Echilibrul magnetohidrostatic (aplicatii in structuri plasmatice solare)

Ecuatiile ce descriu echilibrul magnetohidrostatic sunt:

$$-\nabla p + \frac{1}{\mu}\left(\nabla \times \vec{B}\right)\times \vec{B} + \rho\vec{g} = 0,$$

$$\nabla \cdot \vec{B} = 0,$$

$$p = \rho R T.$$

Inainte de a analiza fenomene specifice, sa vedem care este problema echilibrului presiunii in prezenta unui camp magnetic care nu exercita nici o forta. Presupunem existenta unui camp magnetic uniform orientat dupa axa Oz, in prezenta unui camp termic cunoscut.

$$\mathbf{B} = B_0\hat{\mathbf{z}}, \qquad \mathbf{g} = -g\hat{\mathbf{z}}.$$

Deoarece **j** este nul, nu actioneaza forta Lorentz, deci:

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho(z)g = -\frac{p(z)}{\Lambda(z)},$$

Unde: $\boxed{\Lambda(z) = RT(z)}$ este <u>inaltimea scalei presiunii</u>.

Integram ecuatia diferentiala:

$$\log p = -n(z) + \log p(0),$$

Unde:

$$n(z) = \int_0^z \frac{1}{\Lambda(u)} du,$$

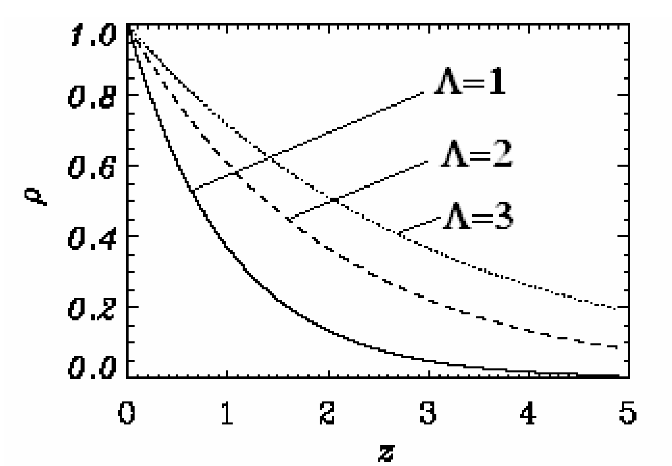
Deci:

$$\boxed{p(z) = p(0)\exp\left[-n(z)\right].}$$

Daca atmosfera este **izoterma**, atunci T si Λ sunt constante si rezulta:

$$\boxed{p(z) = p(0)\exp\left(-z/\Lambda\right), \qquad \rho(z) = \rho(0)\exp\left[-z/\Lambda\right],}$$

Si deci, presiunea scade exponential pe o lungime de scala specifica data de inaltimea Λ a scalei de presiune.



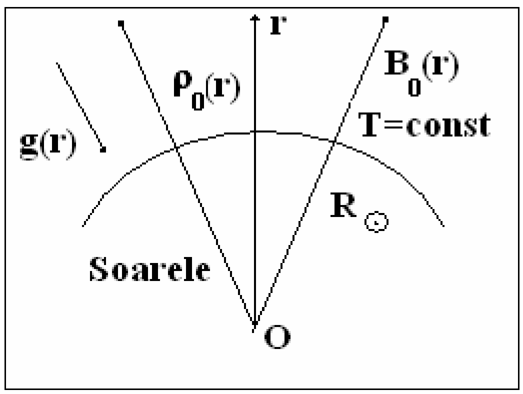
Pentru R=8.3 $10^3 JK^{-1} mol^{-1}$ atunci :

Structura	Acceleratie gravitationala Ms^{-2}	Temperatura K, MK	Masa molara	Λ Km,Mm
Fotosfera	274	6000K	1.3	140Km
Corona	274	1MK	0.6	50Mm
Atmosfera Terrei	9.81	300K	29	8.7Km

Daca se doreste un carcul al stratificarii gravitationale la mari dimensiuni de scala (gaurile coronale) trebuiesc luate in considerare efectele induse de geomatria structurii (in general sferica) si de inaltime in expresia acceleratiei gravitationale:

$$g(r) = \frac{GM_\odot}{r^2}$$

In cazul **geometriei sferice** si al atmosferei izoterme:



$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\rho(r)g(r), \quad p = \frac{\rho \mathrm{R} T}{\tilde{\mu}}$$

Deci:

$$\frac{\mathrm{d}\rho(r)}{\mathrm{d}r} = -\frac{R_\odot^2}{r^2}\frac{1}{\Lambda}\rho(r).$$

$$\Lambda(z) = \frac{\mathrm{R}T(z)}{\tilde{\mu}g},$$

$$g(R_\odot) = \frac{GM_\odot}{R_\odot^2}$$

Integrand ecuatia diferentiala obtinem:

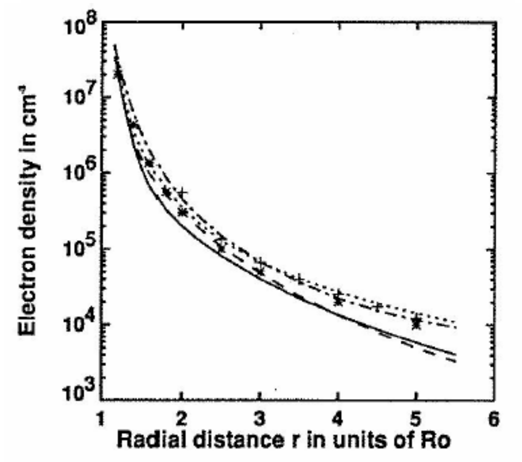
$$\int \frac{\mathrm{d}\rho}{\rho} = -\int \frac{R_\odot^2}{\Lambda r^2}\,\mathrm{d}r \Rightarrow \rho(r) = C\exp\left(\frac{R_\odot^2}{\Lambda r}\right).$$

Determinand constanta c din conditia :

$$\rho(R_\odot) = \rho_0.$$

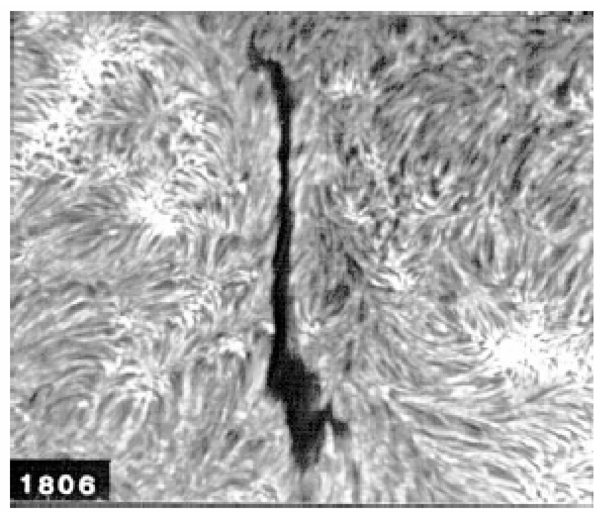
Rezulta imediat:

$$\rho(r) = \rho_0 \exp\left[\frac{R_\odot}{\Lambda}\left(\frac{R_\odot}{r} - 1\right)\right].$$



Magnetostatica prominentelor

Prominentele sunt nori densi si reci de plasma in corona solara plasati in lungul liniilor de inversiune magnetica.



Proprietatile prominentelor	**Proprietati coronale**
Lungime $L \approx 200$ Mm	
Inaltime $h \approx 50$ Mm	
Grosime $w \approx 6$ Mm	
$T \approx 5,000 - 10,000$ K	$T \approx 1 - 3 \times 10^6$ K
$\rho_0 \approx 2 \times 10^{-10}\,\mathrm{kg\,m^{-3}}$	$\rho_0 \approx 2 \times 10^{-12}\,\mathrm{kg\,m^{-3}}$
$\Lambda \approx 0.6\mathrm{Mm}$	$\Lambda \approx 50\mathrm{Mm}$

Ne propunem determonarea structurii liniilor de camp magnetic si a evolutiei densitatii. Ecuatiile ce descriu evolutia prominentelor sunt:

$$-\nabla p + \frac{1}{\mu}\left(\nabla \times \vec{B}\right) \times \vec{B} + \rho\vec{g} = 0,$$

$$\nabla \cdot \vec{B} = 0,$$

$$p = \rho RT.$$

Pentru simplitata: $\vec{B} = \left(B_x, 0, B_z\right)$, $\vec{g} = \left(0,0,g\right)$, $\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$, $T = const.$ ceea ce conduce la

$\nabla \cdot \vec{B} = 0 \Rightarrow B_x = const.$ proiectiile ecuatiei de miscare sunt:

$$RT\frac{\partial \rho}{\partial x} = -\frac{1}{\mu}B_z\frac{\partial B_z}{\partial x},$$

$$0 = \frac{1}{\mu}B_x\frac{\partial B_z}{\partial x} - \rho g$$

Notand: $\gamma = g / RTB_x$ rezula imediat:

$$\frac{\partial^2 B_z}{\partial x^2} + \frac{\gamma}{2}\frac{\partial}{\partial x}B_z^2 = 0 \Rightarrow \begin{cases} B_z = B_z^0 \tanh\left(\frac{1}{2}\gamma B_z^0 z\right) \\ \rho = \frac{1}{\mu RT}\left(\frac{B_z^0}{\cosh\left(\frac{1}{2}\gamma B_z^0 z\right)}\right)^2 \end{cases}$$