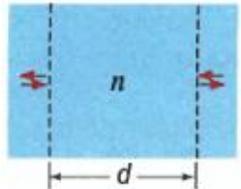MODUL 1 – INTRODUCTION TO NANOPHOTONICS, TRANSFER MATRIX METHOD

Based on Saleh B.E.A.,Teich M.C.Fundamentals of Photonics, Wiley, 2ed, 2007,chap.7

Evaluation tests

1. Homogenuos medium

For a homogenous medium of index n and thickness d show that the transfer matrix M is:

$$\mathbf{M} = \begin{bmatrix} \exp(-j\varphi) & 0 \\ 0 & \exp(j\varphi) \end{bmatrix}, \mathbf{S} = \begin{bmatrix} \exp(-j\varphi) & 0 \\ 0 & \exp(-j\varphi) \end{bmatrix}, \varphi = nk_o d.$$

Hint: Use the definition of scattering matrix S and determine the reflection and transmission coefficients with Fresnel equations.

2. Single Dielectric Boundary

Show that at the boundary between two media of refractive indexes n1 and n2 the scattering and transfer matrix is:
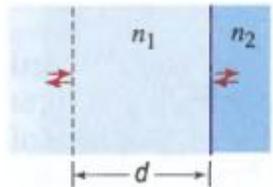
$$\mathbf{S} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{n_1 + n_2} \begin{bmatrix} 2n_1 & n_2 - n_1 \\ n_1 - n_2 & 2n_2 \end{bmatrix}.$$

$$\mathbf{M} = \frac{1}{2n_2} \begin{bmatrix} n_2 + n_1 & n_2 - n_1 \\ n_2 - n_1 & n_2 + n_1 \end{bmatrix}$$

3. Propagation Followed by a Boundary

Show that for a homogeneous medium followed by the boundary between two media of refractive indexes n1 and n2 the scattering and transfer matrix is:

$$\mathbf{M} = \frac{1}{2n_2} \begin{bmatrix} (n_2 + n_1)\,e^{-j\varphi} & (n_2 - n_1)\,e^{j\varphi} \\ (n_2 - n_1)\,e^{-j\varphi} & (n_2 + n_1)\,e^{j\varphi} \end{bmatrix}, \quad \varphi = n_1 k_o d$$

4. Propagation Followed by Transmission Through a Slab.

Show that for a two homogeneous media with refractive indexes n1 and n2 and thicknesses d1 and d2 the transfer matrix is:
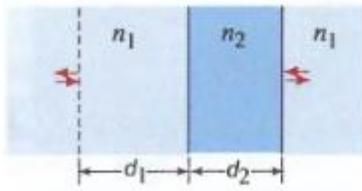
$$M = \frac{1}{4n_1n_2} \begin{bmatrix} (n_1 + n_2)\,e^{-j\varphi_2} & (n_1 - n_2)\,e^{j\varphi_2} \\ (n_1 - n_2)\,e^{-j\varphi_2} & (n_1 + n_2)\,e^{j\varphi_2} \end{bmatrix}$$

$$\times \begin{bmatrix} (n_2 + n_1)\,e^{-j\varphi_1} & (n_2 - n_1)\,e^{j\varphi_1} \\ (n_2 - n_1)\,e^{-j\varphi_1} & (n_2 + n_1)\,e^{j\varphi_1} \end{bmatrix}$$

where:

$$\varphi_1 = n_1 k_o d_1 \ \text{ and } \ \varphi_2 = n_2 k_o d_2.$$

and transmission coeficient is:

$$t = \exp(-j\varphi_1) \frac{4n_1 n_2 \exp(-j\varphi_2)}{(n_1 + n_2)^2 - (n_1 - n_2)^2 \exp(-j2\varphi_2)}$$

Hint: Use the relation between the scattering and transfer matrices.

5. Single Dielectric Boundary. Oblique TE and TM waves

Show that a wave transmitted through a planar boundary between media of refractive indexes n1 and n2 at angles θ1 and θ2, satisfying Snell's law (n1 sin θ1 =n2 sin θ2), is described by a scattering and transfer matrix determined from the Fresnel equations:
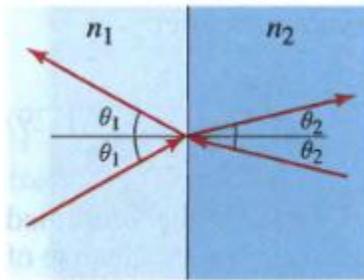


$$S = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{\tilde{n}_1 + \tilde{n}_2} \begin{bmatrix} 2a_{12}\tilde{n}_1 & \tilde{n}_2 - \tilde{n}_1 \\ \tilde{n}_1 - \tilde{n}_2 & 2a_{21}\tilde{n}_2 \end{bmatrix}$$

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2a_{21}\tilde{n}_2} \begin{bmatrix} \tilde{n}_1 + \tilde{n}_2 & \tilde{n}_2 - \tilde{n}_1 \\ \tilde{n}_2 - \tilde{n}_1 & \tilde{n}_1 + \tilde{n}_2 \end{bmatrix}.$$

TE: $\tilde{n}_1 = n_1 \cos\theta_1,$ $\tilde{n}_2 = n_2 \cos\theta_2,$ $a_{12} = a_{21} = 1,$

TM: $\tilde{n}_1 = n_1 \sec\theta_1,$ $\tilde{n}_2 = n_2 \sec\theta_2,$ $a_{12} = \cos\theta_1/\cos\theta_2 = 1/a_{21}.$

6. Off-axis Propagation Followed by Transmission Through a Slab.

Show that for a slab with thickness d and refractive indexes n2 between two homogeneous media with refractive index n1, the transfer matrix is:
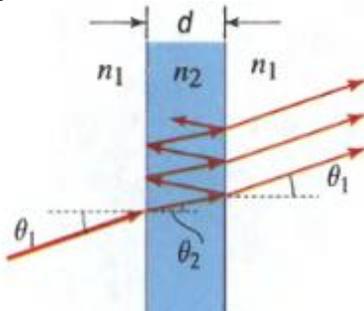
$$M = \frac{1}{4\tilde{n}_1\tilde{n}_2} \begin{bmatrix} (\tilde{n}_1 + \tilde{n}_2)\,e^{-j\tilde{\varphi}_2} & (\tilde{n}_1 - \tilde{n}_2)\,e^{j\tilde{\varphi}_2} \\ (\tilde{n}_1 - \tilde{n}_2)\,e^{-j\tilde{\varphi}_2} & (\tilde{n}_1 + \tilde{n}_2)\,e^{j\tilde{\varphi}_2} \end{bmatrix}$$

$$\times \begin{bmatrix} (\tilde{n}_2 + \tilde{n}_1)\,e^{-j\tilde{\varphi}_1} & (\tilde{n}_2 - \tilde{n}_1)\,e^{j\tilde{\varphi}_1} \\ (\tilde{n}_2 - \tilde{n}_1)\,e^{-j\tilde{\varphi}_1} & (\tilde{n}_2 + \tilde{n}_1)\,e^{j\tilde{\varphi}_1} \end{bmatrix}$$

where for TE waves, respectively TM waves we have:

TE: $\tilde{n}_1 = n_1 \cos\theta_1,$ $\tilde{n}_2 = n_2 \cos\theta_2,$ $\tilde{\varphi}_1 = n_1 k_o d_1 \cos\theta_1$

TM: $\tilde{n}_1 = n_1 \sec\theta_1,$ $\tilde{n}_2 = n_2 \sec\theta_2,$ $\tilde{\varphi}_2 = n_2 k_o d_2 \cos\theta_2$

7. Mirror Fabry-Perot etalon

Consider two lossless partially reflective mirrors with amplitude transmittances t1 and t2 and amplitude reflectances r1 and r2 separated by a distance d filled with a medium of refractive index n. Show that for the overall system the transfer matrix is:

$$M = \begin{bmatrix} 1/t_1^* & r_1/t_1 \\ r_1^*/t_1^* & 1/t_1 \end{bmatrix} \begin{bmatrix} \exp(-j\varphi) & 0 \\ 0 & \exp(j\varphi) \end{bmatrix} \begin{bmatrix} 1/t_2^* & r_2/t_2 \\ r_2^*/t_2^* & 1/t_2 \end{bmatrix}$$

Using this result show that the intensity transmittance is:

$$\mathcal{T} = |t|^2 = \frac{|t_1 t_2|^2}{|1 - r_1 r_2 \exp(-j2\varphi)|^2}.$$

where:

$$\varphi = n k_o d.$$

For two identical mirrors with reflectance R= | r1 | 2 = | r2 | 2 show that we have:

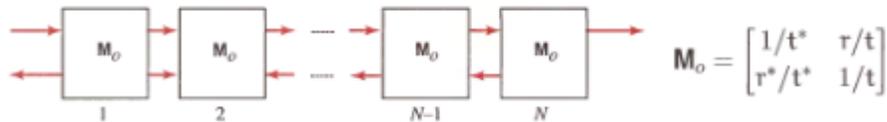$$T_e = \frac{(1-R)^2}{1 + R^2 - 2R\cos(\delta)} = \frac{1}{1 + F\sin^2\left(\frac{\delta}{2}\right)}$$

where:

$$\delta = \left(\frac{2\pi}{\lambda}\right) 2nl \cos\theta.$$

8. Bragg grating

For a system composed from N identical modules with transfer matrix Mo

$$M_o = \begin{bmatrix} 1/t^* & r/t \\ r^*/t^* & 1/t \end{bmatrix}$$

show that the overall transfer matrix MN =MoN is:

$$M_o^N = \Psi_N M_o - \Psi_{N-1} I,$$

$$\Psi_N = \frac{\sin N\Phi}{\sin \Phi} \qquad \cos\Phi = \text{Re}\{1/t\}$$

Hint: Use the unimodularity of matrix Mo (det Mo=1).
Using the fact that:

$$\mathbf{M}_o^N = \begin{bmatrix} 1/t_N^* & r_N/t_N \\ r_N^*/t_N^* & 1/t_N \end{bmatrix}$$

$$\frac{1}{t_N} = \Psi_N \frac{1}{t} - \Psi_{N-1}$$
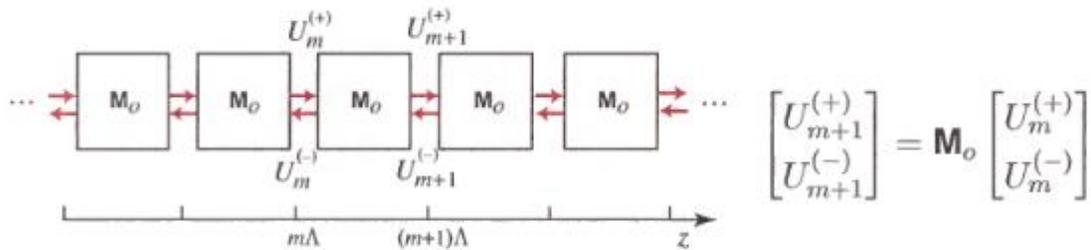
$$\frac{r_N}{t_N} = \Psi_N \frac{r}{t}.$$

$$\mathcal{T}_N = \frac{\mathcal{T}}{\mathcal{T} + \Psi_N^2(1-\mathcal{T})} \cdot \quad \mathcal{R}_N = 1 - \mathcal{T}_N = \frac{\Psi_N^2 \mathcal{R}}{1 - \mathcal{R} + \Psi_N^2 \mathcal{R}}.$$

show that intensity transmittance and reflectance is:
In the limit for the reflectance of a single module show that:

$$\mathcal{R}_N \approx \Psi_N^2 \mathcal{R} = \frac{\sin^2 N\Phi}{\sin^2 \Phi} \mathcal{R}. \quad \mathcal{R} \ll 1 \quad \Psi_N^2 \mathcal{R} \ll 1$$

9. Periodic medium
For a periodic medium composed of identical modules described by transfer matrix Mo

$$\begin{bmatrix} U_{m+1}^{(+)} \\ U_{m+1}^{(-)} \end{bmatrix} = \mathbf{M}_o \begin{bmatrix} U_m^{(+)} \\ U_m^{(-)} \end{bmatrix}$$

we have the eigenmodes that satisfy relation:

$$\begin{bmatrix} U_{m+1}^{(+)} \\ U_{m+1}^{(-)} \end{bmatrix} = e^{-j\Phi} \begin{bmatrix} U_m^{(+)} \\ U_m^{(-)} \end{bmatrix}, \quad m = 1, 2, \ldots; \quad \Phi = K\Lambda.$$

For m=0 we have the eigenvalue problem:

$$\mathbf{M}_o \begin{bmatrix} U_0^{(+)} \\ U_0^{(-)} \end{bmatrix} = e^{-j\Phi} \begin{bmatrix} U_0^{(+)} \\ U_0^{(-)} \end{bmatrix}.$$

Show that we have (use the form of Mo from previous problem):

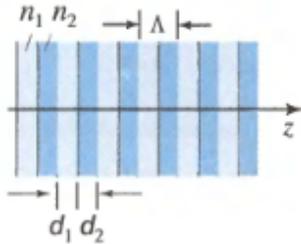$$\cos \Phi = \operatorname{Re} \left\{ \frac{1}{t} \right\}$$

Hint: Use relations:

$$|t|^2 + |r|^2 \doteq 1.$$

$$e^{-j\Phi} = \tfrac{1}{2}(1/t + 1/t^*) \pm j\{1 - [\tfrac{1}{2}(1/t + 1/t^*)]^2\}^{1/2}$$

10. 1D photonic crystal: Alternating dielectric layers

Show that for the system in the figure we have the following dispersion relation:

$$\cos\left(2\pi\frac{K}{g}\right) = \frac{1}{t_{12}t_{21}}\left[\cos\left(\pi\frac{\omega}{\omega_B}\right) - |r_{12}|^2\cos\left(\pi\zeta\frac{\omega}{\omega_B}\right)\right]$$

where:

$$g = 2\pi/\Lambda, \text{ and } \omega_B = c\pi/\Lambda$$

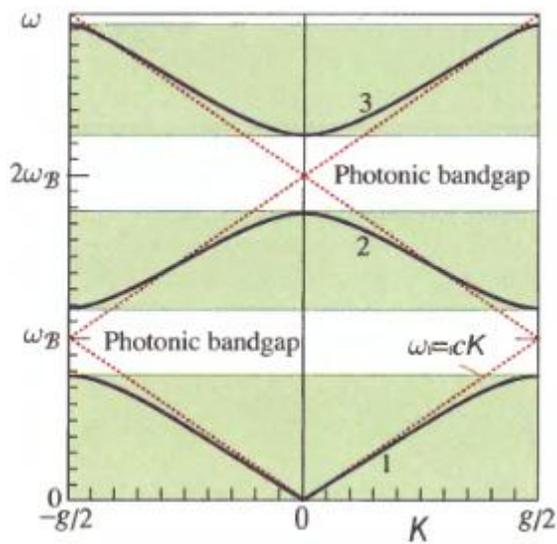$$\omega_B = (c_o/\bar{n})(\pi/\Lambda)$$
$$\Lambda = d_1 + d_2$$
$$\bar{n} = (n_1 d_1 + n_2 d_2)/\Lambda$$
$$\zeta = (n_1 d_1 - n_2 d_2)/(n_1 d_1 + n_2 d_2)$$
$$t_{12}t_{21} = 4n_1 n_2/(n_1 + n_2)^2$$
$$|r_{12}|^2 = (n_2 - n_1)^2/(n_1 + n_2)^2.$$

Plot dispersion relation in coordinates K,ω and show that we have photonic bandgaps around frequency values m*ωB :

Hint: Use the relation obtained in the previous problem:

$$\cos \Phi = \mathrm{Re} \left\{ \frac{1}{t} \right\}$$

and replace transmission t obtained from problem 4 (Propagation Followed by Transmission Through a Slab) by phases:

$$\varphi_1 + \varphi_2 = k_o(n_1 d_1 + n_2 d_2) = \pi\omega/\omega_B$$
$$\varphi_1 - \varphi_2 = \zeta\pi\omega/\omega_B$$

MODUL 2   PLANE-WAVE METHOD

Evaluation tests

1.1D photonic crystal

Using program photo1d3.m obtain the following band structures for 1D photonic crystal (from Joannopoulos book p.46). Explain in what conditions appear the band gap. What represent the blue curves? Define the Brillouin zone in this case.
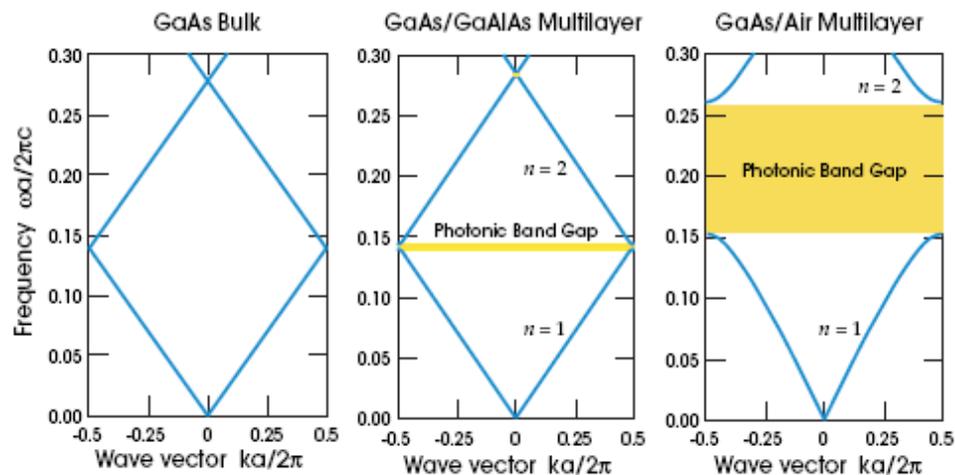

Figure 2: The photonic band structures for on-axis propagation, as computed for three different multilayer films. In all three cases, each layer has a width 0.5a. Left: every layer has the same dielectric constant ε = 13. Center: layers alternate between ε of 13 and 12. Right: layers alternate between ε of 13 and 1.

2. Off-plane propagation

Write PWM equations for TM and TE modes and compare with the code in offplane1dte.m and offplane1dtm.m. Using the previous programs plot the band structure for TM and TE modes. For which frequencies we have band gaps? Define the Brillouin zone in this case.

3. 1D Band structure

What is plotted with blue and green curves in programs gapkyte1d.m and gapkytm1d.m?
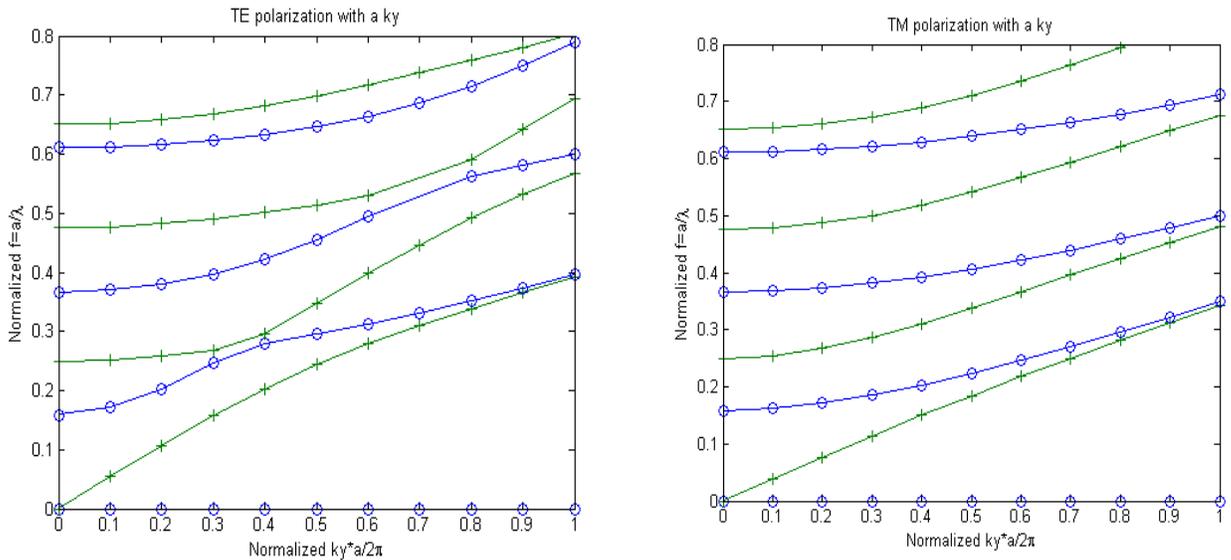
Explain the obtained plots.

4. 2D Brillouin zone.

Derive lattice vectors for direct (a1,a2) and reciprocal lattice (b1,b2) in two cases: square and triangular lattice. Compare the results with eqs.(23-26).

Compare the results with the code for lattice vectors in the programs sqideal2d.m and triang2dte.m.

Take in account that in code the complex plane is used, instead of usual bidimensional real space.

What represents the variable G in the previous code?

Using fig.4 plot the Brillouin zone for square and triangular lattice. Compare the frontier of the Brillouin zone with the code for wave vectors k1,k2,k3 in the programs sqideal2d.m and triang2dte.m..

5. 2D Fourier transform of dielectric constant

Derive Fourier coefficients $K_{m,p}^{\bar{\varepsilon}_r}$ of the inverse of dielectric constant (eq. 34) from its definition (eq. 31). Write down $K_{m,p}^{\bar{\varepsilon}_r}$ for the case of square and triangular lattice (with lattice step a) of cylinders with radius R.

Another derivation of $K_{m,p}^{\bar{\varepsilon}_r}$ it is first to calculate Fourier coefficients of dielectric constant (which in 2D form a matrix) and after to invert the obtained matrix and get the Fourier coefficients of the inverse of dielectric constant. This is the way used in the programs sqideal2d.m and triang2dte.m.

Show that the Fourier transform of dielectric constant is:

$$F(k) = \frac{1}{A} \oiint_A f(r)e^{-i\vec{k}\cdot\vec{r}}\, ds = \varepsilon_b \delta(k) + (\varepsilon_a - \varepsilon_b)\frac{2\pi R^2}{A}\frac{J_1(kR)}{kR}$$

$$f(r) = \begin{cases} \varepsilon_a \cdots r < R \\ \varepsilon_b \cdots r > R \end{cases} = \varepsilon_b + \begin{cases} \varepsilon_a - \varepsilon_b, r < R \\ 0, r > R \end{cases}$$

For the full lattice G=m$_1$b$_1$+m$_2$b$_2$, replaces k (valid only for the unit cell case) , where b1,b2 are the reciprocal lattice vectors.
Compare the result with the code for matrix eps2(x,y) and eps21 in triang2dte.m and respectively sqideal2d.m. What represents the coefficient f in the code?
Hint: look in the PWM manual of Shangping Guo at derivation of equation (42).

6. 2D PWM equation
From relation (7) (in components in eq.(10))

$$\nabla \times \frac{1}{\varepsilon_r} \nabla \times \overline{H} = \frac{\omega^2}{c^2}\overline{H}$$

using the Fourier expansion of inverse of dielectric constant eq.(13) and H field (similar to relation 14 for E field) derive PWM equation for H field (similar to relation 39 for E field):
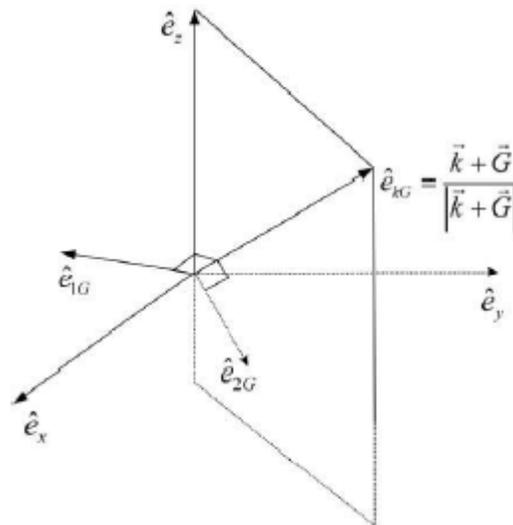
$$\sum_{\vec{G}'} |\vec{k}+\vec{G}||\vec{k}+\vec{G}'|\varepsilon^{-1}(\vec{G}-\vec{G}') \begin{bmatrix} \hat{e}_{2\vec{G}}\cdot\hat{e}_{2\vec{G}'} & -\hat{e}_{2\vec{G}}\cdot\hat{e}_{1\vec{G}'} \\ -\hat{e}_{1\vec{G}}\cdot\hat{e}_{2\vec{G}'} & \hat{e}_{1\vec{G}}\cdot\hat{e}_{1\vec{G}'} \end{bmatrix}\begin{bmatrix} h_{1\vec{G}'} \\ h_{2\vec{G}'} \end{bmatrix} = \left(\frac{\omega}{c}\right)^2 \begin{bmatrix} h_{1\vec{G}} \\ h_{2\vec{G}} \end{bmatrix}$$

Show that the matricial equation reduces to two independent relations for TM and TE waves:

TM: $$\sum_{\vec{G}'} |\vec{k}+\vec{G}||\vec{k}+\vec{G}'|\varepsilon^{-1}(\vec{G}-\vec{G}')h_1(\vec{G}') = \frac{w^2}{c^2}h_1(\vec{G})$$

TE: $$\sum_{\vec{G}'} |\vec{k}+\vec{G}||\vec{k}+\vec{G}'|\varepsilon^{-1}(\vec{G}-\vec{G}')(\hat{e}_1\cdot\hat{e}_1')h_2(\vec{G}') = \frac{w^2}{c^2}h_2(\vec{G})$$

for special orientation of polarization vectors of H as in figure.

Compare with the code in programs sqideal2d.m and triang2dte.m. (take in account that the matrix M for eigenvalue problem is written in complex plane):

$$TE: \quad M=(real(k+G.')*real(k+G)+imag(k+G.')*imag(k+G)).*inv(eps2)$$

$$TM: \quad M=abs(k+G.')*abs(k+G).*inv(eps2)$$

Hint: look in the PWM manual of Shangping Guo at derivation of equation (31).

7. 2D Eigenvalue problem
In the programs sqideal2d.m and triang2dte.m we have the code E=sort(abs(eig(M))) that sorts the absolute value of eigenvalues of matrix M. Write alternative code for solving eigenvalues of matrix M using the Jacobi method.

8. 2D dispersion curves
Obtain the plots in fig.6 and 7 by modifying the programs triang2dte.m and triang2dtm.m. Obtain also dispersion curves for TM modes. Where is located the band gap for TE and TM modes?

9. 2D supercell
Obtain the plot in fig.2.12 by modifying the program sqband.m. What represents the straight line in figure 2.12?

10. 3D diamond Brillouin zone.
Derive lattice vectors for direct (a1,a2,a3) and reciprocal lattice (b1,b2,b3) for diamond lattice. Compare the results with the code for lattice vectors in the program diamond3d.m.
Plot the Brillouin zone for diamond lattice. Compare the frontier of the Brillouin zone with the code for wave vectors k1,k2,k3,k4,k5,k6 in the program diamond3d.m
Plot the dispersion curves. Where is located the band gap?
The simulation is one of the hallmarks in the study of photonic band gap structures and was first made in the article K. M. Ho, C. T. Chan, and C. M. Soukoulis, Phys. Rev. Lett. 65, 3152 (1990). It opened a way for the fabrication of the first photonic structure with a complete photonic band gap (CPBG) and advanced the field considerably. One of its main conclusions is that, regarding a CPBG, the diamond structure fares much better than a simple face-centered-cubic (fcc) one: (i) the threshold value of the di-
electric contrast ε to open a CPBG is 4 (8.2 for an fcc structure), (ii) a CPBG opens between the 2nd and 3rd bands (the 8th-9th bands for an fcc structure), and, consequently, is much more stable against disorder, and (iii) a CPBG is signicantly larger (15% and 5%, for the respective diamond and fcc closed packed lattice of spheres with a dielectric contrast ε = 12.96).

Hint: look in the PWM manual of Shangping Guo at page 21

$$\mathbf{r}_0 = \vec{0}$$
$$\mathbf{r}_1 = (a/4)(\hat{x} + \hat{y} + \hat{z})$$
$$\mathbf{r}_2 = (a/4)(2\hat{x} + 2\hat{y})$$
$$\mathbf{r}_3 = (a/4)(3\hat{x} + 3\hat{y} + \hat{z})$$
$$\mathbf{r}_4 = (a/4)(2\hat{x} + 2\hat{z})$$
$$\mathbf{r}_5 = (a/4)(2\hat{y} + 2\hat{z})$$
$$\mathbf{r}_6 = (a/4)(3\hat{x} + \hat{y} + 3\hat{z})$$
$$\mathbf{r}_7 = (a/4)(\hat{x} + 3\hat{y} + 3\hat{z})$$

0,15 nm
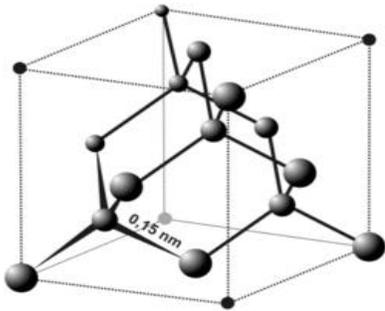
MODUL 3 FINITE-DIFFERENCE TIME DOMAIN METHOD

Evaluation tests

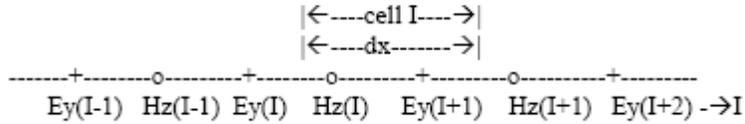**1. Differential Equations (3D to 1D)**
Starting with Maxwell's equations in the time domain (Ampere's and Faraday's laws eq. 1.1), differential form, write the 6 coupled differential equations. (Take the cross products and equate vector components.) Convert these equations to the 1-dimensional TE-to-z case by setting d/dy = d/dz = 0 and Ez = 0. This represents a plane wave propagating in the x-direction. You should end up with equations for Ey and Hz. (The TM-to-z case would have similar equations for Ez and Hy.) Compare the result with eq.1.2.

2. FDTD Equations (1D TE-to-z case)
Convert the 1D TE differential equations above to their FDTD difference form. (Use the central
difference formula to approximate the derivatives, and solve for Ey^{n+1} and $Hz^{n+1/2}$.)
Use the 1D FDTD lattice shown below:

```
                           |←----cell I----→|
                           |←----dx-------→|
          -------+--------o---------+--------o---------+---------o---------+---------
          Ey(I-1)  Hz(I-1) Ey(I)   Hz(I)   Ey(I+1)  Hz(I+1)  Ey(I+2) -→I
```

Let the E fields be defined at times n, n+1, n-1, etc.
Let the H fields be defined at times n-1/2, n+1/2, etc.
Compare the result with eq.1.3.

3. Program the FDTD Equations in air (1D TE case)

Modify the source code fdtd1d.m (written for lossy medium) in order to place a forced 2-GHz sinusoidal source in air (sig=0) on Ey at I=inc: Ey(I=inc) = sin(ωt).

```
       |-----------------------------|-----------------------------|
       0                        Ey(inc)                         nx
```
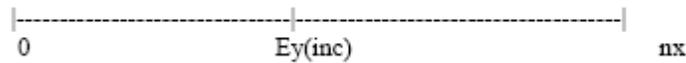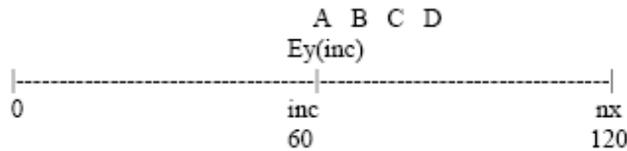
4. Test the FDTD Equations and observe sinusoidal Time Domain Data

Modify the source code fdtd1d.m and use freq = 2GHz, dx = wavelength/20, dt = dx/(2c), nx=120, inc=60. Look at Ey and Hz field at all points as a function of time.
Modify the source code fdtd1d.m and plot the Ey and Hz fields at points A,B,C,D as a function of time for 100 time steps. Give one plot of the four Ey fields, and another of the four Hz fields. Store the Ey fields at point C for use in problem 7.
Plot the Ey field at point D against the analytical value: Ey(x) = sin(ωt-βx), where x is the
distance from the source.

```
                              A  B  C  D
                              Ey(inc)
       |----------------------------|----------------------------|
       0                          inc                          nx
                                   60                          120
```

A is located at I=60, at source
B is located at I=63, 3 cells from source
C is located at I=67, 7 cells from source
D is located at I=90, 30 cells from source

5. Observe Pulsed Time Domain Data

Change the source in fdtd1d.m to a raised cosine pulse:
Ey(inc) = 1-cos(ωt)   , 0<t<1/Fmax
              0   ,   t >1/Fmax
Use Fmax = 2GHz, dx = wavelength/20, dt=dx/(2c), nx=120, inc=60.
Look at the Ey and Hz fields as a function of time along the mesh.
Plot the Ey fields at points A,B,C,D as a function of time for 100 time steps.
Eliminate the boundary conditions. If you run more than 120 time steps you will see the waves reflect off the ends of the FDTD mesh.

5. Numerical Stability

Test your sinusoidal wave simulations with several values of dt = S* dx / c to verify the stability criterion. For 1D you expect your simulations to become unstable when dt > dx / c (Courant factor S>1).

6. Observe Numerical Dispersion
In fdtd1d.m use the raised cosine pulsed source, Fmax = 2GHz, dt=dx/(2c), nx=220, inc=110. Run for 200 time steps.
Plot the Ey fields as a function of time 30 cells from the source for dx = wavelength/60, wavelength/20, wavelenth/10, and wavelength/5.
Plot the Ey fields 30 cells from the source for the sinusoidal source using dx=wavelength/5 and compare result with the values observed at point D in problem 4.

7. 3D FDTD
Modify source code fdtd3D_pec.m and change boundary conditions from perfect electric conductor (PEC) to perfect magnetic conductor (PMC).

8. Differential Equations (3D to 2D)
Starting with Maxwell's equations in the time domain (Ampere's and Faraday's laws eq. 1.1), differential form, write the 6 coupled differential equations. (Take the cross products and equate vector components.) Convert these equations to the 2-dimensional case by setting d/dz = 0. Group Hx, Hy and Ez equations and obtain transverse-magnetic (TM z-polarized) mode equations. Group Ex, Ey and Hz equations and obtain transverse-electric (TE z-polarized) mode equations.

9. FDTD equations (2D TE z-polarized)
Convert the 2D TE differential equations above to their FDTD difference form. Compare with the update equations for E and H fields in the source code fdtd2D.m.

10. 2D Scatterer
Change in the source code fdtd2D.m the metal cylinder with square (size 6x6 cm) dielectric with dielectric constant 12.

11. 2D PML boundary conditions
Compare eqs. (3.11) with 2D PML conditions for X direction in the source code fdtd2D.m. Find in fdtd2D.m the code for eq. (3.12).