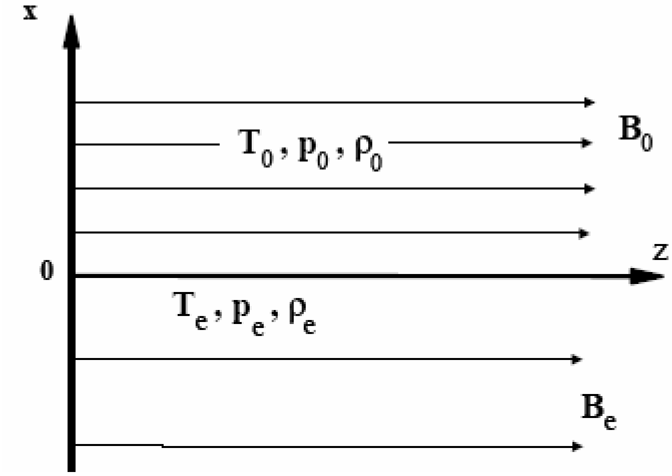
Moduri de oscilatie la o interfata magnetica

Fie o interfata ingusta intre doua straturi plasmatice cu temperaturi, densitati si presiuni diferite, fiecare din ele imersate in campuri magnetice uniforme, dar de valori diferite:



$\mathbf{B} = B_0\mathbf{e_z}$. $\mathbf{B} = B_e\mathbf{e_z}$

Parametrii de echilibru ai celor doua plasme sunt:

$$B_0(x) = \begin{cases} B_0, & x > 0, \\ B_e, & x < 0, \end{cases} \qquad \rho_0(x) = \begin{cases} \rho_0, & x > 0, \\ \rho_e, & x < 0, \end{cases}$$

$$T_0(x) = \begin{cases} T_0, & x > 0, \\ T_e, & x < 0, \end{cases} \qquad p_0(x) = \begin{cases} p_0, & x > 0, \\ p_e, & x < 0. \end{cases} \qquad (1)$$

Este clar ca in cele doua medii plasmatice sitezele sonice sunt : C_{S0}, C_{Se} iar cele Alfvenice C_{A0}, C_{Ae}. In stare stationara toate marimile fizice sunt uniforme cu exceptia presiunii care lainterfata x=0 sufera un salt si astfel trebuie impusa conditia de continuitate:

$$p_0 + \frac{B_0^2}{2\mu} = p_e + \frac{B_e^2}{2\mu}. \qquad (2)$$

Sa simplificam analiza, considerand ca stratul plasmatic superior este rece (p_0=0) iar in stratul inferior este nemagnetizat (field-free), $B_e = 0$.

Sa consideram ca sistemul este perturbat si ca perturbatiile se propaga in directia campului magnetic (|| $\mathbf{e_z}$).

Sa scriem ecuatiile MHD pentru mediul **plasmatic superior:**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \qquad (3)$$

$$\rho \frac{d\mathbf{V}}{dt} = -\frac{1}{\mu}\mathbf{B} \times (\nabla \times \mathbf{B}). \qquad (4)$$

Forma perturbata a acestor ecuatii va fi:

$$\frac{\partial \tilde{\mathbf{B}}}{\partial t} = \nabla \times (\tilde{\mathbf{V}} \times \mathbf{B_0}), \tag{5}$$

$$\rho_0 \frac{\partial \tilde{\mathbf{V}}}{\partial t} = -\frac{1}{\mu} \mathbf{B_0} \times (\nabla \times \tilde{\mathbf{B}}). \tag{6}$$

Sa consideram unde armonice de forma:

$$\propto \exp(i\omega t - ikz), \tag{7}$$

$$\tilde{\mathbf{B}} = (B_x, B_y, B_z),\ \tilde{\mathbf{V}} = (V_x, V_y, V_z)$$

Ecuatiile (5-6) capata forma:

$$i\omega B_x = -ikB_0 V_x, \tag{8}$$

$$i\omega B_z = -B_0 \frac{\partial V_x}{\partial x}, \tag{9}$$

$$i\rho_0 \omega V_x = -\frac{B_0}{\mu}\left(ikB_x + \frac{\partial B_z}{\partial x}\right). \tag{10}$$

Combinand ecuatiile (8-10), ele se pot pune sub forma unei singure ecuatii:

$$\boxed{\frac{\partial^2 V_x}{\partial x^2} - \left(k^2 - \frac{\omega^2}{C_{A0}^2}\right) V_x = 0.} \tag{11}$$

Din ecuatiile (8-9) se pot determina rapid perturbatiile in camp magnetic:

$$B_x = -\frac{kB_0}{\omega} V_x, \tag{12}$$

$$B_z = \frac{iB_0}{\omega}\frac{\partial V_x}{\partial x}. \tag{13}$$

Ecuatiile perturbate pentru **<u>mediul plasmatic inferior</u>** sunt:

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla(\rho_e \mathbf{V}) = 0, \tag{14}$$

$$\rho_e \frac{\partial \mathbf{V}}{\partial t} = -\nabla \tilde{p}, \tag{15}$$

Iar ecuatia de stare:

$$\tilde{p} = C_{se}^2 \tilde{\rho}. \tag{16}$$

Sub forma scalara acestea sunt:

$$i\omega\tilde{\rho}+\rho_e\frac{\partial V_x}{\partial x}-ik\rho_e V_z=0, \tag{17}$$

$$i\omega\rho_e V_x=-C_{se}^2\frac{\partial\tilde{\rho}}{\partial x}, \tag{18}$$

$$i\omega\rho_e V_z=ikC_{se}^2\tilde{\rho}. \tag{19}$$

Aceste ecuatii se pot combina intr-o forma unitara:

$$\boxed{\frac{\partial^2 V_x}{\partial x^2}-\left(k^2-\frac{\omega^2}{C_{se}^2}\right)V_x=0.} \tag{20}$$

Si

$$\tilde{\rho}=\frac{i\rho_e\omega}{\omega^2-C_{se}^2k^2}\frac{\partial V_x}{\partial x}, \tag{21}$$

$$\frac{\partial V_z}{\partial x}=-ikV_x. \tag{22}$$

Rezolvarea EDO poate fi facuta numai daca se cunosc conditiile la limita. De aceea sunt necesare conditiile de continuitate in presiunea totala si componeta normala a vitezei la interfata x=0:

$$p_T^{(0)}(x=0)=p_T^{(e)}(x=0), \tag{23}$$

$$V_x^{(0)}(x=0)=V_x^{(e)}(x=0), \tag{24}$$

Dar si conditia de localizare a modului:

$$V_x^{(0)}(x\to+\infty)\to 0, \qquad V_x^{(e)}(x\to-\infty)\to 0. \tag{25}$$

In mediul superior presiunea termica este nula, ceea ce ne va permite scrierea presiunii totale ca functie de V_x:

$$p_T=\frac{B_x^2+B_y^2+(B_0+B_z)^2}{2\mu}\approx\frac{B_0B_z}{\mu}. \tag{26}$$

Avand in vedere ecuatia (9) rezulta:

$$p_T=\frac{B_0B_z}{\mu}=\frac{iB_0^2}{\mu\omega}\frac{\partial V_x}{\partial x}. \tag{27}$$

In mediul inferior, perturbatia in presiunea magnetica este nula astfel:

$$p_T=\tilde{p}=C_{se}^2\tilde{\rho}=\frac{iC_{se}^2\rho_e\omega}{\omega^2-C_{se}^2k^2}\frac{\partial V_x}{\partial x}, \tag{28}$$

Ecuatia (11) conduce la urmatoarea solutie:

$$V_x^{(0)} = A_1 \exp(m_0 x) + A_2 \exp(-m_0 x), \qquad (29)$$

Unde A_1 si A_2 sunt constante arbitrare iar

$$m_0 = \sqrt{k^2 - \frac{\omega^2}{C_{A0}^2}}. \qquad (30)$$

Ecuatia (20) admite solutia :

$$V_x^{(e)} = B_1 \exp(m_e x) + B_2 \exp(-m_e x), \qquad (31)$$

Unde B_1 si B_2 sunt constante arbitrare iar

$$m_e = \sqrt{k^2 - \frac{\omega^2}{C_{se}^2}}. \qquad (32)$$

Satisfacerea conditiilor la infinit conduce la A_1=0 si B_2=0. Conditia (24) utilizand (29) si (31) conduce la:

Conditia (23) utilizand (27) si (28) conduce la

$$p_T^{(0)}(0) = p_T^{(e)}(0) \;\Rightarrow\; \frac{iB_0^2}{\mu\omega}\frac{\partial V_x^{(0)}}{\partial x}(0) = \frac{iC_{se}^2\rho_e\omega}{\omega^2 - C_{se}^2 k^2}\frac{\partial V_x^{(e)}}{\partial x}(0). \quad (34)$$

Calculul derivatelor in x=0

$$\frac{\partial V_x^{(0)}}{\partial x}(0) = -m_0 A_2 \exp(-m_0 0) = -m_0 A_2, \qquad (35)$$

$$\frac{\partial V_x^{(e)}}{\partial x}(0) = m_e B_1 \exp(m_e 0) = m_e B_1, \qquad (36)$$

Cu utilizarea A_2=B_1 modifica (34) astfel:

$$-\frac{iB_0^2}{\mu\omega} m_0 = \frac{iC_{se}^2\rho_e\omega}{\omega^2 - C_{se}^2 k^2} m_e, \qquad (37)$$

Sau

$$-\rho_0 C_{A0}^2 m_0 = \frac{C_{se}^2\rho_e\omega^2}{\omega^2 - C_{se}^2 k^2} m_e \qquad (38)$$

Care este **ECUATIA DE DISPERSIE A MODURILOR DE INTERFATA MAGNETICA**.

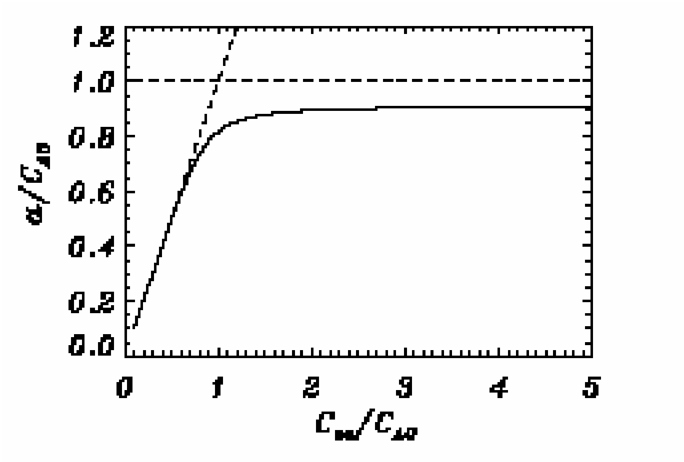
Introducand viteza de faza ***a* = *ω/k*** ecuatia (38) devine:

$$-\frac{\rho_0}{\rho_e}\frac{C_{A0}^2}{C_{se}^2}\sqrt{1 - \frac{a^2}{C_{A0}^2}} = \frac{a^2}{a^2 - C_{se}^2}\sqrt{1 - \frac{a^2}{C_{se}^2}} \qquad (39)$$

Adica o ecuatie algebrica transcendentala.

Analiza relatiei de dispersie

In general $a < C_{A0}$; $a < C_{Se}$ ceea ce inseamna ca modurile de interfata se propaga mai lent decat cele din mediile plasmatice. Observam ca relatia de dispersie (39) nu depinde nici de ω nici de k, ceea ce inseamna ca modurile sunt nedispersive.



In cazul general, cand plasma magnetizata nu este rece $p_0 > 0 \Rightarrow C_{S0} > 0$, relatia de dispersie are aceeasi forma insa

$$m_0 = \frac{(k^2C_{s0}^2 - \omega^2)(k^2C_{A0}^2 - \omega^2)}{(C_{s0}^2 + C_{A0}^2)(k^2C_{T0}^2 - \omega^2)}, \qquad (40)$$

Unde

$$C_{T0} = \frac{C_{s0}C_{A0}}{(C_{s0}^2 + C_{A0}^2)^{1/2}}, \qquad (41)$$

Este viteza ***tubulara sau cusp.***