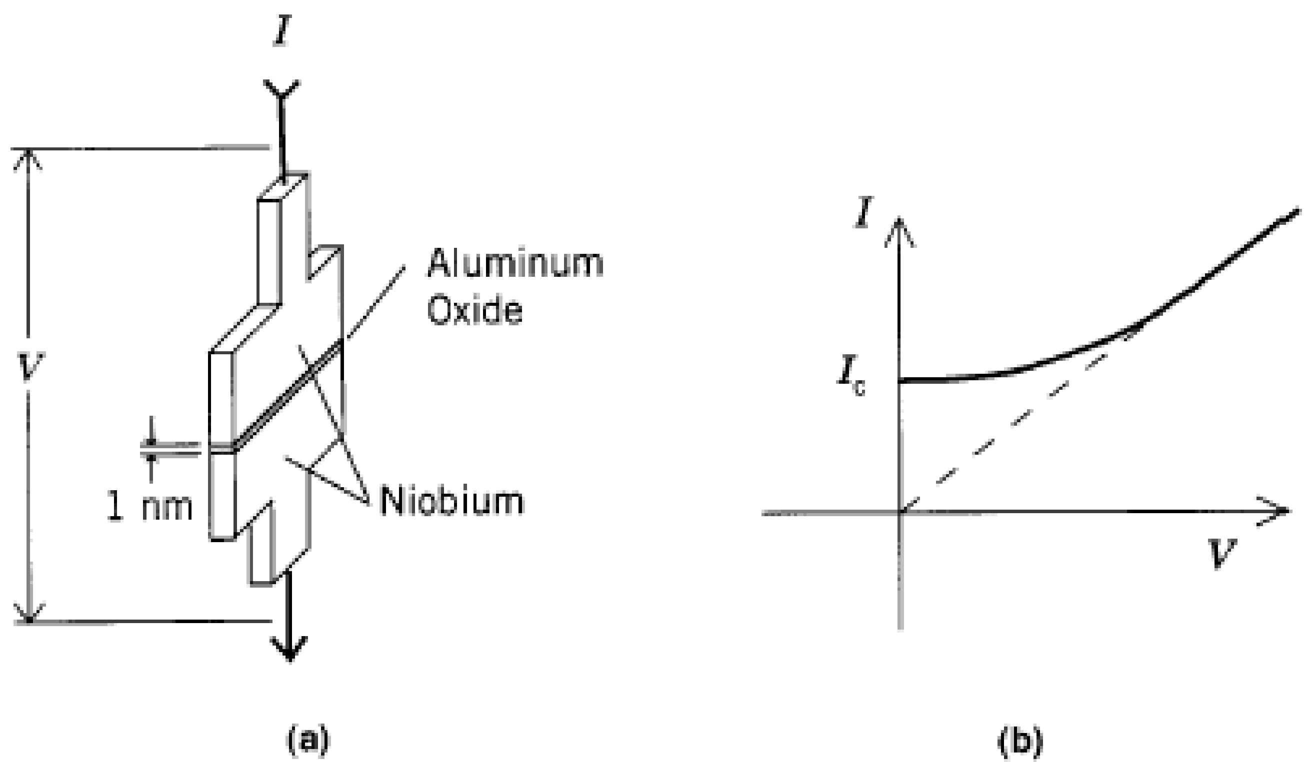


FIGURE 48.11 The Josephson junction in (a) consists of a superconductor such as niobium separated by a thin insulation layer. The voltage (V) vs. current (I) curve in (b) shows that a superconducting current flows through the junction with zero volts across the junction.

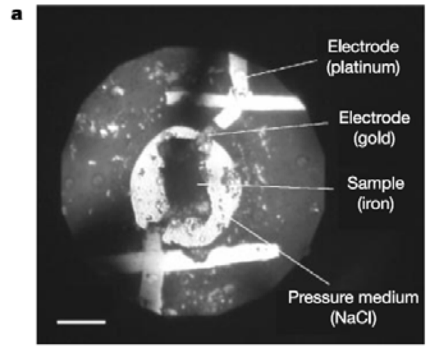
http://web.mit.edu/6.763/www/FT03/Lectures/Lecture11.pdf

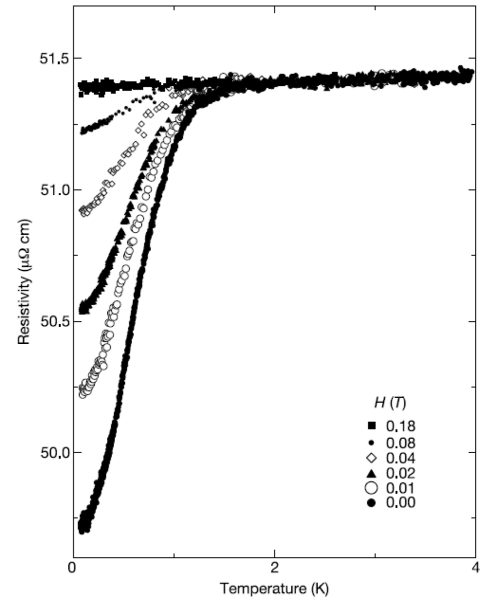
Not superconducting?

- Iron
  - is a ferromagnet (in the ambient pressure phase)
  - all spin are parallel for $T<T_{Curie}=1000K$
- Many elements are insulating
  - but can be doped: boron doped Diamond
- Many elements need high pressure
  - different lattice structure
  - larger coordination number

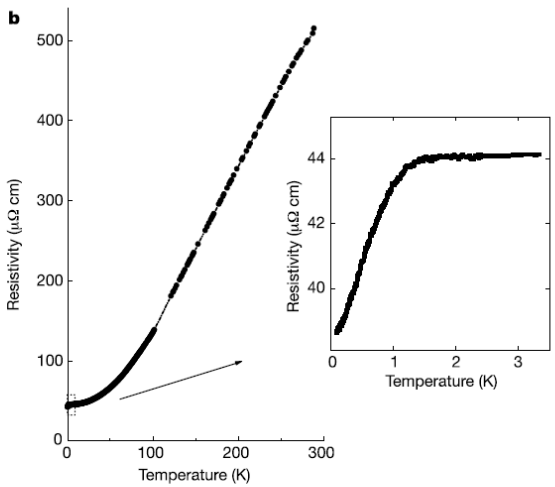
Iron sc exists under pressure

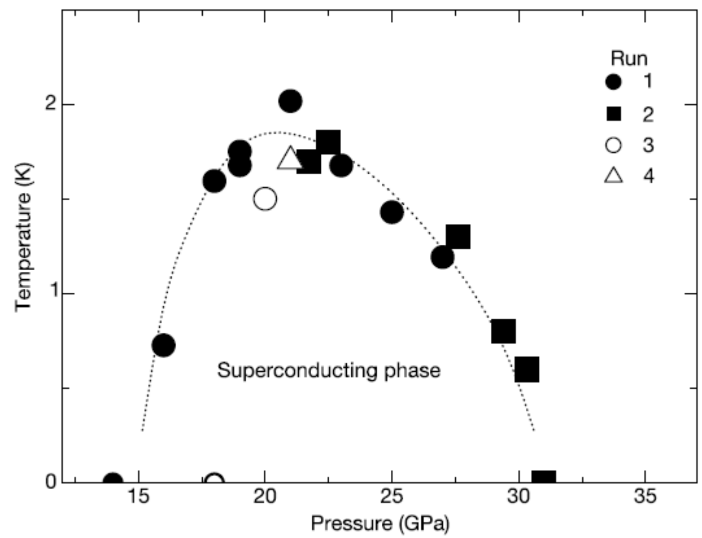
At pressures above 10 GPa, iron is known to transform to a non-magnetic structure and the possibility of superconductivity in this state has been predicted.





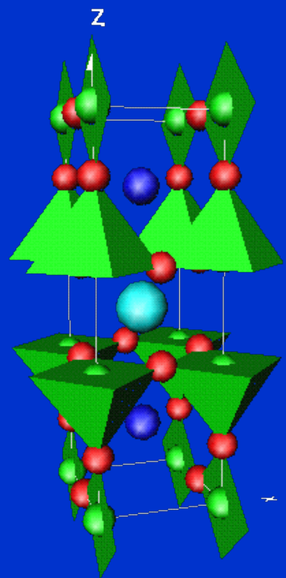
K Shimizu et al. Nature 412 (6844), 316-318. 2001 Jul 19.



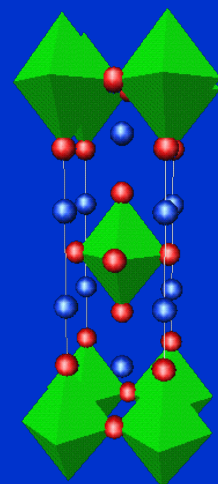


Material Overviewof High-*Tc* Superconductive Copper Oxides, MaaritKARPPINEN

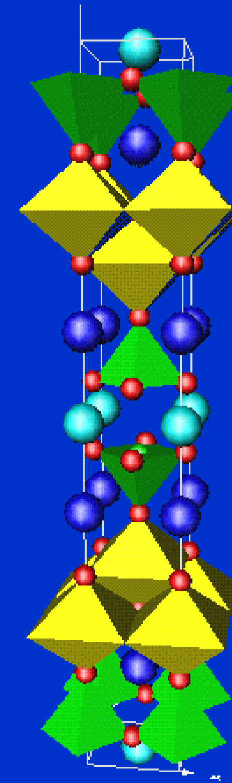
High Temperature Superconductors



$YBCO_7$



LSCO



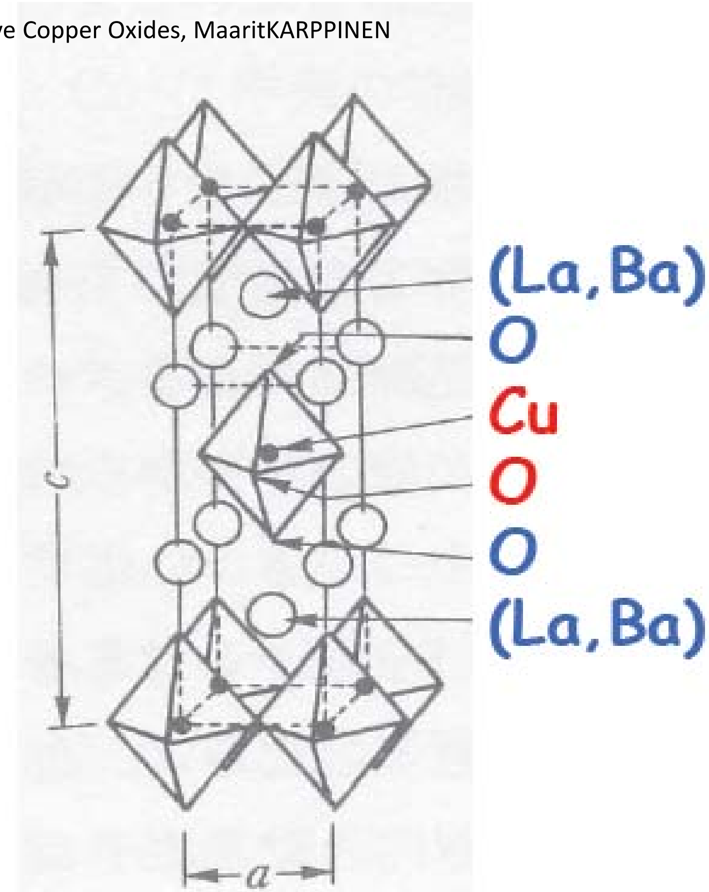
HgCuO

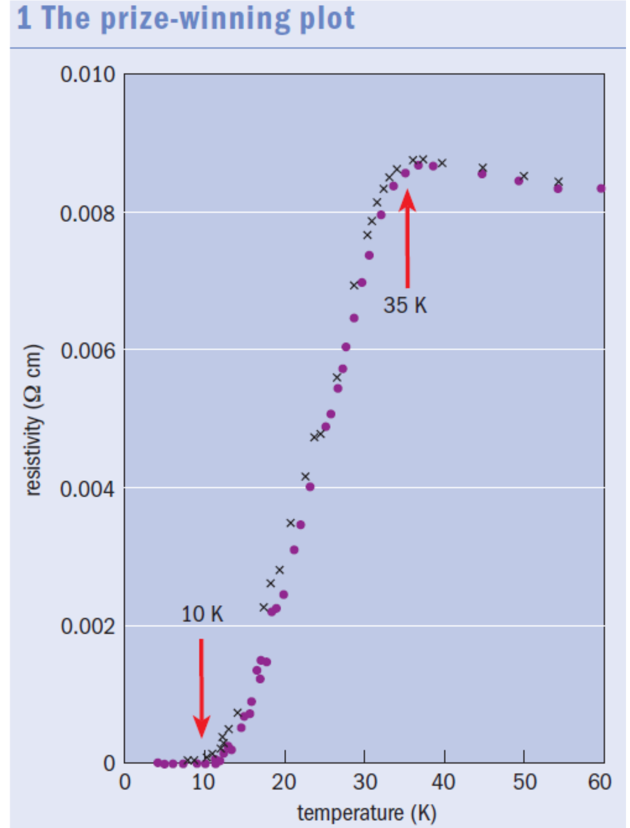
Georg Bednorz
and Alex Muller

received the
Nobel Prize 1987
for discovery of
the first of the
copper-oxide
superconductors

Bednorz Muller 1987

Material Overviewof High-T_c Superconductive Copper Oxides, MaaritKARPPINEN



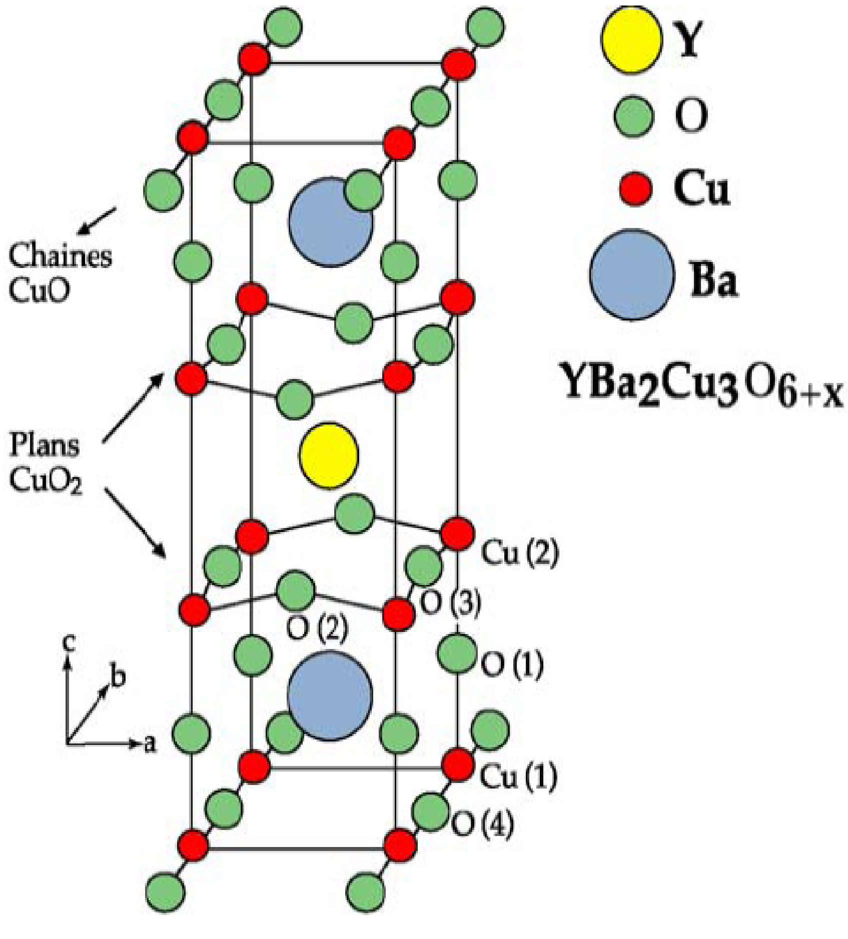


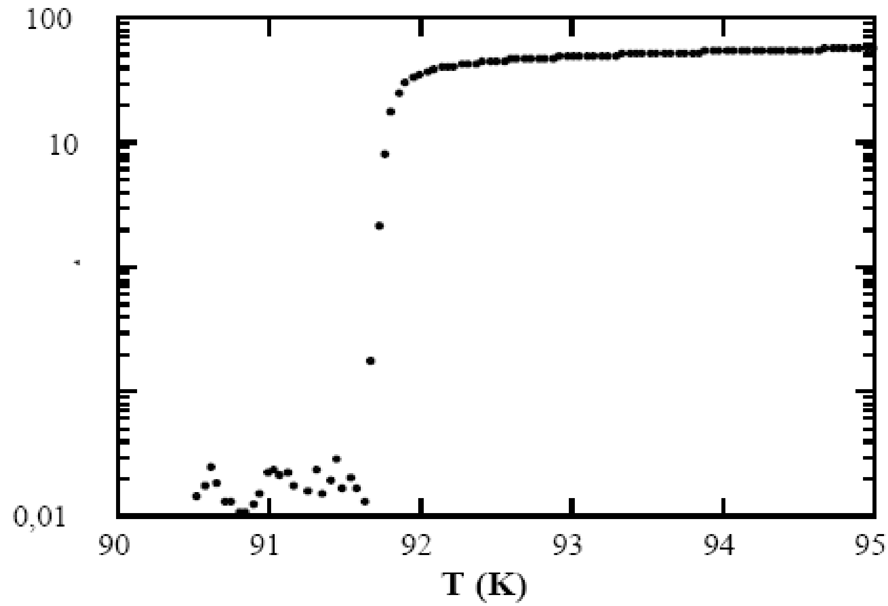
$(La,Ba)_2CuO_{4+\delta}$

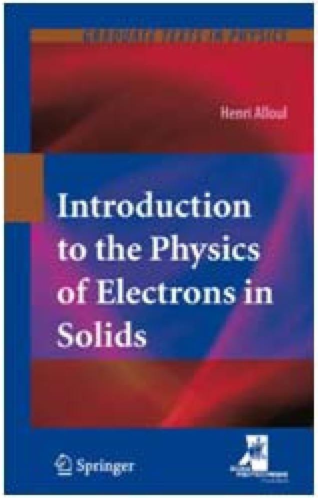
$T_c \approx 35$ K

Material Overviewof High-*Tc* Superconductive Copper Oxides, MaaritKARPPINEN

$YBa_2Cu_3O_7$



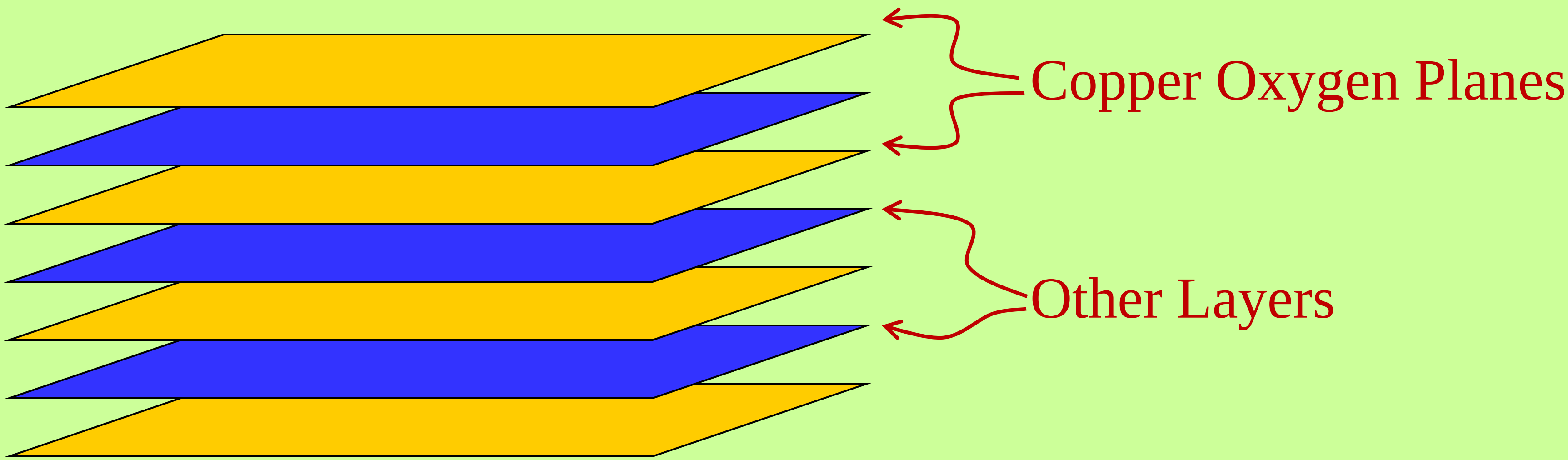




taken from

The levitation of a magnet by a superconductor provides one of the most amazing manifestations of superconductivity, a source of wonder to all that witness it for the first time, and especially to students who have just themselves synthesised the superconducting ceramic $YBa_2Cu_3O_7$. How can three insulating oxides Y_2O_3, CuO, and BaO react in the solid phase to generate a metallic material which becomes superconducting in liquid nitrogen? This illustrates the fact that, in complex systems, properties are very hard to predict from a straightforward understanding of the separate constituents. The richness of condensed matter physics lies in the experimental revelation of spectacular phenomena arising from complex systems, which leads us to seek rational explanations

High Temperature Superconductors

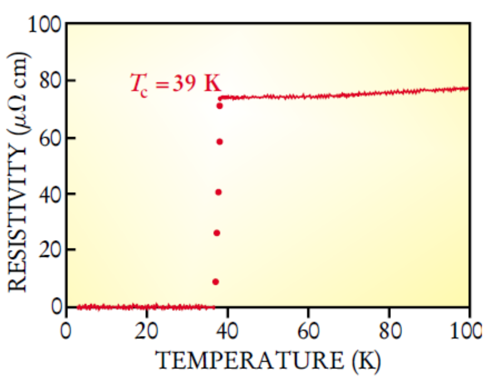


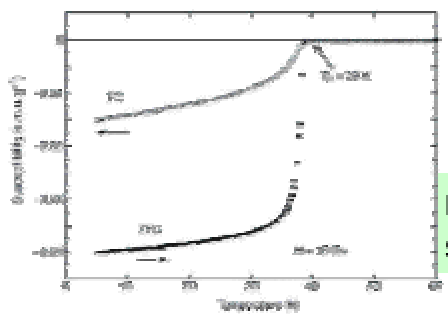
Layered structure → quasi-2D system

H. Kumakura
National Institute for Materials Science
ASC2008-Short course, Chicago, August. 17,2008

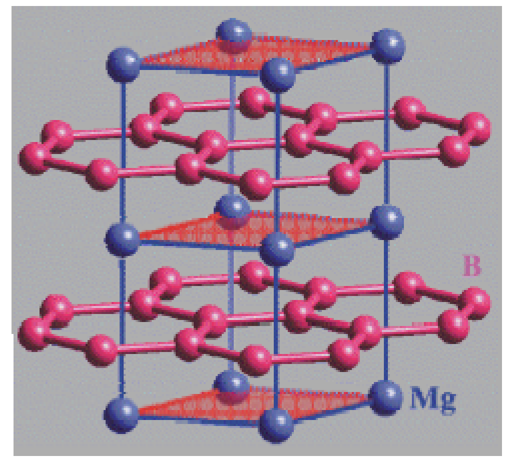
Everyone can "find" a HTSC

- "On January 10, 2001, Prof. Jun Akimitsu (Aoyama Gakuin Univ., Japan) announced, that his students discovered superconductivity in MgB_2 at T_c = 39 K."





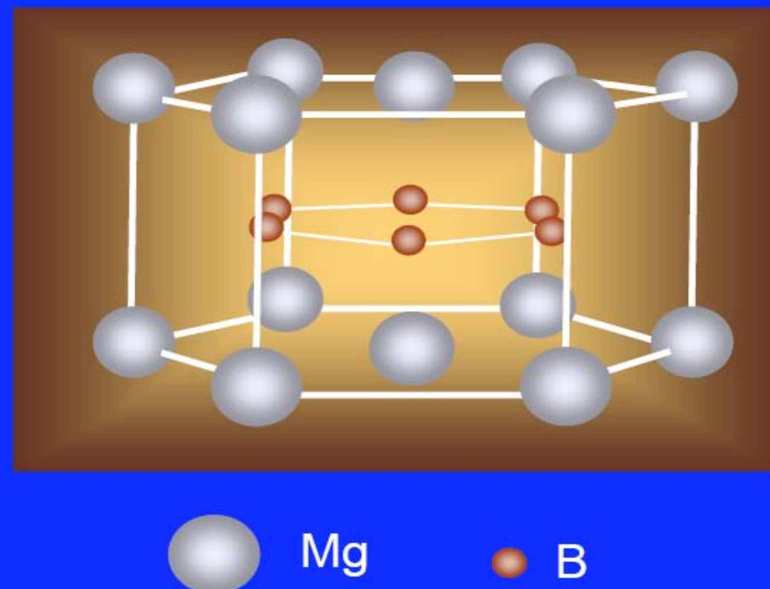
magnetic susceptibility



layered structure

MgB_2

MgB_2

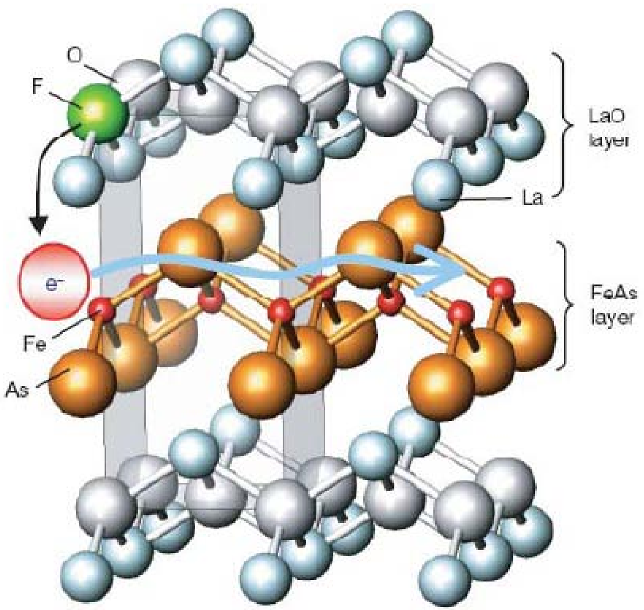


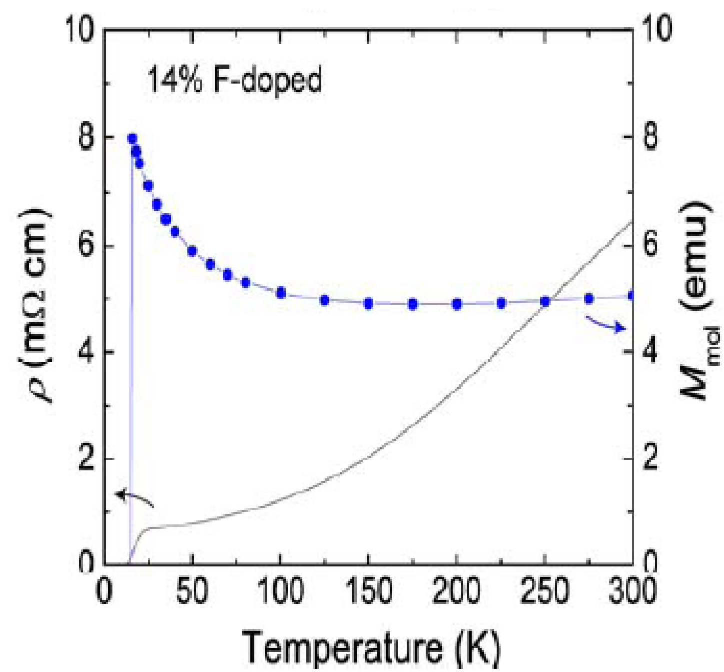
The show must go on…

Pnictide superconductors: $LaFeAsO_{1-x}F_x$

A new class of high-temperature superconductors has been discovered in layered iron arsenic compounds. Results in this rapidly moving field may shed light on the still unsolved problem of high-temperature cuprate superconductivity.

The parent compound, LaOFeAs , was not superconducting, but upon replacing some of the oxygen by fluorine, the material became superconducting.





- replacing O with F, Tc is enhanced;
- T_c~ 26 K when O replaced by F (doping with electrons);
- T_c ~ 55 K when La replaced by Sm;

J|A|C|S COMMUN|CAT|ONS

Published on Web 02/23/2008

Iron-Based Layered Superconductor La[$O_{1-x}F_x$]FeAs ($x = 0.05-0.12$) with $T_c = 26$ K

Yoichi Kamihara,*,† Takumi Watanabe,‡ Masahiro Hirano,†,§ and Hideo Hosono†,‡,§

ERATO-SORST, JST, Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, Materials and Structures Laboratory, Tokyo Institute of Technology, Mail Box R3-1, and Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan

Received January 9, 2008; E-mail: hosono@msl.titech.ac.jp

Sm → $T_C = 55$K

Superconductivity and Phase Diagram in Iron-based Arsenic-oxides $ReFeAsO_{1-\delta}$ (Re = rare earth metal) without Fluorine Doping

Zhi-An Ren*, Guang-Can Che, Xiao-Li Dong, Jie Yang, Wei Lu, Wei Yi, Xiao-Li Shen, Zheng-Cai Li, Li-Ling Sun, Fang Zhou, Zhong-Xian Zhao*

National Laboratory for Superconductivity, Institute of Physics and Beijing National Laboratory for Condensed Matter Physics, Chinese Academy of Sciences, P. O. Box 603, Beijing 100190, P. R. China

April, 2008

Some Known High Temperature Superconductors

$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$	138 K (record-holder)
$HgBa_2Ca_2Cu_3O_8$	133-135 K
$HgBa_2Ca_3Cu_4O_{10+\delta}$	125-126 K
$HgBa_2Ca_{1-x}Sr_xCu_2O_{6+\delta}$	123-125 K
$HgBa_2CuO_{4+\delta}$	94-98 K
$Tl_2Ba_2Ca_2Cu_3O_{10}$	126 K
$Tl_{1.6}Hg_{0.4}Ba_2Ca_2Cu_3O_{10+\delta}$	123 K
$TlBa_2Ca_2Cu_3O_{9+\delta}$	120 K
$Tl_{0.5}Pb_{0.5}Sr_2Ca_2Cu_3O_9$	112 K
$TlBa_2Ca_3Cu_4O_{11}$	112 K
$Tl_2Ba_2Ca_3Cu_4O_{12}$	127 K
$Bi_{1.6}Pb_{0.6}Sr_2Ca_2Sb_{0.1}Cu_3O_y$	115 K (thick film on MgO substrate)
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110 K
$Bi_2Sr_2CaCu_2O_9$	110 K
$Bi_2Sr_2Ca_{0.8}Y_{0.2}Cu_2O_8$	95-96K
$Bi_2Sr_2CaCu_2O_8$	91-92K
$Ca_{1-x}Sr_xCuO_2$	110 K (Highest Tc ternary/quaternary compound)
$TmBa_2Cu_3O_7$	90 - 101 K
$GdBa_2Cu_3O_7$	94 K
$YBa_2Cu_3O_{7+\delta}$	93 K
$Y_2Ba_4Cu_7O_{15}$	93 K
$Yb_{0.9}Ca_{0.1}Ba_{1.8}Sr_{0.2}Cu_4O_8$	86 K
$YbBa_{1.6}Sr_{0.4}Cu_4O_8$	78 K

Superconductivity in various materials

Elements -- many elements superconduct Al, Pb, S, Sn, Ti, Mo, Zr, ….

First observed superconductor -- Mercury $T_c = 4.15$ K

Highest T_c -- Niobium (at zero pressure) $T_c = 9.2$ K

Binary Compounds

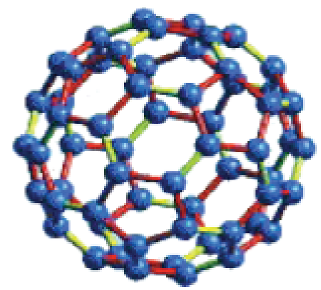
Nb_3Ge thin film with $T_c = 23$ K (held world record for nearly 20 years)

MgB_2 -- Discovered in 2001 has $T_c = 40$ K

Organic Compounds

$(BEDT\text{-}TTF)_2Cu[N(CN)_2]Cl$ -- $T_c = 12.8$ K

C_{60} -- $T_c = 60$ K



Copper Oxides

Superconduct at temperatures as high as 150 K

- Key point is that superconductivity above the boiling point of liquid nitrogen will reduce the cost for technological uses to an acceptable level.

Cost of liquid helium (boils at 4.2 K) = same as good wine

Cost of liquid nitrogen (boils at 77 K) = same as bottled water

Chang; http://phy.ntnu.edu.tw/~changmc/Teach/SS/SS_note/chap10.pdf

Applications

- Wire
  Power Transmission

- Transformers
  Reduce energy loss.

- Transportation
  MagLev trains in Germany and Japan

- Motors and Generators
  Use of superconductors results in less energy loss.

- Magnets
  Can maintain magnetic field forever!

- Ultra sensitive magnetic field detection
  Enables technologies such as MRI.

Magnetic Levitation

The Yamanashi MLX01 Maglev Train achieved a speed of 343 miles/h on April 17 1999.

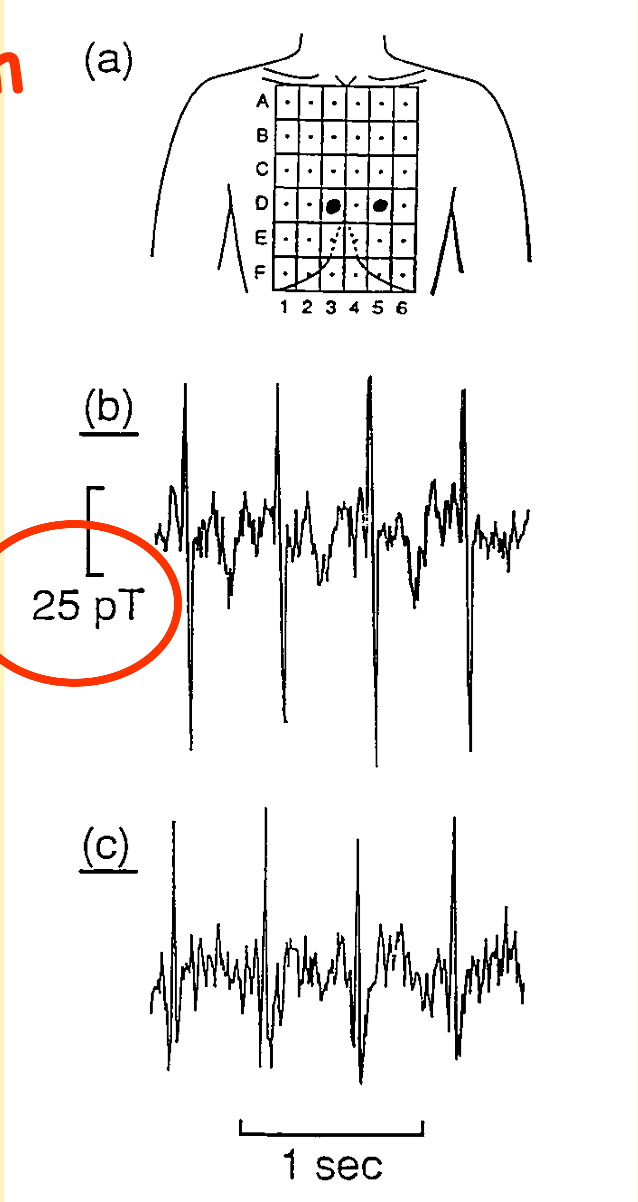
www.rtri.or.jp/rd/maglev/html/english/maglev_frame_E.html

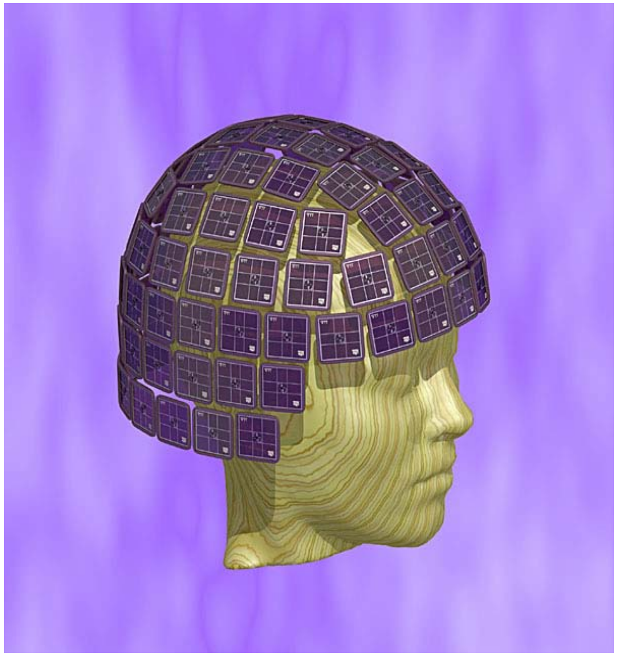
The application of Quantum Mechanics to measure tiny magnetic fields: SQUIDs

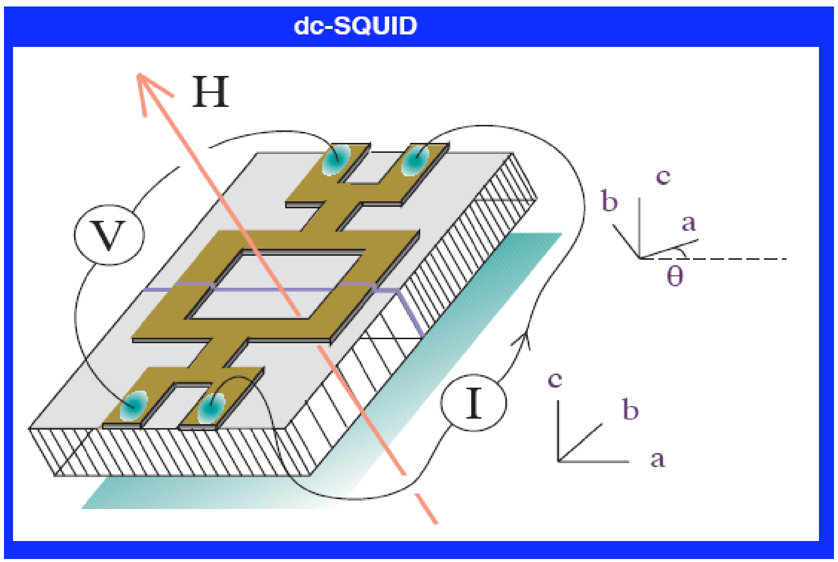
The quantum of flux, $\Phi_o = h / 2e$, has the tiny value 2×10^{-15} T.m^2

This is one millionth of the flux due to the earth's field through the hole with 1cm diameter!

Superconducting Quantum Interference Devices can measure tiny fields – such as those due to currents flowing in your heart muscle



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Superconductivity/fluidity

- 1913 Heike Kamerlingh-Onnes, ".. properties of matter at low temperatures, .. production of liquid helium"

  and superconductivity in Hg in 1911

- 1972 Bardeen (1st: 1956: transistor), Cooper, Schrieffer, ".. **BCS**-theory"
- 1973 Esaki, Giaever, Josephson, ".. tunneling in semi- and superconductors"
- 1978 Kapitsa, ".. basic inventions and discoveries .. low-temperature physics"

- 1987 Bednorz, Müller, ".. superconductivity in ceramic materials"

- 2003 Abrikosov, Ginzburg, Leggett, ".. theory of superconductors and superfluids"

Related and Fundamental

- 1962 Landau, “.. theories for condensed matter, especially liquid helium"
- 1977 Anderson, Mott, van Vleck, “.. electronic structure of magnetic and disordered systems"
- 1996 Lee, Osheroff, Richardson, “.. superfluidity in helium-3"
- 1998 Laughlin, Störmer, Tsui, “.. quantum fluid with fractionally charged excitations". (see Quantum Hall effect).
- 2001 Cornell, Ketterle, Wieman, “.. Bose-Einstein condensation in dilute gases .. "

These are 12 Nobel prizes related to superconductivity and quantum states of matter.

Landau

Mott

Anderson

van Vleck

Laughlin

Superconductivity phenomena

- Perfect conductivity $\sigma = \infty$
- Perfect diamagnetism B = 0
- Magnetic field suppresses superconductivity $H_c(T)$, $H_{c1}(T)$, $H_{c2}(T)$
- Magnetic flux is quantized in units of h/2e
- Dynamics of the lattice is important $T_c \propto M^{-\alpha}$
- Energy gap 2Δ
- T_c and energy gap are related
- Superconductivity mechanism in HTS is different from LTS