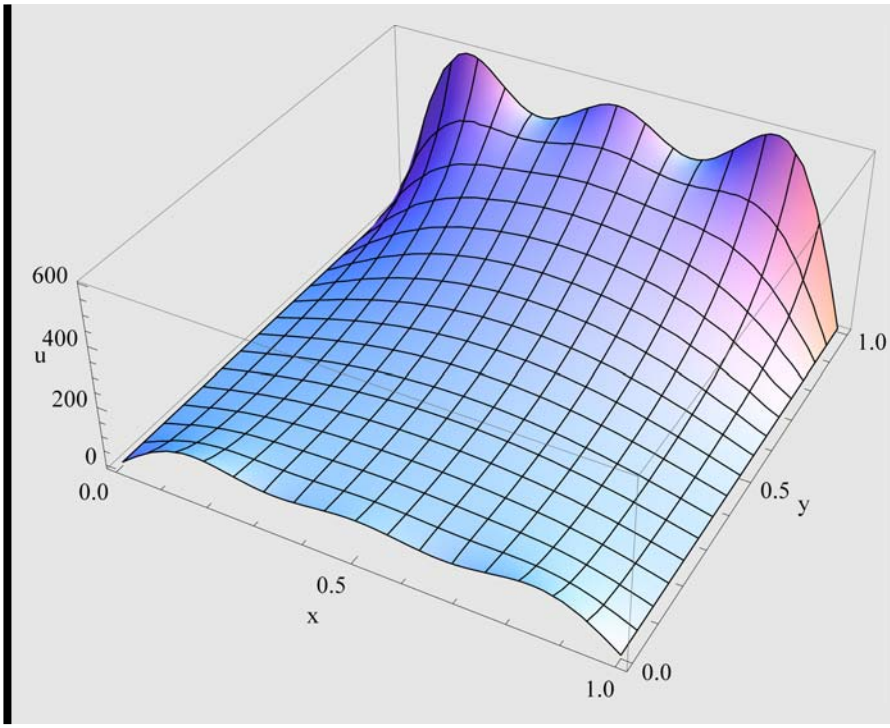
Laplace's Equation

In a Rectangle

Example 1

```
Clear[a, b, A, B, n, u, x, y];
a = 1;
b = 1;
f1[x_] = 100;
f2[x_] = 500;
λ[n_] = n π / a;
A[n_] = 2/a ∫_0^a f1[x] Sin[λ[n] x] dx;
B[n_] = (2/a ∫_0^a f2[x] Sin[λ[n] x] dx - A[n] Cosh[λ[n] b]) / Sinh[λ[n] b];
u[x_, y_, n_] := Sum_{i=1}^{n} Sin[λ[i] x] (N[A[i]] Cosh[λ[i] y] + N[B[i]] Sinh[λ[i] y])
```

```
Plot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"}]
```

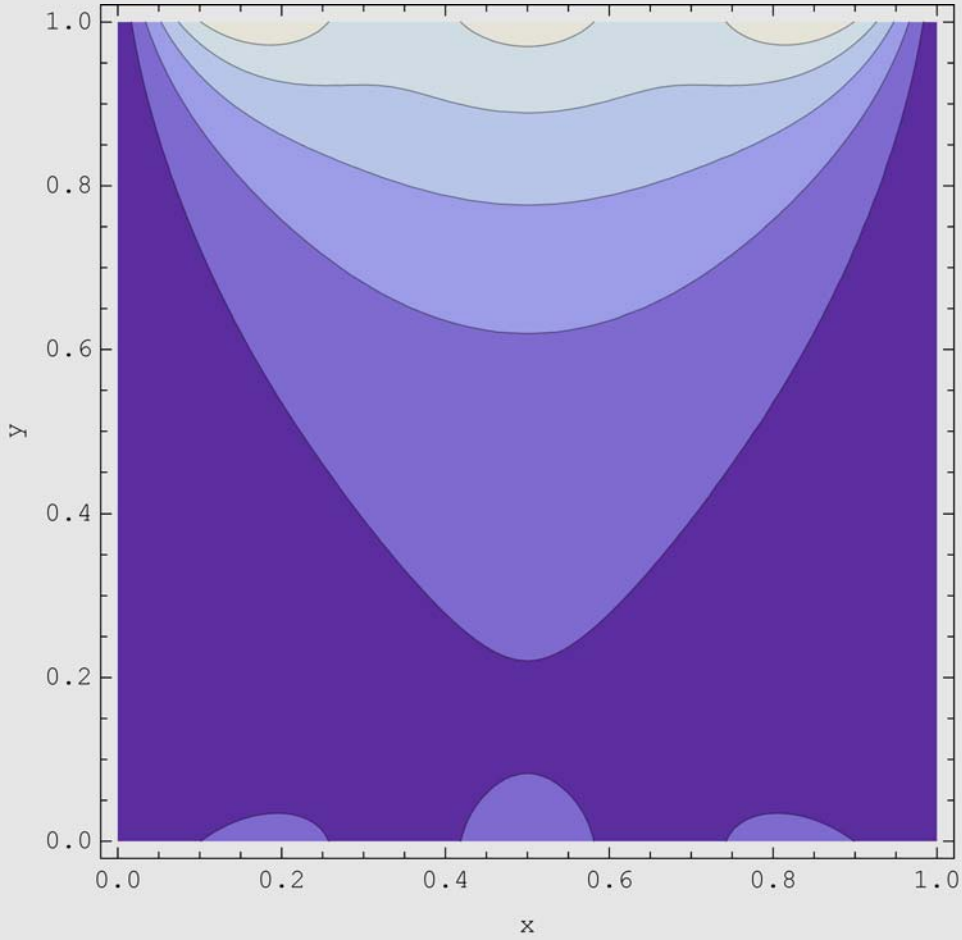



```
Needs["BarCharts`"]; Needs["Histograms`"]
```

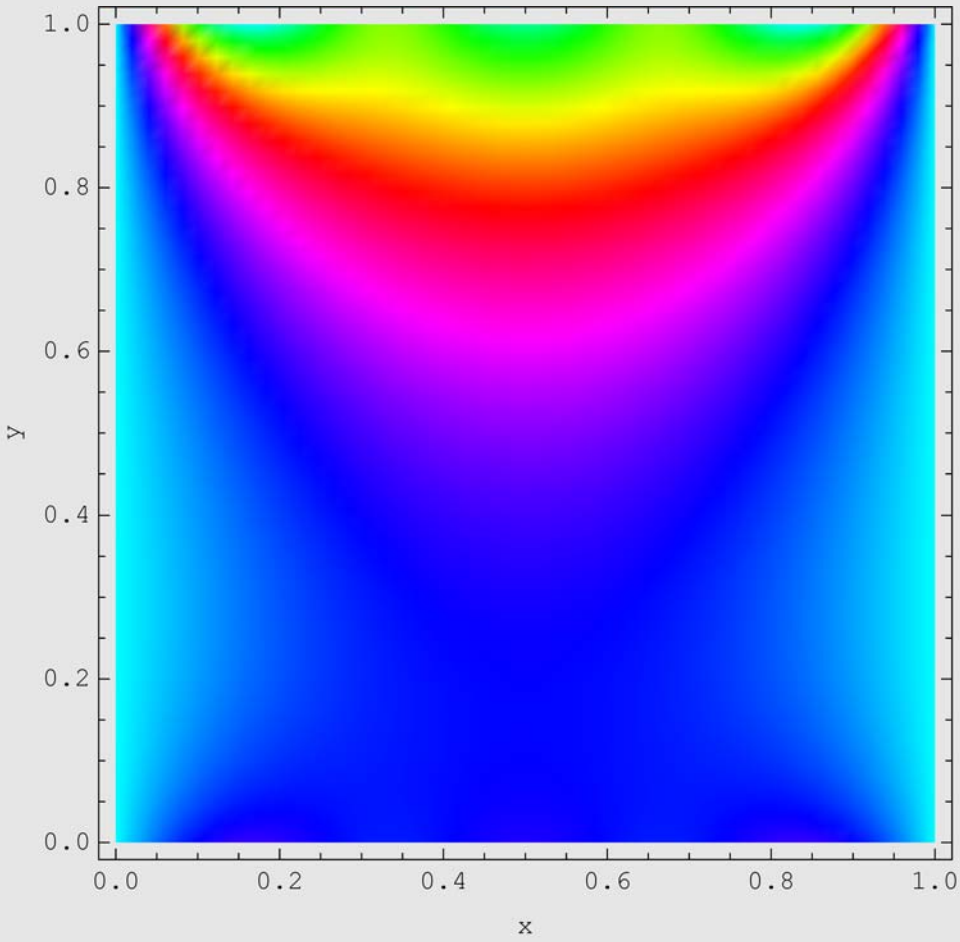
```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25];
```

```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25, ShadowPosition → 1];
```

```
ContourPlot[u[x, y, 5], {x, 0, a}, {y, 0, b}, FrameLabel → {"x", "y"}]
```



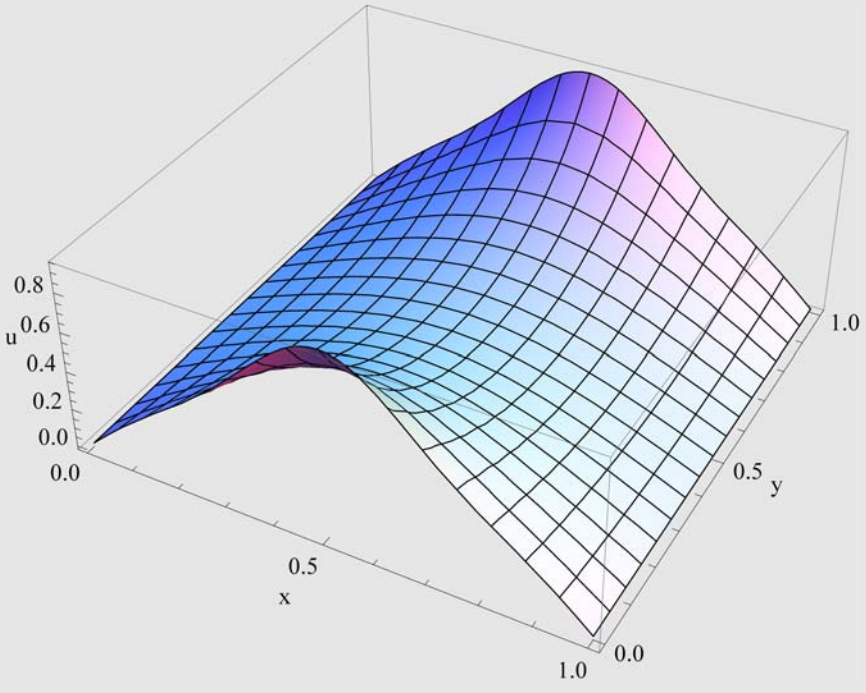
```
DensityPlot[u[x, y, 5], {x, 0, a}, {y, 0, b},
 ColorFunction → (Hue[0.5` + #1] &), PlotPoints → 50, Mesh → False,
 FrameLabel → {"x", "y"}]
```



Example 2

```
Clear[a, b, A, B, λ, n, u, x, y];
a = 1;
b = 1;
λ[n_] = n π / a;
A[n_] = 2/a (∫_0^(a/2) 2 x Sin[λ[n] x] dx + ∫_(a/2)^a 2 (1 - x) Sin[λ[n] x] dx);
B[n_] =
  (2/a (∫_0^(a/2) 2 x Sin[λ[n] x] dx + ∫_(a/2)^a 2 (1 - x) Sin[λ[n] x] dx) - A[n] Cosh[λ[n] b]) / Sinh[λ[n] b];
u[x_, y_, n_] := Sum_(i=1)^n Sin[λ[i] x] (N[A[i]] Cosh[λ[i] y] + N[B[i]] Sinh[λ[i] y])
```

```
Plot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"}]
```

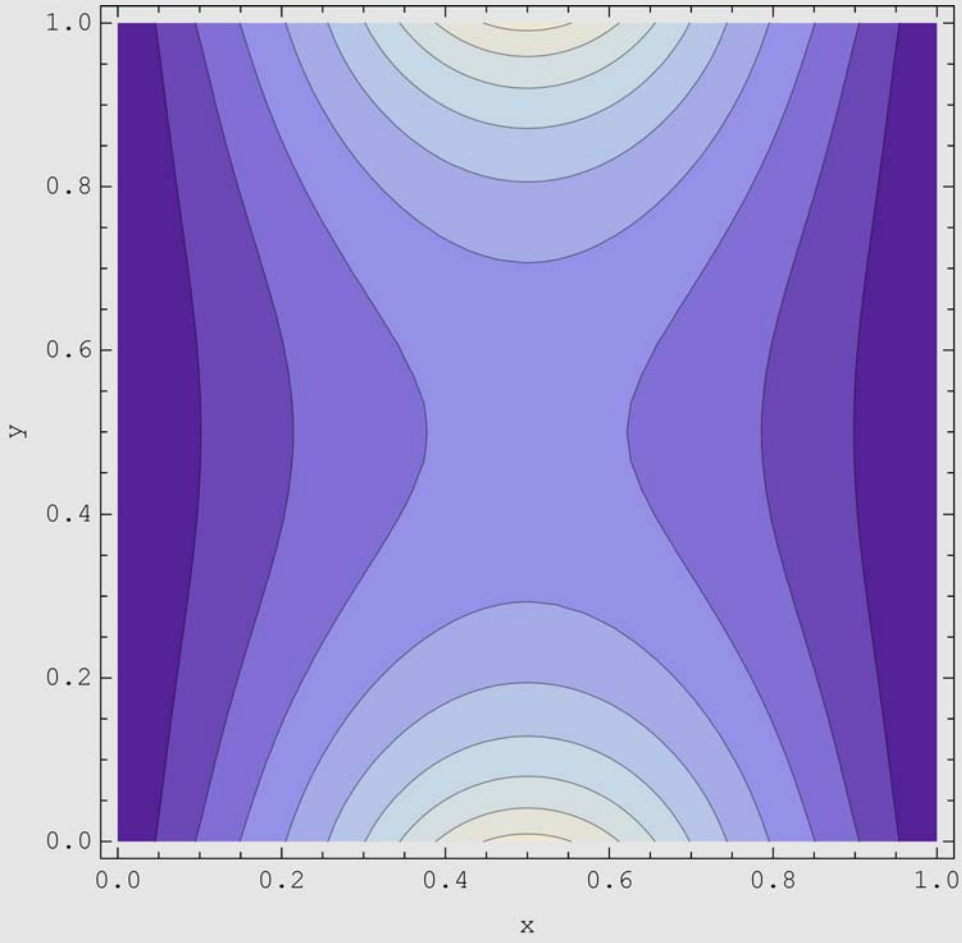



```
Needs["BarCharts`"]; Needs["Histograms`"]
```

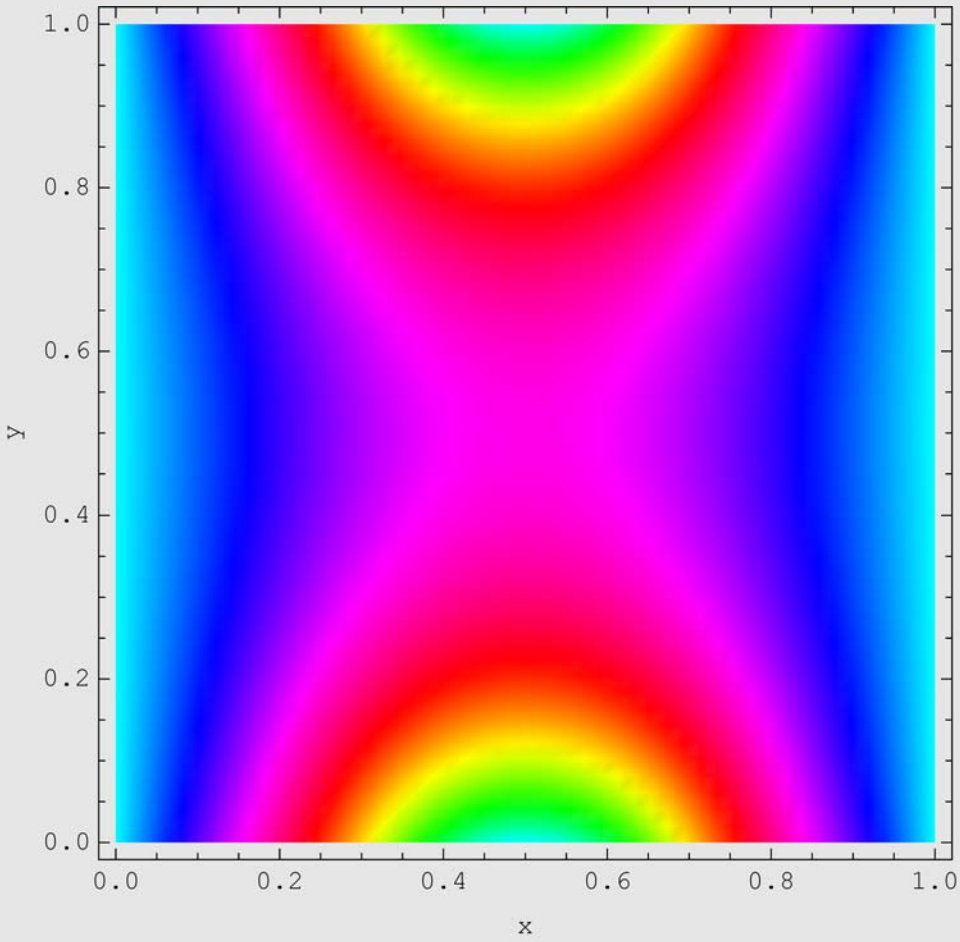
```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25];
```

```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25, ShadowPosition → 1];
```

```
ContourPlot[u[x, y, 5], {x, 0, a}, {y, 0, b}, FrameLabel → {"x", "y"}]
```



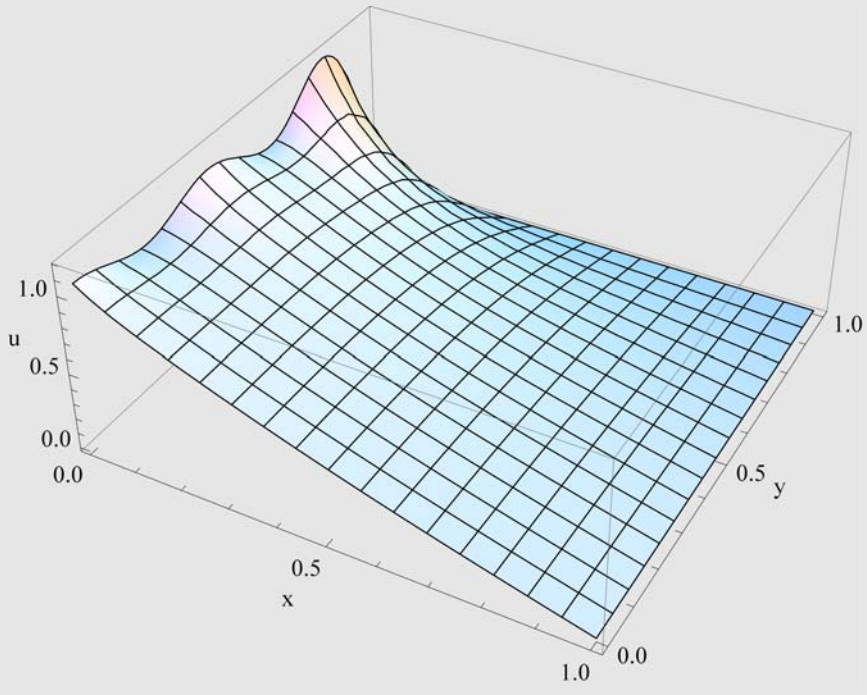
```
DensityPlot[u[x, y, 5], {x, 0, a}, {y, 0, b},
 ColorFunction → (Hue[0.5` + #1] &), PlotPoints → 50, Mesh → False,
 FrameLabel → {"x", "y"}]
```



Example 3

```
Clear[a, b, A, B, λ, n, u, x, y];
a = 1;
b = 1;
λ[n_] = (2 n - 1) π / (2 b);
A[n_] = 2/a ∫_0^a Cos[λ[n] y] dy;
B[n_] = -A[n] Cosh[λ[n] a] / Sinh[λ[n] a];
u[x_, y_, n_] := Σ_{i=1}^{n} Cos[λ[i] y] (N[A[i]] Cosh[λ[i] x] + N[B[i]] Sinh[λ[i] x])
```

```
Plot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"}]
```

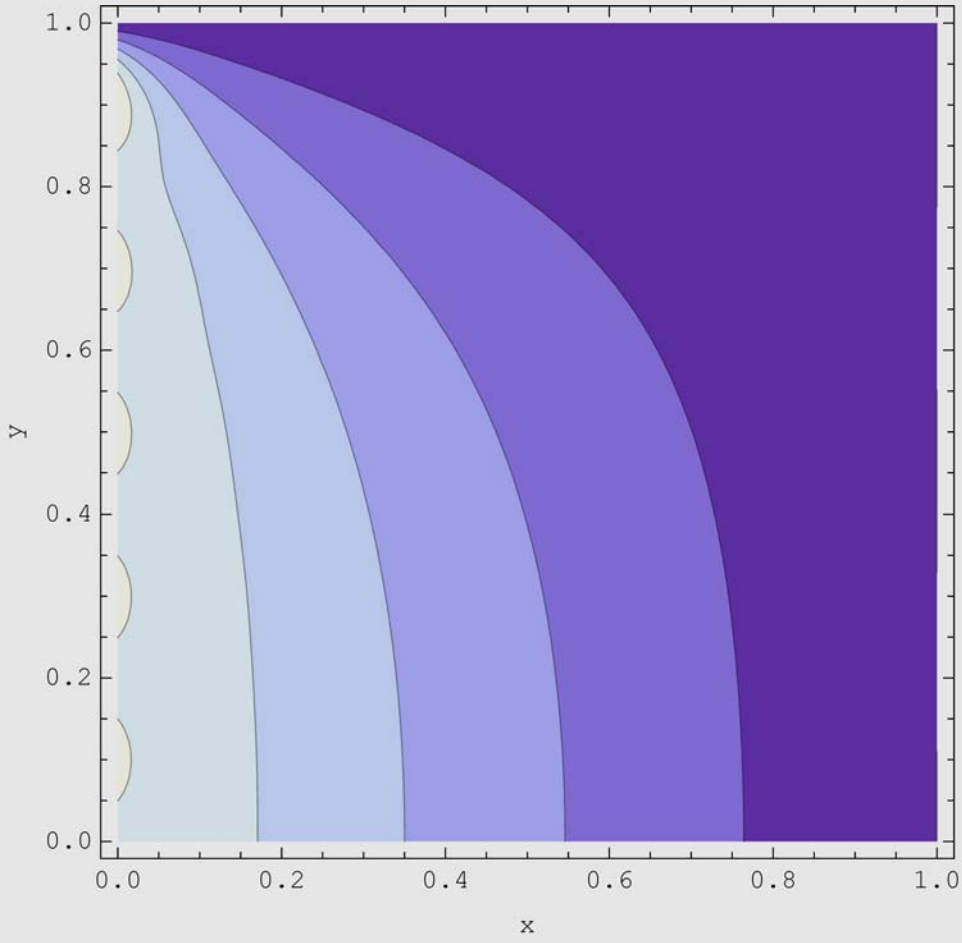



```
Needs["BarCharts`"]; Needs["Histograms`"]
```

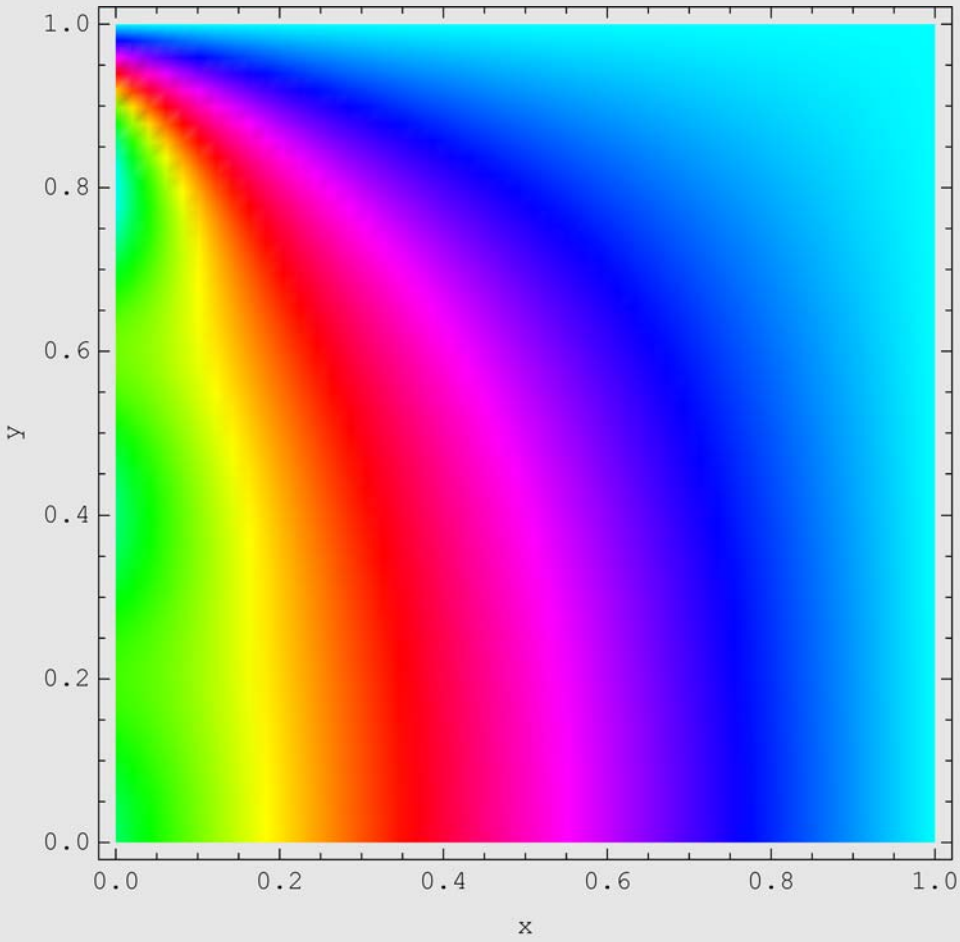
```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25];
```

```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25, ShadowPosition → 1];
```

```
ContourPlot[u[x, y, 10], {x, 0, a}, {y, 0, b}, FrameLabel → {"x", "y"}]
```



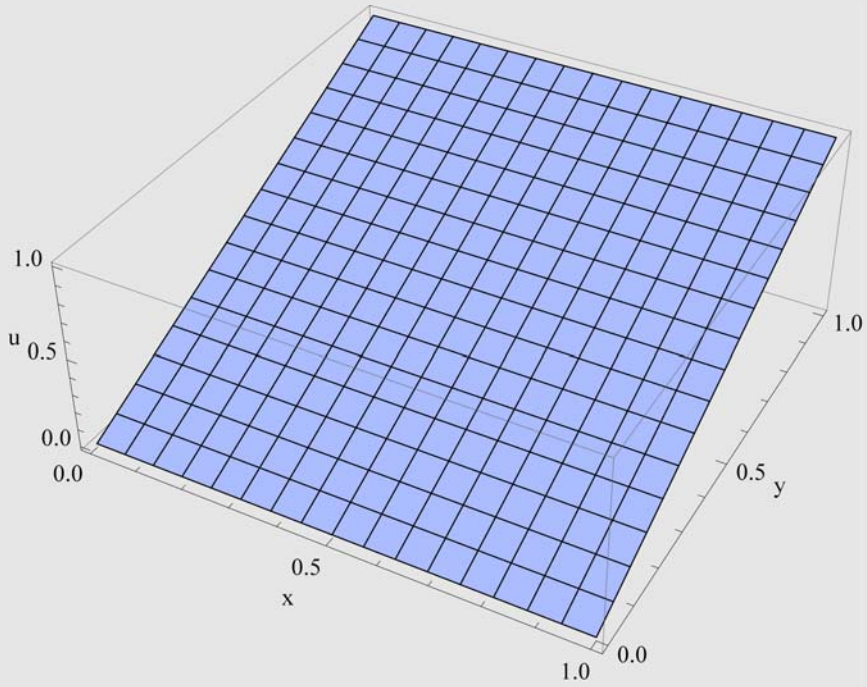
```
DensityPlot[u[x, y, 5], {x, 0, a}, {y, 0, b},
 ColorFunction → (Hue[0.5` + #1] &), PlotPoints → 50, Mesh → False,
 FrameLabel → {"x", "y"}]
```



Example 4

```
Clear[a, b, A, B, λ, n, u, x, y];
a = 1;
b = 1;
λ[n_] = n π / a;
B[n_] = 2 / (a Sinh[λ[n] b]) ∫_0^a Cos[λ[n] x] dx;
u[x_, y_, n_] := ∑_{i=1}^{n} (Cos[λ[i] x] N[B[i]] Sinh[λ[i] y]) + y/b
```

```
Plot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"}]
```

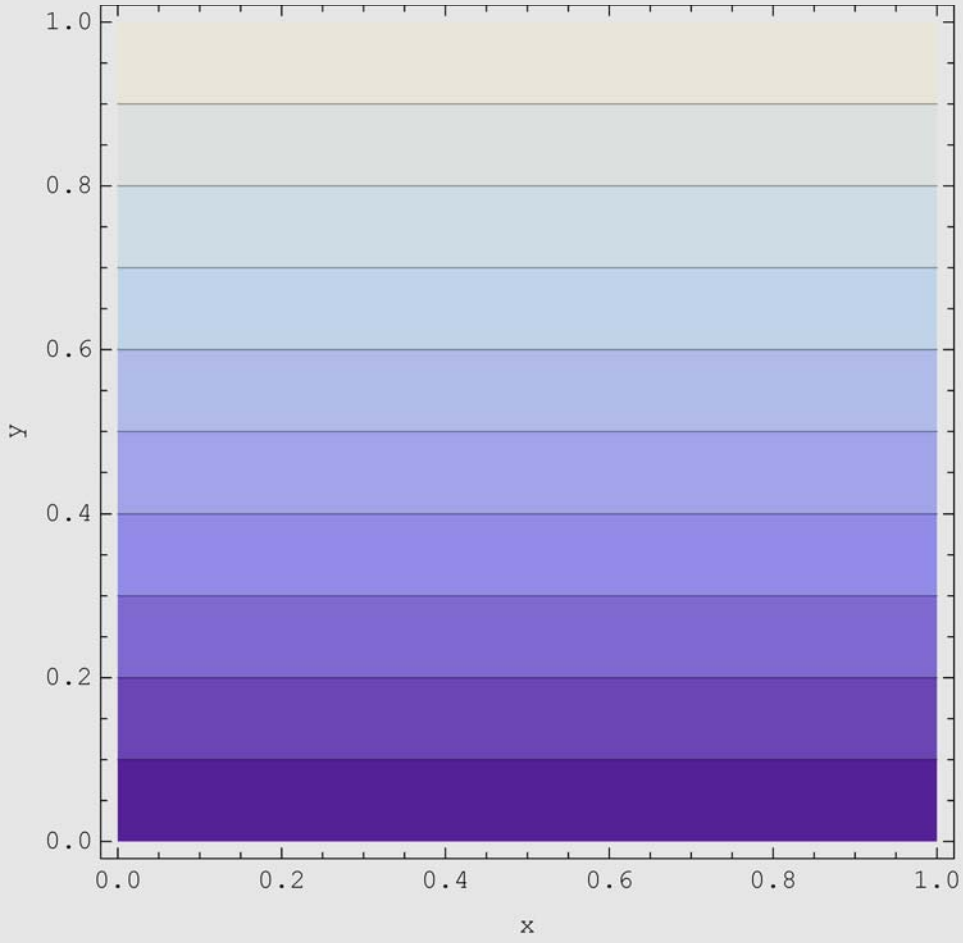



```
Needs["BarCharts`"]; Needs["Histograms`"]
```

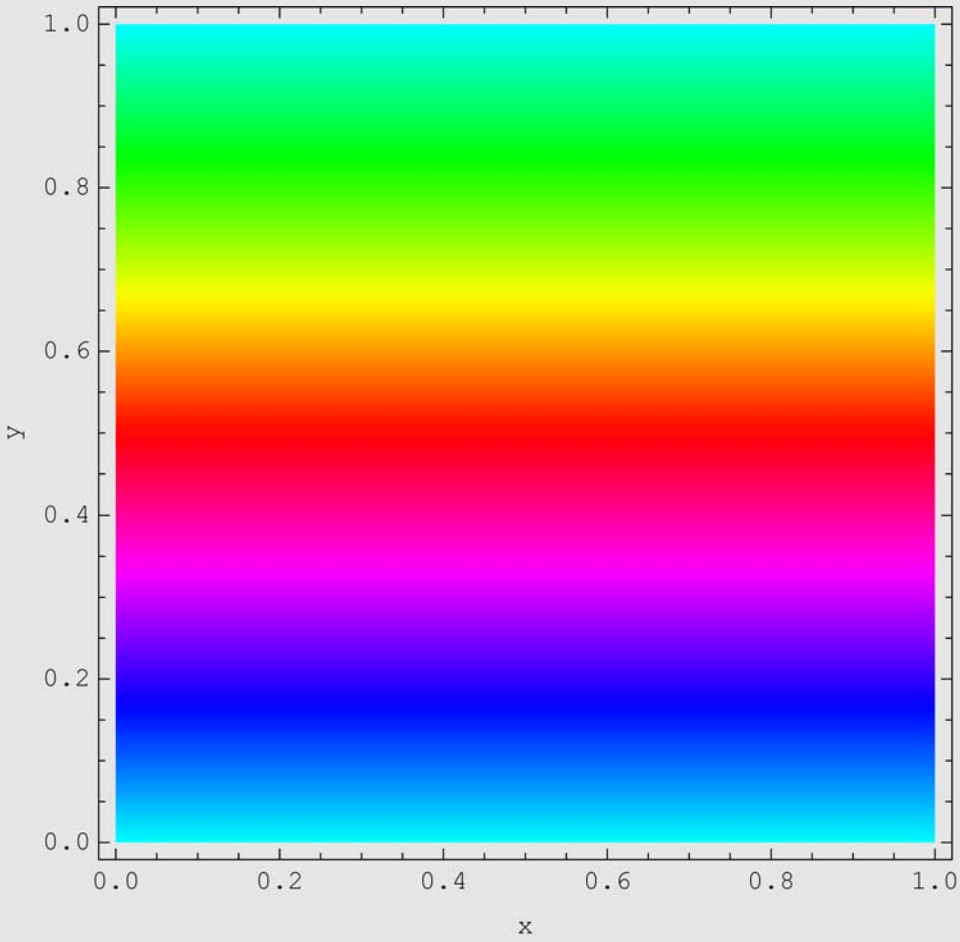
```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25, SurfaceMesh → False,
  ShadowMesh → False];
```

```
ShadowPlot3D[u[x, y, 5], {x, 0, a}, {y, 0, b}, AxesLabel → {"x", "y", "u"},
  ColorFunction → (Hue[.5 + #] &), PlotPoints → 25, ShadowPosition → 1];
```

```
ContourPlot[u[x, y, 5], {x, 0, a}, {y, 0, b}, FrameLabel → {"x", "y"}]
```



```
DensityPlot[u[x, y, 5], {x, 0, a}, {y, 0, b},
 ColorFunction → (Hue[0.5` + #1] &), PlotPoints → 50, Mesh → False,
 FrameLabel → {"x", "y"}]
```



In a Disk

Example 1

```
Clear[c, f, A, B, v, r, θ, n];
c = 1;
f[θ_] = Abs[θ / 2];
A0 = 1/(2 π) ∫_-π^π f[θ] dθ;
A[n_] = 1/(π c^n) ∫_-π^π f[θ] Cos[n θ] dθ;
B[n_] = 1/(π c^n) ∫_-π^π f[θ] Cos[n θ] dθ;
v[r_, θ_, n_] := A0 + Sum[(N[A[i]] r^i Cos[i θ] + N[B[i]] r^i Sin[i θ]), {i, 1, n}]
```

```
Needs["BarCharts`"]; Needs["Histograms`"]
```

```
Needs["Graphics`ParametricPlot3D`"]
```

```
polar = RevolutionPlot3D[v[r, θ, 5], {r, 0, 1}, {θ, -π, π}];
```

```
Shadow[polar, XShadow → False, YShadow → False];
```

Example 2

```
Clear[c, f, A, B, v, r, θ, n];
c = 1;
f[θ_] = -Exp[θ / 2];
A0 = 1/(2 π) ∫_-π^π Cos[θ] dθ;
A[n_] = 1/(π c^n) ∫_-π^π f[θ] Cos[n θ] dθ;
B[n_] = 1/(π c^n) ∫_-π^π f[θ] Cos[n θ] dθ;
v[r_, θ_, n_] := A0 + Sum[(N[A[i]] r^i Cos[i θ] + N[B[i]] r^i Sin[i θ]), {i, 1, n}]
```

```
Needs["BarCharts`"]; Needs["Histograms`"]
```

```
Needs["Graphics`ParametricPlot3D`"]
```

```
polar = RevolutionPlot3D[v[r, θ, 5], {r, 0, 1}, {θ, -π, π}];
```

```
Shadow[polar, XShadow → False, YShadow → False];
```

Example 3

```
Clear[c, f, A, B, v, r, θ, n];
c = 1;
f[θ_] = -Log[θ];
A0 = 1/(2 π) ∫_(-π)^π Cos[θ] dθ;
A[n_] = 1/(π c^n) ∫_(-π)^π f[θ] Cos[n θ] dθ;
B[n_] = 1/(π c^n) ∫_(-π)^π f[θ] Cos[n θ] dθ;
v[r_, θ_, n_] := A0 + Sum[(N[A[i]] r^i Cos[i θ] + N[B[i]] r^i Sin[i θ]), {i, 1, n}]
```

```
Needs["BarCharts`"]; Needs["Histograms`"]
```

```
Needs["Graphics`ParametricPlot3D`"]
```

```
polar = RevolutionPlot3D[v[r, θ, 5], {r, 0, 1}, {θ, 0, 2 π}];
```

```
Shadow[polar, XShadow → False, YShadow → False];
```

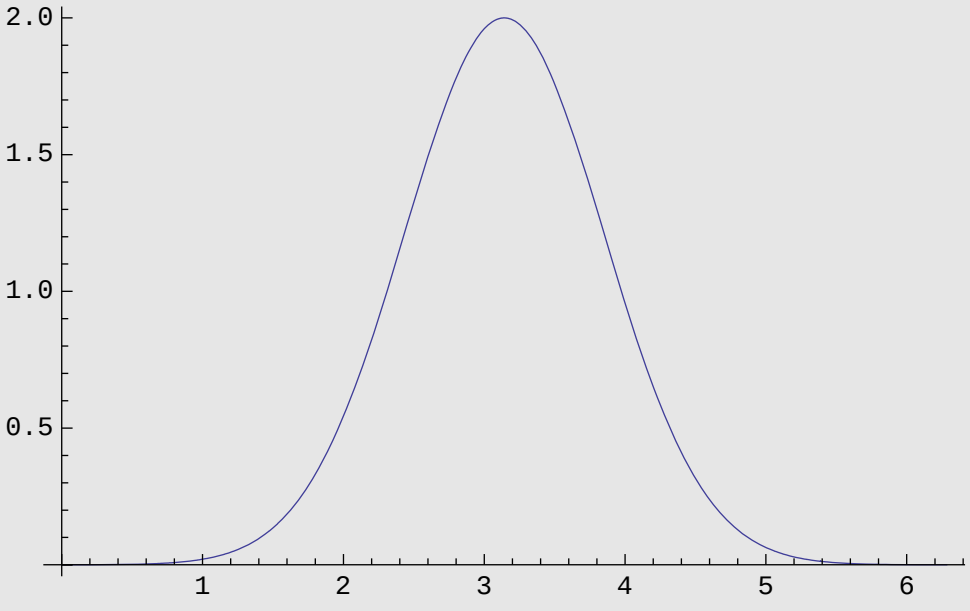
Example 4

```
Clear[c, f, A, B, v, r, θ, n];
c = 1;
f[θ_] = 2 Exp[-(θ - π)^2];
A0 = 1/(2 π) ∫_-π^π Cos[θ] dθ;
A[n_] = 1/(π c^n) ∫_-π^π f[θ] Cos[n θ] dθ;
B[n_] = 1/(π c^n) ∫_-π^π f[θ] Cos[n θ] dθ;
v[r_, θ_, n_] := A0 + Sum[(N[A[i]] r^i Cos[i θ] + N[B[i]] r^i Sin[i θ]), {i, 1, n}]
```

```
Needs["BarCharts`"]; Needs["Histograms`"]
```

```
Needs["Graphics`ParametricPlot3D`"]
```

```
Plot[2 Exp[-(θ - π)^2], {θ, 0, 2 π}]
```



```
polar = RevolutionPlot3D[v[r, θ, 5], {r, 0, 1}, {θ, 0, 2 π},
    ViewPoint → {2.422`, 1.341`, 1.945`}];
```

```
Shadow[polar, ViewPoint -> {2.422, 1.341, 1.945}, XShadow → False,
  YShadow → False];
```